Matching of transverse momentum dependent distributions at twist-3 to NLO

Alexey Vladimirov
Institut für Theoretische Physik,
Universität Regensburg,
*D-93040 Regensburg, Germany**

This text is notes on my calculation of NLO matching of Sivers function. Use it for educational purpose, and if (occasionally) you find it instructive and helpful, please, cite our work [I.Scimemi,A.Tarasov,A.Vladimirov, 1901.04519]. Take care that this is technical notes, and thus, I did not care about explanations, and language issues :). Also there are possible misprints, but they should be easily detectable from the surrounding.

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*Electronic address: alexey.vladimirov@physik.uni-regensburg.de

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I. DEFINITION OF TMD DISTRIBUTIONS

We outline our work for the transverse momentum dependent parton distribution functions (TMDPDFs) which, in the quark case, are defined by the matrix element [1–3]

$$\Phi_{q\leftarrow h,ij}(x,\boldsymbol{b}) \;=\; \int \frac{dz}{2\pi}e^{-ixz(pn)}\langle P,S|\bar{T}\{\bar{q}_j\,(zn+\boldsymbol{b})\,[\lambda n+\boldsymbol{b},\pm\infty n]\}T\{[\pm\infty n,0]q_i(0)\}|P,S\rangle, \tag{1.1}$$

where $\pm\infty n$ indicates different light-cone infinities. The TMD distributions which appear in SIDIS have Wilson lines pointing to $+\infty n$, while in Drell-Yan they point to $-\infty n$. The Wilson lines within the TMD operator are along a light-like direction n. Another light-like vector is associated with the large-component of the hadron momentum P,

$$p^\mu = (np)\bar{n}^\mu = P^\mu - \frac{n^\mu}{2}\frac{M^2}{(nP)}, \tag{1.2}$$

where $(nP) = (np)$, and M is the mass of hadron ($P^2 = M^2$). Together vectors n and $\bar{n}$ define the scattering plane. The relative normalization of vectors is

$$n^2 = \bar{n}^2 = 0, \qquad (n\bar{n}) = 1. \tag{1.3}$$

Thus, any four-vector can be decomposed into the components

$$v^\mu = v^+\bar{n}^\mu + v^- n^\mu + v_T^\mu, \tag{1.4}$$

where $v^+ = (nv)$, $v^- = (\bar{n}v)$, and v_T is the transverse component orthogonal to the scattering plane $(v_T n) = (v_T\bar{n}) = 0$. To specify the reference frame we state that $v^\pm = (v^0 \pm v^3)/\sqrt{2}$.

The transverse components play a special role in our consideration. The transverse subspace is projected out by the transverse part of the metric tensor

$$g_T^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu p^\nu + p^\mu n^\nu}{(np)}. \tag{1.5}$$

There are only two non-zero components, $g_T^{11} = g_T^{22} = -1$. In the following, we also need the transverse part of the Levi-Civita tensor

$$\epsilon_T^{\mu\nu} = \frac{n_\alpha p_\beta}{(np)}\epsilon^{\alpha\beta\mu\nu}, \tag{1.6}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is defined in the Bjorken convention ($\epsilon_{0123} = -\epsilon^{0123} = 1$). Consequently, we have $\epsilon_T^{12} = -\epsilon_T^{21} = 1$, which coincides with the definition [1, 4, 5], despite the opposite normalization of the four-dimension ϵ-tensor. The tensor $\epsilon_T^{\mu\nu}$ does not change sign when both indices are down, $\epsilon_{T\mu\nu} = \epsilon_T^{\mu\nu}$, and $\epsilon_T^{\mu\nu}\epsilon_{T,\mu\rho} = \delta_{T,}{}^{\nu}{}_{\rho}$. Since the transverse subspace is Euclidian, the scalar product transverse vectors is negative, $v_T^2 < 0$. In the following, we use the bold font notation to designate the Euclidian scalar product of transverse vectors, i.e. $\boldsymbol{b}^2 = -b^2 > 0$, when it is convenient.

The spin of the hadron is parameterized by the spin-vector S,

$$S^2 = -1, \qquad (PS) = 0. \tag{1.7}$$

The light-cone decomposition of the spin vector is

$$S^\mu = \frac{\lambda}{M}p^\mu - \frac{\lambda}{2}\frac{M}{(np)}n^\mu + s_T^\mu, \tag{1.8}$$

where the helicity λ of the hadron is

$$\frac{(nS)}{(np)} = \frac{\lambda}{M}. \tag{1.9}$$

The vector s_T^μ is the transverse component of the spin, $s_T^2 = \lambda^2 - 1$. With the help of ϵ_T-tensor we can introduce another useful vector

$$\tilde{s}_T^\mu = \epsilon_T^{\mu\nu}S_\nu, \tag{1.10}$$

and it implies $\tilde{s}_T^2 = s_T^2$.

The open spinor indices (ij) of the TMD operator in eq. (1.1) are to be contracted with different gamma-matrices, which we denote generically as Γ. The gamma-matrices that appear at the leading order of TMD factorization are

$$\Gamma = \{\gamma^+, \gamma^+\gamma_5, i\sigma_T^{\alpha+}\gamma_5\}, \tag{1.11}$$

where $\sigma_T^{\alpha+} = g_T^{\alpha\beta}\sigma_{\beta\gamma}n^\gamma$, and

$$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu), \qquad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\epsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta. \tag{1.12}$$

In the naive parton model interpretation, these gamma-structures are related to the observation of unpolarized (γ^+), longitudinally polarized $(\gamma^+\gamma^5)$ and transversely polarized $(i\sigma_T^{\alpha+}\gamma^5)$ quarks within the hadron. Beyond the leading order factorization one expects that the power suppressed terms of TMD show also different gamma structures. However, currently, the TMD factorization theorem is not established beyond the leading order. Moreover, it is known that TMD distributions with different from (1.11) gamma-structure contain rapidity divergences that are not renormalized by the standard TMD soft factor [6].

Historically, the TMD distributions have been introduced and parameterized in momentum space. Denoting

$$\Phi_{q\leftarrow h}^{[\Gamma]} = \frac{1}{2}\mathrm{Tr}\left(\Phi_{q\leftarrow h}\Gamma\right), \tag{1.13}$$

we have [5, 7]

$$\Phi_{q\leftarrow h}^{[\gamma^+]}(x,p_T) = f_1(x,p_T) - \frac{\epsilon_T^{\mu\nu}p_{T\mu}s_{T\nu}}{M}f_{1T}^\perp(x,p_T), \tag{1.14}$$

$$\Phi_{q\leftarrow h}^{[\gamma^+\gamma_5]}(x,p_T) = \lambda\, g_{1L}(x,p_T) - \frac{p_{T\mu}s_T^\mu}{M}g_{1T}(x,p_T), \tag{1.15}$$

$$\begin{aligned}\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma_5]}(x,p_T) = {}& s_T^\alpha h_1(x,p_T) + \lambda\frac{p_T^\alpha}{M}h_{1L}^\perp(x,p_T)\\ &-\frac{\epsilon_T^{\alpha\mu}p_{T\mu}}{M}h_1^\perp(x,p_T) + \frac{p_T^2}{M^2}\left(\frac{g_T^{\alpha\mu}}{2} - \frac{p_T^\alpha p_T^\mu}{p_T^2}\right)s_{T\mu}h_{1T}^\perp(x,p_T),\end{aligned} \tag{1.16}$$

where $p_T^2 = -\boldsymbol{p}_T^2 < 0$. Note, that the functions $f(x,p_T)$ depend only on the modulus of p_T, but not on the direction. The functions presented here are traditionally called unpolarized (f_1), Sivers $(f_{1T}^\perp)$, helicity (g_{1L}), worm-gear T (g_{1T}), transversity (h_1), worm-gear L $(h_{1L}^\perp)$, Boer-Mulders $(h_1^\perp)$ and pretzelosity $(h_{1T}^\perp)$ distributions.

For practical calculations it is convenient to write TMD distributions in momentum space as Fourier transform of distributions in position space in the usual manner

$$\Phi_{q\leftarrow h,ij}(x,p_T) = \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}e^{+i(\boldsymbol{b}\boldsymbol{p}_T)}\Phi_{q\leftarrow h,ij}(x,\boldsymbol{b}), \tag{1.17}$$

where the scalar product $(\boldsymbol{b}\boldsymbol{p}_T)$ is Euclidian. The decomposition in eq. (1.14-1.16) is then replaced by its analog it position space,

$$\Phi_{q\leftarrow h}^{[\gamma^+]}(x,\boldsymbol{b}) = f_1(x,\boldsymbol{b}) + i\epsilon_T^{\mu\nu}b_\mu s_{T\nu}Mf_{1T}^\perp(x,\boldsymbol{b}), \tag{1.18}$$

$$\Phi_{q\leftarrow h}^{[\gamma^+\gamma_5]}(x,\boldsymbol{b}) = \lambda g_{1L}(x,\boldsymbol{b}) + ib_\mu s_T^\mu M g_{1T}(x,\boldsymbol{b}), \tag{1.19}$$

$$\begin{aligned}\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma_5]}(x,\boldsymbol{b}) = {}& s_T^\alpha h_1(x,\boldsymbol{b}) - i\lambda b^\alpha M h_{1L}^\perp(x,\boldsymbol{b})\\ &+i\epsilon_T^{\alpha\mu}b_\mu M h_1^\perp(x,\boldsymbol{b}) + \frac{M^2\boldsymbol{b}^2}{2}\left(\frac{g_T^{\alpha\mu}}{2} + \frac{b^\alpha b^\mu}{\boldsymbol{b}^2}\right)s_{T\mu}h_{1T}^\perp(x,\boldsymbol{b}).\end{aligned} \tag{1.20}$$

This parameterization coincides[1] with the parameterization given in [8]. The explicit transformation rules for all these functions can be found in appendix I 1.

[1] Comparing parameterization one should take into account that the TMD operator in [8] is taken with the vector b oriented in the opposite direction.

1. Relation between TMD distributions in momentum and coordinate spaces

The momentum and coordinate representations are related by Fourier transformation (1.17),

$$\Phi^{[\Gamma]}_{q\leftarrow h}(x,p_T)=\int \frac{d^2\boldsymbol{b}}{(2\pi)^2}e^{+i(\boldsymbol{b}\boldsymbol{p}_T)}\Phi^{[\Gamma]}_{q\leftarrow h}(x,\boldsymbol{b}). \tag{1.21}$$

Performing the Fourier transformation of the parameterizations in eq. (1.18, 1.19,1.20) and comparing it to the parameterizations in eq. (1.14, 1.15, 1.16) we find the relation between momentum and position space representations. They are conventionally presented using

$$\widehat{F}^{(n)}(x,p_T) \;=\; \frac{M^{2n}}{n!}\int_0^\infty \frac{|\boldsymbol{b}|d|\boldsymbol{b}|}{2\pi}\left(\frac{|\boldsymbol{b}|}{|\boldsymbol{p}_T|}\right)^n J_n(|\boldsymbol{b}||\boldsymbol{p}_T|)F(x,\boldsymbol{b}). \tag{1.22}$$

The inverse transformation is

$$\widehat{F}^{(n)}(x,\boldsymbol{b})=2\pi\frac{n!}{M^{2n}}\int_0^\infty |\boldsymbol{p}_T|d|\boldsymbol{p}_T|\left(\frac{|\boldsymbol{p}_T|}{|\boldsymbol{b}|}\right)^n J_n(|\boldsymbol{b}||\boldsymbol{p}_T|)F(x,p_T). \tag{1.23}$$

Correspondingly, all TMDPDFs are split into three classes which transforms in the same manner,

$$f_1=\widehat{f}_1^{(0)},\qquad g_{1L}=\widehat{g}_{1L}^{(0)},\qquad h_1=\widehat{h}_1^{(0)}, \tag{1.24}$$

$$f_{1T}^{\perp}=\widehat{f}_{1T}^{\perp(1)},\qquad g_{1T}=\widehat{g}_{1T}^{(1)},\qquad h_{1L}^{\perp}=\widehat{h}_{1L}^{\perp(1)},\qquad h_1^{\perp}=\widehat{h}_1^{\perp(1)}, \tag{1.25}$$

$$h_{1T}^{\perp}=\widehat{h}_{1T}^{\perp(2)}. \tag{1.26}$$

II. LIGHT-CONE EXPANSION FOR TMD OPERATOR

The small-b matching of TMD distribution to the integrated distributions is obtained by the operator product expansion (OPE) at small-b. The OPE is independent from the hadronic states and for this reason universal. Let us introduce a separate notation for the TMD operators. The operator that produces TMD distributions in the Drell-Yan case is

$$\mathcal{U}^{\Gamma}_{\rm DY}(z,\boldsymbol{b}) \;=\; \bar q(zn+\boldsymbol{b})[zn+\boldsymbol{b},-\infty n+\boldsymbol{b}]\Gamma[-\infty n-\boldsymbol{b},-zn-\boldsymbol{b}]q(-zn-\boldsymbol{b}), \tag{2.1}$$

where Γ represents the gamma-matrices of the leading set (1.11), and the half-infinite Wilson lines are defined as

$$[a_1n+\boldsymbol{b},a_2n+\boldsymbol{b}] \;=\; P\exp\left(ig\int_{a_2}^{a_1}d\sigma n^\mu A_\mu(\sigma n+\boldsymbol{b})\right). \tag{2.2}$$

Here and in the following we also omit the T-ordering of the fields, since it is irrelevant in the tree order approximation. The operator that produces the TMD distributions for SIDIS reads

$$\mathcal{U}^{\Gamma}_{\rm DIS}(x,\boldsymbol{b}) \;=\; \bar q(zn+\boldsymbol{b})[zn+\boldsymbol{b},+\infty n+\boldsymbol{b}]\Gamma[+\infty n-\boldsymbol{b},-zn-\boldsymbol{b}]q(-zn-\boldsymbol{b}). \tag{2.3}$$

Generally, the links which connect the end points of Wilson lines at a distant transverse plane must be added in both operators (for DY and for SIDIS). Here, we omit them for simplicity, assuming that some non-singular gauge (e.g. covariant gauge) is in use. In non-singular gauges the field nullifies at infinities, $A_\mu(\pm\infty n)=0$, and the contribution of distant gauge links vanish.

The relation between the TMD distribution (1.1) and the TMD operator (2.1) is

$$\Phi^{[\Gamma]}_{q\leftarrow h}(x,\boldsymbol{b})=\int\frac{dz}{2\pi}e^{-2ixzp^+}\langle P,S|\mathcal{U}^{\Gamma}\left(z,\frac{\boldsymbol{b}}{2}\right)|P,S\rangle. \tag{2.4}$$

The light-cone expansion of the TMD operators corresponds to the expansion in the variable $\boldsymbol{b}$. The OPE has a generic schematic form

$$\mathcal{U}(z,\boldsymbol{b})=\sum_i\left[C_i*\mathcal{O}_i^{\rm tw2}\right](z)+\boldsymbol{b}^\mu\sum_i\left[\tilde C_i*\mathcal{O}_{\mu,i}^{\rm tw3}\right](z)+O(\boldsymbol{b}^2), \tag{2.5}$$

where C's are Wilson coefficient functions which depend on $\ln \boldsymbol{b}^2$, $\mathcal{O}$'s are light-cone operators, and the symbol $*$ denotes some integral convolution between coefficient function and operators. Here, the superscripts tw2 and tw3 indicate the *collinear twist*, which in principle differs from the *geometrical twist*. We remind that the term *collinear twist* indicates the distributions which enter the same order of momentum expansion. It is not a well-defined quantum number, in contrast to the *geometrical twist*. The later is defined by "dimension-spin" value, and is a well-defined quantum number, in the sense that e.g. it conserves and does not mix under scaling transformations. As we will see the operators $\mathcal{O}^{\text{tw3}}$ are compositions of geometrical twist-2 and twist-3 operators. The coefficient functions are perturbatively calculable. In this work, we study the matching only at order α_s^0.

At leading order in α_s the quantum fields can be considered as classical fields, that satisfy QCD equations of motion. In this approximation, the small-b OPE is just the Taylor expansion at $b = 0$. Expanding up $\mathcal{U}$ to the linear order in $\boldsymbol{b}$ we obtain

$$\mathcal{U}^\Gamma(z,\boldsymbol{b}) = \mathcal{U}^\Gamma(z,\boldsymbol{0}) + b^\mu \frac{\partial}{\partial b^\mu}\mathcal{U}^\Gamma(z,\boldsymbol{b})\Big|_{\boldsymbol{b}=0} + O(\boldsymbol{b}^2). \tag{2.6}$$

The leading term reads

$$\mathcal{U}^\Gamma_{\text{DY}}(z,\boldsymbol{0}) = \bar q(zn)[zn,-\infty n]\Gamma[-\infty n,-zn]q(-zn) = \bar q(zn)[zn,-zn]\Gamma q(-zn), \tag{2.7}$$

where the half-infinite segments of Wilson line compensate each other due to the unitarity of a Wilson line. The same holds for the SIDIS operator

$$\mathcal{U}^\Gamma_{\text{DIS}}(z,\boldsymbol{0}) = \bar q(zn)[zn,+\infty n]\Gamma[+\infty n,-zn]q(-zn) = \bar q(zn)[zn,-zn]\Gamma q(-zn). \tag{2.8}$$

Therefore, we obtain that $\mathcal{U}^\Gamma_{\text{DY}}(z,\boldsymbol{0}) = \mathcal{U}^\Gamma_{\text{DIS}}(z,\boldsymbol{0})$, which is well known.

The term linear in $\boldsymbol{b}^\mu$ is given by the derivative of the operator. Explicitly, it reads

$$\frac{\partial}{\partial b^\mu}\mathcal{U}^{\text{DY}}_\Gamma(z,\boldsymbol{b})\Big|_{\boldsymbol{b}=0} = \bar q(zn)[zn,-\infty n](\overleftarrow{\partial_{T\mu}} - \overrightarrow{\partial_{T\mu}})\Gamma[-\infty n,-z\lambda]q(-z), \tag{2.9}$$

where $\partial_{T\mu}$ is the derivative with respect to the transverse components only. This expression can be written as

$$\begin{aligned}\frac{\partial}{\partial b^\mu}\mathcal{U}^\Gamma_{\text{DY}}(z,\boldsymbol{b})\Big|_{\boldsymbol{b}=0} =\; & \bar q(zn)\overleftarrow{D_\mu}[zn,-\infty n]\Gamma[-\infty n,-zn]q(-zn) \\ & +ig\int_{-\infty}^{z}\bar q(zn)[zn,\tau n]F_{\mu+}(\tau n)[\tau n,-\infty n]\Gamma[-\infty n,-zn]q(-zn) \\ & -ig\int_{-z}^{-\infty}\bar q(zn)[zn,-\infty n]\Gamma[-\infty n,\tau n]F_{\mu+}(\tau n)[\tau n,-zn]q(-zn) \\ & -\bar q(zn)[zn,-\infty n]\Gamma[-\infty n,-zn]\overrightarrow{D_\mu}q(-zn),\end{aligned} \tag{2.10}$$

where the covariant derivative and the field-strength tensor are defined as usual

$$\overrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - igA_\mu, \qquad \overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + igA_\mu, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]. \tag{2.11}$$

To obtain the expression (2.10) we have used the assumption that[2] $A(-\infty n) = 0$, and the explicit expression for the total derivative of a Wilson line,

$$\begin{aligned}\partial_\mu\{[z_1n,z_2n]\} &= \frac{d}{dy^\mu}[z_1n+y,z_2n+y]\Big|_{y=0} \\ &= ig\left(A_\mu(z_1n)[z_1n,z_2n] - [z_1n,z_2n]A_\mu(z_2n) + \int_{z_2}^{z_1} d\tau[z_1n,\tau n]F_{\mu+}(\tau n)[\tau n,z_2n]\right),\end{aligned} \tag{2.12}$$

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[2] In singular gauges, one generally cannot expect the boundary condition $A(\pm\infty n) = 0$, but $A(\pm\infty n,\boldsymbol{\infty}) = 0$. In this case the TMD operator receives the transverse link to corresponding infinity, which preserves the gauge invariance (for a discussion on the role of singular gauge see f.i. [9–12]). Therefore, transverse derivative operator $(\overleftarrow{\partial_T} - \overrightarrow{\partial_T})$ is inserted at far-end of Wilson lines at $\pm n\infty + \boldsymbol{\infty}$ (compare to (2.10)), and as a result it also differentiates transverse links. Then the expansion formula (2.13) obtains an extra term

$$\begin{aligned}\frac{\partial}{\partial b^\mu}\mathcal{U}^\Gamma_{\text{DY}}(z,\boldsymbol{b})\Big|_{\boldsymbol{b}=0} =\; & (2.13) - 2ig\lim_{b\to 0}\Big\{\int_0^b d\boldsymbol{\tau}^\nu\; \bar q(zn)[zn,-\infty n][-\infty n,-\infty n+\boldsymbol{\tau}] \\ & \Gamma F_{\nu\mu}(-\infty n+\boldsymbol{\tau})[\boldsymbol{\tau}-\infty n,-\infty n][-\infty n,-zn]q(-zn)\Big\},\end{aligned}$$

where $\boldsymbol{\tau} = \tau\boldsymbol{b}/|b|$. The limit $b\to 0$ is smooth and thus produces zero. In this way, the result in a singular gauge coincides with the result in a regular gauge. The similar consideration holds for SIDIS operators with replacement $-\infty n \to +\infty n$.

where the vector n can be arbitrary.

The segments of Wilson line between $-\infty$ and τ cancel and we obtain

$$\frac{\partial}{\partial b^\mu}\mathcal{U}^\Gamma_{\text{DY}}(z,\boldsymbol{b})\Big|_{\boldsymbol{b}=0} = \bar q(zn)\left(\overleftarrow{D_\mu}[zn,-zn]-[zn,-zn]\overrightarrow{D_\mu}\right)\Gamma q(-zn) + ig\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\tau\ \bar q(zn)[zn,\tau n]\Gamma F_{\mu+}(\tau n)[\tau n,-zn]q(-zn). \tag{2.13}$$

In the case of SIDIS kinematics the Wilson lines point the future light-like infinity, and therefore, the same derivation gives

$$\frac{\partial}{\partial b^\mu}\mathcal{U}^\Gamma_{\text{DIS}}(z,\boldsymbol{b})\Big|_{\boldsymbol{b}=0} = \bar q(zn)\left(\overleftarrow{D_\mu}[zn,-zn]-[zn,-zn]\overrightarrow{D_\mu}\right)\Gamma q(-zn) - ig\left(\int_{z}^{\infty}+\int_{-z}^{\infty}\right)d\tau\ \bar q(zn)[zn,\tau n]\Gamma F_{\mu+}(\tau n)[\tau n,-zn]q(-zn). \tag{2.14}$$

Comparing the results for DY in eq. (2.13) and SIDIS in eq. (2.14) kinematics we observe that the first term is the same, while the last terms differ because of the limits of integration and a common sign. Therefore, already at this stage it is clear that the operator in the first term does not contribute to P-odd distributions (i.e. Sivers and Boer-Mulders functions) which is known to change sign in different kinematics.

As expected, the non-compact (in the sense that it spans an infinite range of position space) TMD operator is expressed via a set of compact light-cone operators. The light-cone operators in eq. (2.13, 2.14) are not very well defined, in the sense, that they are of indefinite *geometrical twist* (more specifically, this issue concerns the first terms of eq. (2.13, 2.14)). At the next stage of the OPE we need to classify the contributions with respect to twist and decompose over independent components. These components are parameterized via parton distributions functions, which are universal and can be measured in different experiment.

As the key point here is the twist-expansion we provide some additional discussion. The standard approach to twist-decomposition of operators is to consider their local expansion. In the local expansion the contributions of different twists can be separated by the permutation algebra, and summed back to a non-local representation, see e.g. the detailed decomposition of similar operators in [13]. However, a much simpler approach consists in taking the operator directly in a non-local form [14, 15]. In this approach, one starts with operators off the light-cone, and makes the twist-decomposition, and then perform the limit to the light-cone.

In principle, the procedure of twist-decomposition can be made at the level of operators, see e.g.[14]. However, practically it is involved, especially for tensor gamma-structure. The evaluation is significantly simpler instead in the terms of distributions, e.g. as it is done in ref. [15]. Here we are going to follow this second approach. In fact, the derivation presented in the following sections closely follows the procedure described in details in [15] for the case of meson distribution amplitudes. The difference in kinematics does not allow us to use the powerful method of conformal basis, but there is no principle difference in other aspects.

Prior to the parameterization and twist-decomposition let us prepare the operator for this procedure, and make its off-light-cone generalization. At our order of accuracy (twist-3) we do not need the most general form of the three-point operators, since they are already of *geometrical twist*-3 and do not contain admixture with twist-2 operators. Therefore, the generalization should be done only for the two-point operators, and it can be simply achieved by the replacement $zn^\mu \to y^\mu$ with $y^2 \neq 0$. The result is conveniently re-written in the following form

$$\bar q(y)\left(\overleftarrow{D_\mu}[y,-y]-[y,-y]\overrightarrow{D_\mu}\right)\Gamma q(-y) = \frac{\partial}{\partial y^\mu}\bar q(y)[y,-y]\Gamma q(-y) - ig\int_{-1}^{1}dv\, vy^\nu \bar q(y)[y,vy]\Gamma F_{\mu\nu}(vy)[vy,-y]q(-y), \tag{2.15}$$

where we have used the formula for the stretch derivative of the Wilson line

$$\frac{\partial}{\partial y^\mu}[y,-y] = ig\left(A_\mu(y)[y,-y]+[y,-y]A_\mu(-y)+\int_{-1}^{1}dvvy^\nu[y,vy]F_{\mu\nu}(vy)[vy,-y]\right). \tag{2.16}$$

Note, that this expression is the same for DY and SIDIS operators. The last term of (2.15) is again pure twist-3 operator, and thus one can set it directly on the light-cone.

Let us conclude this section with an intermediate summary of our main results. For convenience we introduce the generic notation for two- and three-point operators

$$\mathcal{O}_\Gamma(z) = \bar q(zn)[zn,-zn]\Gamma q(-zn), \tag{2.17}$$

$$\mathcal{T}^\mu_\Gamma(z_1,z_2,z_3) = g\bar q(z_1n)[z_1n,z_2n]\Gamma F^{\mu+}(z_2n)[z_2n,z_3n]q(z_3n). \tag{2.18}$$

The expression for the first terms of small-b expansion for TMD operator reads (at leading order in α_s)

$$\mathcal{U}^{\Gamma}_{\rm DY}(z,\boldsymbol{b}) = \mathcal{O}_\Gamma(z)+b_\mu\Big\{\lim_{y\to zn}\frac{\partial}{\partial y_\mu}\mathcal{O}_\Gamma(y)-i\int_{-1}^1 dv\, vz\,\mathcal{T}^\mu_\Gamma(z,vz,-z) +i\left(\int_{-\infty}^z+\int_{-\infty}^{-z}\right)d\tau\mathcal{T}^\mu_\Gamma(z,\tau,-z)\Big\}+O(\boldsymbol{b}^2), \qquad (2.19)$$

$$\mathcal{U}^{\Gamma}_{\rm DIS}(z,\boldsymbol{b}) = \mathcal{O}_\Gamma(z)+b_\mu\Big\{\lim_{y\to zn}\frac{\partial}{\partial y_\mu}\mathcal{O}_\Gamma(y)-i\int_{-1}^1 dv\, vz\,\mathcal{T}^\mu_\Gamma(z,vz,-z) -i\left(\int_{z}^\infty+\int_{-z}^{\infty}\right)d\tau\mathcal{T}^\mu_\Gamma(z,\tau,-z)\Big\}+O(\boldsymbol{b}^2). \qquad (2.20)$$

The limit $y\to zn$ implies $y^2\to 0$ such that the light-like separation between fields is z.

III. COLLINEAR DISTRIBUTIONS

Evaluating the matrix elements of eq. (2.19, 5.74), and hence the matching of TMD distributions we meet with a number of collinear parton distributions. In this section we present the parameterization of two and three point parton distributions that appear in the final result. In fact, the functions that we find represent a complete set of *geometrical twist* 2 and 3 quark distributions. For the two-point functions we use the standard parameterization by [16]. For the three point functions there is not a commonly accepted parameterization, therefore, we introduce a parameterization inspiring in [17].

A. Parameterization of quark-quark correlators

The standard parameterization of light-cone quark-quark correlators is given [16] and reads

$$\langle P,S|O^{\gamma^\mu}(z)|P,S\rangle = 2\int dx e^{2ixzp^+}\Big\{p^\mu f_1(x)+\frac{n^\mu}{(np)}M^2f_4(x)\Big\}, \qquad (3.1)$$

$$\langle P,S|O^{\gamma^\mu\gamma^5}(z)|P,S\rangle = 2\int dx e^{2ixzp^+}\Big\{\lambda p^\mu g_1(x)+s_T^\mu M g_T(x)+\lambda M^2\frac{n^\mu}{(np)}g_3(x)\Big\}, \qquad (3.2)$$

$$\langle P,S|O^{i\sigma^{\mu\nu}\gamma^5}(z)|P,S\rangle = 2\int dx e^{2ixzp^+}\Big\{(s_T^\mu p^\nu-p^\mu s_T^\nu)h_1(x)+\lambda\frac{M}{(np)}(p^\mu n^\nu-n^\mu p^\nu)h_L(x) +(s_T^\mu n^\nu-n^\mu s_T^\nu)\frac{M^2}{(np)}h_3(x)\Big\}, \qquad (3.3)$$

where the operators $\mathcal{O}^\Gamma$ are defined in eq. (2.17). The twist-2 PDFs f_1, g_1 and h_1 are known as unpolarized, helicity and transversity PDFs. The PDFs g_T and h_L are of *collinear* twist-3. The PDFs f_4, g_3 and h_3 are of *collinear* twist-4, and do not appear in the current final results. The *collinear* twist-3 PDFs are not independent as they are combinations of PDFs of twist-2 and three-point PDFs. The derivation of this relation can be done with the help of QCD equations of motion and is presented in the appendix **??**.

The PDF defined by eq. (3.1, 3.2, 3.3) are non-zero for $-1<x<1$ and zero for $|x|>1$ [18]. They can be represented by

$$f_1(x)=\theta(x)q(x)-\theta(-x)\bar q(x), \qquad (3.4)$$

where $q(x)$ and $\bar q(x)$ are the usual quark and anti-quark parton densities in the infinite momentum frame. A similar interpretation holds for g_1 and h_1.

At $z\to 0$ the operators turn to local operators. The matrix elements of local operator can be parameterized in terms of the corresponding charges. This implies the existence of exact relations relations among the first moments of PDFs. In the present case the important relations are

$$\int_{-1}^1 dx g_1(x)=\int_{-1}^1 dx g_T(x), \qquad \int_{-1}^1 dx h_1(x)=\int_{-1}^1 dx h_L(x), \qquad (3.5)$$

and they are another form of the Burkhard-Cottingham sum rule [19].

In order to proceed with the matching, we need also a parameterization of off light-cone collinear functions. In general, the parameterization of matrix elements off light-cone does not coincide with the parameterization of light-cone matrix elements, which is given in eq. (3.1, 3.2, 3.3). However, on and off light-cone parameterizations can be related each other order by order in the expansion over y^2 (where y is the distance between quark fields), see e.g. discussion in [20]. Such relations up to linear terms in y are presented in appendix III A 1. Using the off-light-cone parameterization of eq. (3.9, 3.10, 3.11) we derive the matrix elements of the first terms in the small-b OPE in eq. (2.19, 5.74). We find

$$n_\alpha g_T^{\mu\nu}\lim_{y\to zn}\frac{\partial}{\partial y^\nu}\langle P,S|O^{\gamma^\alpha}(y)|P,S\rangle \;=\; 0, \tag{3.6}$$

$$n_\alpha g_T^{\mu\nu}\lim_{y\to zn}\frac{\partial}{\partial x^\nu}\langle P,S|O^{\gamma^\alpha\gamma^5}(y)|P,S\rangle \;=\; 2s_T^\mu M\int due^{2ixzp^+}\frac{g_1(x)-g_T(x)}{z}, \tag{3.7}$$

$$n_\gamma g_T^{\alpha\beta}g_T^{\mu\nu}\lim_{y\to zn}\frac{\partial}{\partial y^\nu}\langle P,S|O^{i\sigma^{\beta\gamma}\gamma^5}(y)|P,S\rangle \;=\; 2\lambda M g_T^{\mu\alpha}\int dxe^{2ixzp^+}\frac{h_1(x)-h_L(x)}{z}. \tag{3.8}$$

Moreover these expressions depend on the particular off-light-cone parameterization that is used. In any case, the functions g_T and h_L are not independent, and must be expressed in the terms of distributions with definite *geometrical twist*. Such a re-expression is also dependent on the parameterization. In the final result all (intermediate and off-light-cone) parameterization dependence cancels, and the result is uniquely defined using definite twist distributions.

1. *Matrix element off-light cone*

The rules for working with matrix element off light-cone are discussed in details in [15, 20]. For completeness we present here the intermediate steps which lead to equations (3.6, 3.7, 3.8).

The initial step is the parameterization of matrix element of the operator off light-cone, in terms of four-dimensional vectors, y^μ, P^μ and S^μ, as well as, tensors $g^{\mu\nu}$ and $\epsilon^{\mu\nu\rho\sigma}$. Naturally, such a parameterization structurally repeats the parameterization of light-cone matrix element (3.1, 3.2, 3.3), with the replacement $p\to P$, $z\to y$ and $s_T\to S$,

$$\langle P,S|O^{\gamma^\mu}(y)|P,S\rangle \;=\; 2\int dxe^{2ix(yP)}\Big\{P^\mu A_1(x)+\frac{y^\mu}{(yP)}M^2A_3(x)\Big\}, \tag{3.9}$$

$$\langle P,S|O^{\gamma^\mu\gamma^5}(y)|P,S\rangle \;=\; 2\int dxe^{2ix(yP)}\Big\{P^\mu\frac{(yS)}{(yP)}MB_1(x)+S^\mu MB_2(x)+\frac{(yS)}{(yP)}M^3\frac{y^\mu}{(yP)}B_3(x)\Big\}, \tag{3.10}$$

$$\langle P,S|O^{i\sigma^{\mu\nu}\gamma^5}(y)|P,S\rangle \;=\; 2\int dxe^{2ix(yP)}\Big\{(S^\mu P^\nu-P^\mu S^\nu)C_1(x)+\frac{(yS)}{(yP)^2}M^2(P^\mu y^\nu-y^\mu P^\nu)C_2(x) +(S^\mu y^\nu-y^\mu S^\nu)\frac{M^2}{(yP)}C_3(x)\Big\}. \tag{3.11}$$

The parameterization (3.9, 3.11) is given in the space of physical vectors (P^μ, S^μ, y^μ) whereas the parameterization (3.1, 3.2, 3.3) is given in the space of light-cone vectors (p^μ, s_T^μ, n^μ). To connect these parameterizations we should relate the factorization frame to the physical frame. Assuming that $y^\mu\to zn^\mu$ in the limit $y^2\to 0$, we obtain the following decomposition of n^μ

$$zn^\mu = y^\mu-\frac{P^\mu}{M^2}\left((yP)-\sqrt{(yP)^2-y^2M^2}\right). \tag{3.12}$$

Using this relation we decompose the vector y^μ over basis of (p^μ, s_T^μ, n^μ) and the small parameter y^2,

$$y^\mu = z\Big[\frac{n^\mu}{2}\left(1+\sqrt{1+\frac{y^2M^2}{z^2(np)^2}}\right)-\frac{p^\mu}{M^2}(np)\left(1-\sqrt{1+\frac{y^2M^2}{z^2(np)^2}}\right)\Big] \tag{3.13}$$

The momentum and spin vectors are given by the definitions in eq. (1.2) and (1.8),

$$P^\mu \;=\; p^\mu+\frac{n^\mu}{2}\frac{M^2}{(np)}, \tag{3.14}$$

$$S^\mu \;=\; \frac{\lambda}{M}p^\mu-\frac{\lambda}{2}\frac{M}{(np)}n^\mu+s_T^\mu. \tag{3.15}$$

Therefore, the scalar products with the vector y^μ are

$$(yP) = z(np)\sqrt{1+\frac{y^2M^2}{z^2(np)^2}}, \tag{3.16}$$

$$(yS) = \frac{\lambda}{M}z(np). \tag{3.17}$$

Using this translation dictionary we can compare the parameterizations (3.9, 3.11) and (3.1, 3.2, 3.3) order-by-order in the parameter y^2. At the order $O(y^2)$ we obtain

$$A_1 = f_1, \qquad A_3 = f_4 - \frac{f_1}{2}, \tag{3.18}$$

$$B_1 = g_1 - g_T, \qquad B_2 = g_T, \qquad B_3 = g_3 + g_T - \frac{g_1}{2}, \tag{3.19}$$

$$C_1 = h_1, \qquad C_2 = h_L - h_3 - \frac{h_1}{2}, \qquad C_3 = h_3 - \frac{h_1}{2},$$

where we omit arguments of functions (x) on both sides.

Obviously, the generalization off-light-cone is not unique. In particular, one can add terms power-suppressed in y^2 terms, to the definition in eq. (3.12). However, the reparameterization affects all intermediate steps of calculation and the difference should disappear in the final definite geometrical twist composition. On top of this, such modifications are invisible at our level of accuracy.

B. Parameterization of quark-gluon-quark correlators

The parameterization of matrix elements of a three-point operator has the following general structure

$$\langle P,S|\mathcal{T}^\mu_\Gamma(z_1,z_2,z_3)|P,S\rangle = \sum_i t_\Gamma^{i;\mu\ldots}(P,S,n,g_T,\epsilon_T)\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}F(x_1,x_2,x_3), \tag{3.20}$$

where the integration measure is [18]

$$[dx] = dx_1dx_2dx_3\delta(x_1+x_2+x_3), \qquad -1 < x_1,x_2,x_3 < 1. \tag{3.21}$$

In the rest of the paper we use the tilde notation for Fourier images of the functions

$$\tilde{F}(z_1,z_2,z_3) = \int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}F(x_1,x_2,x_3)\ . \tag{3.22}$$

In eq. (3.20) we have introduced a tensor t built of P^μ, (single entry of)S^μ, n^μ, $g_T^{\mu\nu}$ and $\epsilon_T^{\mu\nu}$, and their scalar products, such that it preserves the permutation symmetry of indices on left-hand side, and it is invariant under rescaling $z \to \alpha z$. Such a tensor contains significant number of terms, which can be restricted by discrete symmetries, such as parity, time-reversal and charge-conjugation (which can be replaced by hermiticity due to CPT theorem). The parity invariance results into a relation among the terms of eq. (3.20)

$$t_\Gamma^{i;\mu\ldots}(P,S,n,g_T,\epsilon_T)F(x_1,x_2,x_3) = \eta_P^{\Gamma;\mu\ldots}t_\Gamma^{i;\mu\ldots}(\bar{P},s_T,\bar{n},g_T,-\epsilon_T)F(x_1,x_2,x_3), \tag{3.23}$$

where the bar denotes the parity transformation of a vector $\bar{v}^\mu = v_\mu$, and $\eta_P^{\Gamma,\mu\ldots}$ is the sign factor that appears in the parity transformation of the operator $\mathcal{P}\mathcal{T}_\Gamma\mathcal{P}^\dagger = \eta_P^\Gamma\mathcal{T}_\Gamma$. The time reversal transformation results into

$$t_\Gamma^{i;\mu\ldots}(P,S,n,g_T,\epsilon_T)F(x_1,x_2,x_3) = \eta_T^{\Gamma;\mu\ldots}t_\Gamma^{i;\mu\ldots}(\bar{P},-s_T,-\bar{n},-g_T,-\epsilon_T)F(-x_3,-x_2,-x_1), \tag{3.24}$$

where $\eta_T^{\Gamma,\mu\ldots}$ is the sign factor that appears in the time-reversal transformation of the operator $T\mathcal{T}_\Gamma T^\dagger = \eta_T^\Gamma\mathcal{T}_\Gamma$. In contrast to the two-point functions the time-reversal symmetry does not restrict the number of tensor structures t_i, because the functions on left- and right-hand sides of eq. (3.24) are of different arguments. Additionally one has the hermiticity relation which gives

$$\eta_H^\Gamma F^*(-x_1,-x_2,-x_3) = F(x_3,x_2,x_1), \tag{3.25}$$

where η_H is sign of hermitian conjugation of the operator $(\mathcal{T}_\Gamma)^\dagger = \eta_H^\Gamma \mathcal{T}_\Gamma$ (here we expect that the tensors t are real). Together the time-reversal (3.24) and hermiticity (3.25) relations dictates the complex and symmetry properties of the functions F.

In general the number of tensors t is very large. However, for the current work we need only the tensors which are non-zero if open indices are transverse, and the rest of indices are contracted with n^μ. In other words, we require the tensor structure of collinear twist-3. We find four such functions

$$\langle P,S|\mathcal{T}^\mu_{\gamma^+}|P,S\rangle = 2(p^+)^2\tilde{s}^\mu_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}T(x_1,x_2,x_3), \tag{3.26}$$

$$\langle P,S|\mathcal{T}^\mu_{\gamma^+\gamma^5}|P,S\rangle = 2i(p^+)^2 s^\mu_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\Delta T(x_1,x_2,x_3), \tag{3.27}$$

$$\begin{aligned}\langle P,S|\mathcal{T}^\mu_{i\sigma^{\alpha+}\gamma^5}|P,S\rangle &= 2(p^+)^2\epsilon^{\mu\alpha}_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\delta T_\epsilon(x_1,x_2,x_3)\\ &\quad +2i(p^+)^2\lambda g^{\mu\alpha}_T M\int [dx]e^{-ip^+(x_1z_1+x_2z_2+x_3z_3)}\delta T_g(x_1,x_2,x_3).\end{aligned} \tag{3.28}$$

Here, the factors M are set to have dimensionless three-point PDFs T. The definition of distributions T and ΔT coincides[3] with the definition used in [17], up to a factor M. The comparison to ETQS[4] functions (here we compare to definitions in eq. (12) and eq. (21) of [23]) gives

$$\widetilde{\mathcal{T}}_{q,F}(x,x+x_2) = MT(-x-x_2,x_2,x), \qquad \widetilde{\mathcal{T}}_{\Delta q,F}(x,x+x_2) = M\Delta T(-x-x_2,x_2,x). \tag{3.29}$$

The distribution T are real dimensionless functions. According to eq. (3.24) they obey the following symmetry properties

$$T(x_1,x_2,x_3) = T(-x_3,-x_2,-x_1), \tag{3.30}$$
$$\Delta T(x_1,x_2,x_3) = -\Delta T(-x_3,-x_2,-x_1), \tag{3.31}$$
$$\delta T_\epsilon(x_1,x_2,x_3) = \delta T_\epsilon(-x_3,-x_2,-x_1), \tag{3.32}$$
$$\delta T_g(x_1,x_2,x_3) = -\delta T_g(-x_3,-x_2,-x_1). \tag{3.33}$$

The Fourier transform of these distributions obey the same symmetry properties. These four functions are the only genuine twist-3 distributions in the quark sector.

It appears very convenient to introduce the following integral combinations,

$$T^{(n)}(x) = \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)+(-1)^n\delta(x+x_1)\right)T(x_1,x_2,x_3), \tag{3.34}$$

$$\Delta T^{(n)}(x) = \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)-(-1)^n\delta(x+x_1)\right)\Delta T(x_1,x_2,x_3), \tag{3.35}$$

$$\delta T^{(n)}_\epsilon(x) = \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)+(-1)^n\delta(x+x_1)\right)\delta T_\epsilon(x_1,x_2,x_3), \tag{3.36}$$

$$\delta T^{(n)}_g(x) = \int \frac{[dx]}{x_2^n}\left(\delta(x-x_3)-(-1)^n\delta(x+x_1)\right)\delta T_g(x_1,x_2,x_3). \tag{3.37}$$

The one-variable functions $T^{(n)}$, $\Delta T^{(n)}$ and $\delta T^{(n)}$ are in some aspects similar to the usual PDFs. For example, they have zero boundary conditions,

$$T^{(n)}(\pm 1) = 0, \qquad \Delta T^{(n)}(\pm 1) = 0, \qquad \delta T^{(n)}_\epsilon(\pm 1) = 0, \qquad \delta T^{(n)}_g(\pm 1) = 0. \tag{3.38}$$

In the following, we intensively use the functions in eq. (3.34-3.37), since they naturally arise and describe the worm-gear functions and allow a simplification of formulas.

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[3] To compare with ref.[17], we note that their definition of $\tilde{s}$ has opposite to us sign. Also during comparison we facilitate $s^2 = -1$.
[4] ETQS is acronym for Efremov-Teryaev-Qiu-Sterman [21, 22].

IV. LEADING MATCHING OF TMD DISTRIBUTIONS

In this section we assemble the result for the leading matching of TMD distributions up to terms linear in $\boldsymbol{b}$. For this purpose we need to evaluate the matrix element of the operators in eq. (2.19, 5.74) using the parameterizations introduced in previous section. Here we should take into account the decomposition of *collinear* twist-3 distributions over the distributions with definite *geometrical twist*. In the following subsections we consider each gamma-structure individually, and discuss the features of its evaluation. For convenience we also collect the final results in sec. **??**.

A. Vector operator

We start with the study of the vector operator, i.e. with $\Gamma = \gamma^+$, in the DY kinematics. Taking the forward matrix element of the operator relation in eq. (2.19) we obtain

$$\langle P,S|\mathcal{U}^{\gamma^+}_{\rm DY}(z,\frac{\boldsymbol{b}}{2})|P,S\rangle = 2p^+\int dx e^{2ixzp^+}f_1(x)+2(p^+)^2M\tilde s^\mu\frac{b_\mu}{2}\Big[-i\int_{-1}^1 dv vz\tilde T(z,vz,-z)+i\left(\int_{-\infty}^z+\int_{-\infty}^{-z}\right)d\tau\tilde T(z,\tau,-z)\Big]+O(\boldsymbol{b}^2), \tag{4.1}$$

where the contribution of the two-point correlator vanishes in accordance to eq. (3.6).

The function $T(z,vz-z)$ is symmetric in v due to the symmetry relation in eq. (3.30). Therefore, the anti-symmetric integral, which is the first in the square brackets of eq. (4.1), vanishes,

$$\int_{-1}^1 dv\, vz\,\tilde T(z,vz,-z) = 0. \tag{4.2}$$

In this way, the contributions linear in b are represented by a single entry, namely, by the last term of eq. (4.1). Using the reflection of coordinates in eq. (3.30) we present it as

$$\left(\int_{-\infty}^z+\int_{-\infty}^{-z}\right)d\tau\tilde T(z,\tau,-z) = \int_{-\infty}^{\infty}d\tau\tilde T(z,\tau,-z). \tag{4.3}$$

Taking into account these simplifications we find

$$\langle P,S|\mathcal{U}^{\gamma^+}_{\rm DY}(z,\frac{\boldsymbol{b}}{2})|P,S\rangle = 2p^+\int dx e^{2ixzp^+}f_1(x)+i(p^+)^2M\tilde s^\mu b_\mu\int_{-\infty}^{\infty}d\tau\tilde T(z,\tau,-z)+O(\boldsymbol{b}^2). \tag{4.4}$$

In the case of SIDIS kinematic the operators are given by eq. (5.74). Applying the same procedure we find

$$\langle P,S|\mathcal{U}^{\gamma^+}_{\rm DIS}(z,\frac{\boldsymbol{b}}{2})|P,S\rangle = 2p^+\int dx e^{2ixzp^+}f_1(x)-i(p^+)^2M\tilde s^\mu b_\mu\int_{-\infty}^{\infty}d\tau\tilde T(z,\tau,-z)+O(\boldsymbol{b}^2), \tag{4.5}$$

where we have used

$$\left(\int_z^\infty+\int_{-z}^{\infty}\right)d\tau\tilde T(z,\tau,-z) = \int_{-\infty}^{\infty}d\tau\tilde T(z,\tau,-z). \tag{4.6}$$

The only difference between Drell-Yan, eq. (4.4) and SIDIS, eq. (4.5) cases is the sign of the linear term. It corresponds to the famous process dependence of the Sivers function [24].

The TMD distribution is obtained by Fourier transformation over the light-cone distance, eq. (2.4). Performing it we obtain

$$\text{(DY)}\qquad \Phi^{[\gamma^+]}_{q\leftarrow h}(x,\boldsymbol{b}) = f_1(x)+ib_\mu\tilde s^\mu_T M\,\pi T(-x,0,x)+O(\boldsymbol{b}^2), \tag{4.7}$$

$$\text{(SIDIS)}\qquad \Phi^{[\gamma^+]}_{q\leftarrow h}(x,\boldsymbol{b}) = f_1(x)-ib_\mu\tilde s^\mu_T M\,\pi T(-x,0,x)+O(\boldsymbol{b}^2). \tag{4.8}$$

Here we have used,

$$\int_{-\infty}^{\infty}\frac{dz}{2\pi}\int_{-\infty}^{\infty}d\tau e^{-2ixzp^+}\tilde T(z,\tau,-z)=\frac{\pi}{(p^+)^2}T(-x,0,x). \tag{4.9}$$

These expressions represent the leading matching of vector TMD distribution. Comparing it to the parameterization in eq. (1.18) we find the matching of individual functions. Naturally, the unpolarized TMDPDF matches the unpolarized PDF, $f_1(x,\boldsymbol{b}) = f_1(x) + O(\boldsymbol{b}^2)$. The Sivers function matching is process dependent and it reads

$$\text{(DY)} \qquad f_{1T}^{\perp}(x,\boldsymbol{b}) = \pi T(-x,0,x) + O(\boldsymbol{b}^2), \tag{4.10}$$

$$\text{(SIDIS)} \qquad f_{1T}^{\perp}(x,\boldsymbol{b}) = -\pi T(-x,0,x) + O(\boldsymbol{b}^2). \tag{4.11}$$

Note, that the correction term is proportional to $\boldsymbol{b}^2$, and therefore, generically, contains twist-5 functions (and twist-4 functions for unpolarized distribution).

These expression, albeit in the different form, are well-known. In the two-point notation for ETQS function (3.29), the central value of three-point function $T(-x,0,x)$ corresponds to the diagonal value $\widetilde{\mathcal{T}}_{q,F}(x,x)$. Therefore, we can compare (4.10,4.11) to the expressions given in literature, where certain momentum space moments are calculated. Using the transformation rules presented in appendix I 1, one can check that

$$\int d^2\boldsymbol{p}_T \frac{\boldsymbol{p}_T^2}{M^2} f_{1T}^{\perp}(x,p_T) = 2\pi T(-x,0,x), \qquad \int d^2\boldsymbol{p}_T e^{-i(\boldsymbol{b}\boldsymbol{p}_T)} \frac{p_T^\alpha}{M} f_{1T}^{\perp}(x,p_T) = i\pi b^\alpha T(-x,0,x). \tag{4.12}$$

Here the sign is given for the DY case, and should be changed for the SIDIS case. To our best understanding[5] these expression coincide with ones presented in [25–28].

B. Axial operator

Taking the forward matrix element of the operator in eq. (2.19) with $\Gamma = \gamma^+\gamma^5$, we obtain

$$\begin{aligned}\langle P,S|\mathcal{U}_{\text{DY}}^{\gamma^+\gamma^5}(z,\frac{\boldsymbol{b}}{2})|P,S\rangle &= 2\lambda p^+ \int dx e^{2ixzp^+} g_1(x) + 2Ms_T^\mu \frac{b_\mu}{2}\Big[\int du e^{2iuzp^+}\frac{g_1(u)-g_T(u)}{z} \\ &+(p^+)^2\int_{-1}^1 dv vz\Delta\tilde{T}(z,vz,-z) - (p^+)^2\left(\int_{-\infty}^{z} + \int_{-\infty}^{-z}\right) d\tau \Delta\tilde{T}(z,\tau,-z)\Big] + O(\boldsymbol{b}^2),\end{aligned} \tag{4.13}$$

where we have used the parameterizations in eq. (3.2, 3.27) and the relation of eq. (3.7).

To proceed further we take the inverse Fourier transform. We have observed that these integrals naturally enter into the moments of the three-point functions, which are defined in eq. (3.34-3.37). Moreover, it is convenient to present them as a Mellin convolution. Using these tricks we find

$$\int \frac{dz}{2\pi} e^{-2ixzp^+} \int_{-1}^1 dv vz\Delta\tilde{T}(z,vz,-z) = \frac{i}{(p^+)^2}\Big[\frac{\Delta T^{(1)}(x)}{2} + \int_{-1}^1 du \int_0^1 dy\, u\Delta T^{(2)}(u)\delta(x-yu)\Big], \tag{4.14}$$

$$\int \frac{dz}{2\pi} e^{-2ixzp^+} \int du e^{2iuzp^+}\frac{g_1(u)-g_T(u)}{z} = i\int_{-1}^1 du \int_0^1 dy\, u(g_1(u)-g_T(u))\delta(x-uy). \tag{4.15}$$

The last integral in eq. (4.13) over the process-dependent term does not vanish,

$$\int \frac{dz}{2\pi} e^{-2ixzp^+}\left(\int_{-\infty}^{z} + \int_{-\infty}^{-z}\right) d\tau \Delta\tilde{T}(z,\tau,-z) = \frac{i}{(p^+)^2}\frac{\Delta T^{(1)}(x)}{2}, \tag{4.16}$$

$$\int \frac{dz}{2\pi} e^{-2ixzp^+}\left(\int_{z}^{\infty} + \int_{-z}^{\infty}\right) d\tau \Delta\tilde{T}(z,\tau,-z) = \frac{-i}{(p^+)^2}\frac{\Delta T^{(1)}(x)}{2}, \tag{4.17}$$

where we have used the assumption that the integrand goes to zero at infinity. The sign difference between these integrals, is compensated by the common sign difference in the operators for DY, eq. (2.19) and SIDIS, eq. (5.74) kinematics. Therefore, the contribution of seemingly process-dependent terms is the same for both operators. It is exactly compensated by the contribution of eq. (4.14), and thus the function $\Delta T^{(1)}$ drops out of calculation.

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[5] The comparison can not be done accurately in all cases, since some articles do not provide the full details on sign conventions and definitions.

Combining all together we obtain the same result for DY and SIDIS kinematics, which is

$$\Phi^{[\gamma^+\gamma^5]}(x,\boldsymbol{b}) = \lambda g_1(x) + i b_\mu s_T^\mu M \int_{-1}^{1} du \int_0^1 dy \delta(x-uy) u \Big(g_1(u) - g_T(u) + \Delta T^{(2)}(u)\Big). \tag{4.18}$$

Comparing to parameterizations in eq. (1.19) we find that the matching for the helicity TMD distribution $g_{1L}(x,\boldsymbol{b}) = g_1(x) + O(\boldsymbol{b}^2)$, and for the worm-gear-T distribution is

$$g_{1T}(x,\boldsymbol{b}) = \int_{-1}^{1} du \int_0^1 dy \delta(x-uy) u \Big(g_1(u) - g_T(u) + \Delta T^{(2)}(u)\Big) + O(\boldsymbol{b}^2). \tag{4.19}$$

The expression (4.19) is not the final one, because the function g_T can be rewritten via functions of definite twist,

$$g_T(x) = \int_0^1 dy \int_{-1}^{1} du \delta(x-yu) \Big[g_1(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}(1-\delta(\bar y)) + \Delta T^{(2)}(u)\Big], \tag{4.20}$$

where $\varepsilon_+ = 2m/M$ with m being the mass of a quark and $\bar y = 1-y$. The derivation of this decomposition is given in the appendix IV B 1. It is straightforward to check that it obeys the Burkhard-Cottingham sum rule eq. (3.5). Inserting the function g_T into eq. (4.19) and using the associativity of Mellin transformation (see also (4.29)) we obtain

$$g_{1T}(x,\boldsymbol{b}) = x \int_{-1}^{1} du \int_0^1 dy \delta(x-uy) \left(g_1(u) + \Delta T^{(2)}(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}\right) + O(\boldsymbol{b}^2). \tag{4.21}$$

This is the final form of the matching of the worm-gear function to the twist-2 and twist-3 functions. The Mellin convolution, which is presented in eq. (4.21) by δ-function, can be explicitly integrated. It gives the following representation

$$g_{1T}(x,\boldsymbol{b}) = x \int_x^1 \frac{du}{u} \left(g_1(u) + \Delta T^{(2)}(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}\right) + O(\boldsymbol{b}^2), \qquad \text{for } x>0, \tag{4.22}$$

$$g_{1T}(x,\boldsymbol{b}) = x \int_{-1}^x \frac{du}{|u|} \left(g_1(u) + \Delta T^{(2)}(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}\right) + O(\boldsymbol{b}^2), \qquad \text{for } x<0, \tag{4.23}$$

The obtained result can be compared to the first transverse momentum moments of TMD distribution derived in ref. [29] (see eqn.(47)), and agrees with it for $x>0$.

1. *Function* g_T

As it is demonstrated in ref. [20] the relation between g_T and definite twist functions is found with the help of the following operator relation

$$\begin{aligned}
\bar q(y)\gamma_\mu\gamma^5[y,-y]q(-y) = & \int_0^1 dt \frac{\partial}{\partial y^\mu} \bar q(ty)\, \not{y}\gamma^5[ty,-ty]q(-ty) \\
& - g\int_0^1 dt t \int_{-t}^{t} dv \frac{\epsilon_{\mu\nu\sigma\rho} y^\nu}{2} \bar q(ty)[ty,vy]F^{\sigma\rho}\, \not{y}[vy,-ty]q(-ty) \\
& - ig \int_0^1 dt \int_{-t}^{t} dv v \bar q(ty)[ty,vy]F_{\mu\nu}y^\nu\, \not{y}\gamma^5[vy,-ty]q(-ty) \\
& - i\epsilon_{\mu\nu\rho\sigma}\int_0^1 dt t y^\nu \partial^\rho \{\bar q(ty)\gamma^\sigma[ty,-ty]q(-ty) \\
& + 2my^\nu \int_0^1 dt t\, \bar q(ty)\sigma_{\nu\mu}\gamma^5[ty,-ty]q(-ty)\},
\end{aligned} \tag{4.24}$$

where m is the mass of quark. This is an exact operator relation, and is the consequence of QCD equations of motion [14]. Next, we evaluate the forward matrix element of equation (4.24) using the parameterizations (3.10, 3.26, 3.27). This operation transfers the variable y^μ into the elementary function, and the derivative over

y^μ can be done. Next we take the limit $y^2 \to 0$ as it is described in sec. III A 1, and apply eq. (3.19). After that procedure we obtain the vector equation which contains the functions of different twists. Its $(2Ms_T^\mu)$-component is

$$\int due^{2ixzp^+}g_T(x) = \int_0^1 dt \int dxe^{2ixtzp^+}g_1(x) + i\varepsilon_+(zp^+)\int_0^1 dt\int dxe^{2ixtzp^+}t\,h_1(x) \tag{4.25}$$
$$+(zp^+)^2\int_0^1 dt\int_{-t}^t dvv\Delta\tilde{T}(tz,vz,-tz) - (zp^+)^2\int_0^1 dtt\int_{-t}^t dv\tilde{T}(tz,vz,-tz),$$

where $\varepsilon_+ = 2m/M$. This equation relates collinear twist-3 function g_T to the functions with geometrical twist 2 and 3.

To obtain the function g_T explicitly, we perform the Fourier transformation for the equation (4.25). It is convenient to write the result in the following form

$$\begin{aligned} g_T(x) \;=\; & \int_0^1 dy\int_{-1}^1 du\delta(x-yu)\Big\{g_1(u) - \varepsilon_+\frac{h_1(u)}{2u}(1-\delta(\bar{y}))\Big\} \\ & + \int_0^1 dy\int [dx]\Big\{\frac{1}{2}T(x_1,x_2,x_3)\Big[\frac{\delta(x-x_3y)(1-\delta(\bar{y}))}{x_2x_3} + \frac{\delta(x+x_1y)(1-\delta(\bar{y}))}{x_1x_2}\Big] \\ & - \frac{1}{2}\Delta T(x_1,x_2,x_3)\Big[\frac{\delta(x-x_3y)(1-\delta(\bar{y}))}{x_2x_3} - \frac{\delta(x+x_1y)(1-\delta(\bar{y}))}{x_1x_2}\Big] \\ & + \frac{\Delta T(x_1,x_2,x_3)}{x_2^2}\Big[\delta(x-yx_3) - \delta(x+yx_1)\Big]\Big\}. \end{aligned} \tag{4.26}$$

In this form it is simple to check the Burkhard-Cottingham sum rule

$$\int_{-1}^1 dxg_T(x) = \int_{-1}^1 dxg_1(x). \tag{4.27}$$

The equation (4.26) has natural substructures in the form of x_2-moments introduced in eq. (3.34, 3.35). Using the notation in eq. (3.34, 3.35) we present the function g_T as a Mellin convolution integral

$$g_T(x) \;=\; \int_0^1 dy\int_{-1}^1 du\delta(x-yu)\Big[g_1(u) + \Delta T^{(2)}(u) + \frac{T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u)}{2u}(1-\delta(1-y))\Big]. \tag{4.28}$$

Using this notation and the associativity of Mellin convolution it is simple to take the integral in eq. (4.15). It reads

$$\begin{aligned} \int_{-1}^1 du\int_0^1 dyu(g_1(u) - g_T(u))\delta(x-yu) \;=\; & \int_{-1}^1 du\int_0^1 dy\delta(x-yu)\Big[uyg_1(u) \\ & -2\bar{y}u\Delta T^{(2)}(u) + y(T^{(1)}(u) - \Delta T^{(1)}(u) - \varepsilon_+ h_1(u))\Big]. \end{aligned} \tag{4.29}$$

C. Tensor operator

The matrix element of the tensor operator in eq. (2.19) (i.e. with $\Gamma = i\sigma_T^{\alpha+}\gamma^5$) has a more complicated form

$$\begin{aligned} \langle P,S|\mathcal{U}_{\rm DY}^{i\sigma_T^{\alpha+}\gamma^5}(z,\frac{\boldsymbol{b}}{2})|P,S\rangle \;=\; & 2s_T^\mu p^+\int dxe^{2ixzp^+}h_1(x) + 2M\frac{b_\mu}{2}\Big[\lambda g_T^{\mu\alpha}\int due^{2iuzp^+}\frac{h_1(u)-h_L(u)}{z} \\ & +(p^+)^2\int_{-1}^1 dvvz\left(\lambda g_T^{\mu\alpha}\delta\tilde{T}_g(z,vz,-z) - i\epsilon_T^{\mu\alpha}\delta\tilde{T}_\epsilon(z,vz,-z)\right) \\ & -(p^+)^2\left(\int_{-\infty}^z + \int_{-\infty}^{-z}\right)d\tau\left(\lambda g_T^{\mu\alpha}\delta\tilde{T}_g(z,\tau,-z) - i\epsilon_T^{\mu\alpha}\delta\tilde{T}_\epsilon(z,\tau,-z)\right)\Big] + O(\boldsymbol{b}^2), \end{aligned} \tag{4.30}$$

where we have used the parameterizations of eq. (3.3, 3.28) and the relation (3.8). Its structure repeats the structure discussed during evaluations of the vector operator (for terms proportional to δT_ϵ) and axial operator (for terms proportional to δT_g). Therefore, we skip the discussion on the Fourier integrals and write down the final expression for the matching of transversally polarized TMD distribution. We obtain (compare to eq.(4.8, 4.7) and (4.18))

$$\begin{aligned}\Phi^{[i\sigma^{\alpha+}\gamma^5]}(x,\boldsymbol{b}) &= s_T^\alpha h_1(x) \pm i b_\mu \epsilon_T^{\mu\alpha}\pi\delta T_\epsilon(-x,0,x) \\ &+ i\lambda b^\alpha M \int_{-1}^1 du \int_0^1 dy \delta(x-uy)u\Big(h_1(u)-h_L(u)+\delta T_g^{(2)}(u)\Big)+O(\boldsymbol{b}^2),\end{aligned} \tag{4.31}$$

where the upper sign should be taken for the DY kinematics, and lower for the SIDIS kinematics.

Comparing eq. (4.31) with the parameterization of eq. (1.20) we obtain the matching of individual TMD distributions. The transversity distribution is $h_1(x,\boldsymbol{b})=h_1(x)+O(\boldsymbol{b}^2)$. The Boer-Mulders functions depends on the underling process and reads

$$\text{(DY)} \qquad h_1^\perp(x,\boldsymbol{b}) = -\pi\delta T_\epsilon(-x,0,x)+O(\boldsymbol{b}^2), \tag{4.32}$$

$$\text{(SIDIS)} \qquad h_1^\perp(x,\boldsymbol{b}) = \pi\delta T_\epsilon(-x,0,x)+O(\boldsymbol{b}^2). \tag{4.33}$$

The worm-gear L function is independent on the process and has the expression

$$h_{1L}^\perp(x,\boldsymbol{b}) = -\int_{-1}^1 du \int_0^1 dy\delta(x-uy)u\Big(h_1(u)-h_L(u)+\delta T_g^{(2)}(u)\Big)+O(\boldsymbol{b}^2). \tag{4.34}$$

The pretzelosity distribution has no matching at this level of accuracy, despite the fact that the matrix element over free quarks is non-zero at $\boldsymbol{b}^2\to 0$ [30]. It is expected that the first non-zero contribution to the pretzelosity is of twist-4.

As in the case of the worm-gear T function, the expression for the worm-gear L function should be rewritten via a definite twist function. The derivation of function h_L is given appendix IV C 1. It reads

$$h_L(x) = -\int_0^1 dy \int_{-1}^1 du\delta(x-yu)\Big[2y(h_1(u)+\delta T_g^{(2)}(u))-\frac{\delta T_g^{(1)}(u)}{u}(2y-\delta(\bar y))\Big]. \tag{4.35}$$

We note that there is no term proportional to the quark mass, since it cancels in the equations of motion. Consequently, the worm-gear L function is

$$h_{1L}^\perp(x,\boldsymbol{b}) = -x\int_{-1}^1 du \int_0^1 dy\delta(x-uy)y\Big(h_1(u)+\delta T_g^{(2)}(u)-\frac{\delta T_g^{(1)}(u)}{u}\Big)+O(\boldsymbol{b}^2). \tag{4.36}$$

Let us point that the expressions for worm-gear L and worm-gear T functions has very similar structure, compare eq. (4.36) to eq. (4.21). The main difference is the factor y that appears in eq. (4.21). The integral over δ-function could be evaluated with the result

$$h_{1L}^\perp(x,\boldsymbol{b}) = -x^2\int_x^1 \frac{du}{u^2}\left(h_1(u)+\delta T_g^{(2)}(u)-\frac{\delta T_g^{(1)}(u)}{u}\right)+O(\boldsymbol{b}^2), \qquad \text{for } x>0, \tag{4.37}$$

$$h_{1L}^\perp(x,\boldsymbol{b}) = x^2\int_{-1}^x \frac{du}{u^2}\left(h_1(u)+\delta T_g^{(2)}(u)-\frac{\delta T_g^{(1)}(u)}{u}\right)+O(\boldsymbol{b}^2), \qquad \text{for } x<0. \tag{4.38}$$

This expression can be compared to the first transverse momentum moment of TMD distribution derived in ref. [29] (see eq.(49) in this reference). We find that our expression for $x>0$ agrees with the result of ref. [29], apart from the quark mass contribution, which is absent in our expression but present in [29].

1. *Function h_L*

The convenient form of the equation of motions for the derivation of function h_L as given in ref. [20],

$$\frac{\partial}{\partial y^\mu}\{\bar q(y)[y,-y](i\sigma^{\mu\nu}\gamma^5)y_\nu q(-y)\} = ig\int_{-1}^1 dv\, vy^\nu y_\alpha \bar q(y)[y,vy]F_{\mu\nu}(vy)(i\sigma^{\mu\alpha}\gamma^5)[vy,-y]q(-y) \tag{4.39}$$

$$+y^\nu\partial_\nu\{\bar q(y)[y,-y]\gamma^5 q(-y)\}. \tag{4.40}$$

We note that the quark mass term is proportional to $\varepsilon_- = (m_{\bar{q}} - m_q)/M$, and thus, vanishes for quarks of the same flavor. Making the same steps as in the evaluation of the function g_T, i.e. considering matrix element with parameterizations as in eq. (3.11) and (3.28), taking derivative and limit $y^2 \to 0$, we obtain

$$\int dx e^{2ixzp^+} \left[-ixzp^+ h_L(x) + (h_1(x) - h_L(x))\right] = -(zp^+)^2 \int_{-1}^{1} dv\, v\delta\tilde{T}_g(z, vz, -z). \tag{4.41}$$

The Fourier transform of this equation leads to the differential equation

$$\begin{aligned} x\partial_x h_L(x) - h_L(x) + 2h_1(x) &= -4p^+ \int \frac{dz}{2\pi} e^{-2ixzp^+} \int_{-1}^{1} dvv(zp^+)^2 \delta\tilde{T}_g(z, vz, -z) \\ &= \partial_x \delta T_g^{(1)}(x) - 2\delta T_g^{(2)}(x). \end{aligned} \tag{4.42}$$

The solution of this differential equation is

$$h_L(x) = \int_0^1 dy \int_{-1}^{1} du\delta(x - yu) y(2h_1(u) - \mathrm{RHS}(u)),$$

where the r.h.s. denotes the right-hand side of eq. (4.42). Performing an integration by parts we obtain

$$h_L(x) \;=\; \int_0^1 dy \int_{-1}^{1} du\delta(x - yu)\Big[2y(h_1(u) + \delta T_g^{(2)}(u)) - \frac{\delta T_g^{(1)}(u)}{u}(2y - \delta(1 - y))\Big]. \tag{4.43}$$

Clearly, it satisfies the Burkhard-Cottingham sum rule

$$\int_{-1}^{1} dx h_L(x) = \int_{-1}^{1} dx h_1(x). \tag{4.44}$$

It is intriguing to observe that the expression for h_L (4.43) is structurally very similar to the expression for g_T in eq. (4.28).

Here the new part starts

V. GLUON OPERATORS

A. Twist-2 operator

The indices $\{\mu\nu\}$ can be projected into 3 independent structures symmetric, anti-symmetric, and symmetric-traceless, by means of the projector

$$g_T^{\mu\mu'}g_T^{\nu\nu'} = P_S^{\mu\nu,\mu'\nu'} + P_A^{\mu\nu,\mu'\nu'} + P_T^{\mu\nu,\mu'\nu'}, \tag{5.1}$$

where

$$P_S^{\mu\nu,\mu'\nu'} = \frac{g_T^{\mu\nu}g_T^{\mu'\nu'}}{2(1-\epsilon)}, \tag{5.2}$$

$$P_A^{\mu\nu,\mu'\nu'} = \frac{g_T^{\mu\mu'}g_T^{\nu\nu'} - g_T^{\mu\nu'}g_T^{\nu\mu'}}{2}, \tag{5.3}$$

$$P_T^{\mu\nu,\mu'\nu'} = \frac{1}{2}\left(g_T^{\mu\mu'}g_T^{\nu\nu'} + g_T^{\mu\nu'}g_T^{\nu\mu'} - \frac{g_T^{\mu\nu}g_T^{\mu'\nu'}}{(1-\epsilon)}\right). \tag{5.4}$$

These projectors satisfy

$$P_I{}^{\mu\nu}{}_{\alpha\beta}P^{\alpha\beta\mu'\nu'} = \delta_{IJ}P_I^{\mu\nu,\mu'\nu'}. \tag{5.5}$$

The gluon twist-2 distribution is defined as

$$\frac{1}{xp^+}\int\frac{d\xi^-}{2\pi}e^{-ix\xi p^+}\langle p|F^{\mu+}(\xi^-)[\xi^-,0]F^{\nu+}(0)|p\rangle = \frac{g_T^{\mu\nu}}{2(1-\epsilon)}g(x)\pm\frac{\epsilon_T^{\mu\nu}}{2}\Delta g(x) + \tau^{\mu\nu}{}_{\mu'\nu'}\delta g^{\mu'\nu'}(x), \tag{5.6}$$

where

$$\tau^{\mu\nu;\alpha\beta} = \frac{g_T^{\mu\alpha}g_T^{\nu\beta} + g_T^{\mu\beta}g_T^{\nu\alpha}}{2} - \frac{g_T^{\mu\nu}g_T^{\alpha\beta}}{2(1-\epsilon)}. \tag{5.7}$$

This tensor is symmetric and traceless:

$$\tau^{\mu\nu;\alpha\beta} = \tau^{\alpha\beta;\mu\nu}, \qquad \tau^{\mu\nu}{}_{\sigma\gamma}\tau^{\sigma\gamma}{}_{\alpha\beta} = \tau^{\mu\nu}{}_{\alpha\beta}, \qquad \tau^{\mu\mu}{}_{\alpha\beta} = 0, \qquad \tau^{\mu\nu}{}_{\mu\nu} = (2-\epsilon)(1-2\epsilon). \tag{5.8}$$

To write down this definition I have combined expressions from [Collins'book], [0504030](see sec.3.1.2-3.2.6), and [1708.03528]. The unpolarized distribution coincides with [Collins'book].Definition of the blue term to be checked later.
Note, that $g^{\mu\nu}(0) = 0$ in the absence of transverse components. E.g. it has non-zero GPD component.

The inverse relation is

$$\langle p|F^{\mu+}(z_1)[z_1,z_2]F^{\nu+}(z_2)|p\rangle = 2p^+\int dx e^{i(z_1-z_2)xp^+}\frac{xp^+}{2}\left[\frac{g_T^{\mu\nu}}{2(1-\epsilon)}g(x)\pm\frac{\epsilon_T^{\mu\nu}}{2}\Delta g(x) + \tau^{\mu\nu}{}_{\mu'\nu'}\delta g^{\mu'\nu'}(x)\right] \tag{5.9}$$

B. Decomposition of 3-tensor in d dimension

In order to write parameterization of the operator we build the projectors to irreducible representations. There are 7 of then

$$g_T^{\mu\mu'} g_T^{\nu\nu'} g_T^{\lambda\lambda'} = \sum_{n=1}^{7} P_i^{\mu\nu\lambda;\mu'\nu'\lambda'}, \tag{5.10}$$

such that

$$\begin{aligned}
\text{symmetric-traceless} \quad P_1^{\mu\lambda\nu;\mu'\lambda'\nu'} &= S^{\mu\nu\lambda;\mu'\nu'\lambda'} - P_2^{\mu\lambda\nu;\mu'\lambda'\nu'}, &(5.11)\\
\text{symmetric} \quad P_2^{\mu\lambda\nu;\mu'\lambda'\nu'} &= \frac{3}{d+2} S^{\mu\nu\lambda;\alpha\beta\beta} S^{\alpha\gamma\gamma;\mu'\nu'\lambda'}, &(5.12)\\
\mu\nu\text{-symmetric-traceless} \quad P_3^{\mu\lambda\nu;\mu'\lambda'\nu'} &= \frac{4}{3} S^{\mu\nu;\alpha\beta} A^{\beta\lambda;\gamma\lambda'} S^{\alpha\gamma;\mu'\nu'} - P_4^{\mu\lambda\nu;\mu'\lambda'\nu'}, &(5.13)\\
\mu\nu\text{-symmetric} \quad P_4^{\mu\lambda\nu;\mu'\lambda'\nu'} &= \frac{2}{d-1} S^{\mu\nu;\alpha\beta} A^{\beta\lambda;\alpha\gamma} A^{\rho\gamma;\sigma\lambda'} S^{\rho\sigma;\mu'\nu'}, &(5.14)\\
\mu\nu\text{-antisymmetric-traceless} \quad P_5^{\mu\lambda\nu;\mu'\lambda'\nu'} &= \frac{4}{3} A^{\mu\nu;\alpha\beta} S^{\beta\lambda;\gamma\lambda'} A^{\alpha\gamma;\mu'\nu'} - P_6^{\mu\lambda\nu;\mu'\lambda'\nu'}, &(5.15)\\
\mu\nu\text{-antisymmetric} \quad P_6^{\mu\lambda\nu;\mu'\lambda'\nu'} &= \frac{2}{d-1} A^{\mu\nu;\alpha\lambda} A^{\alpha\lambda';\mu'\nu'}, &(5.16)\\
\text{anti-symmetric} \quad P_7^{\mu\lambda\nu;\mu'\lambda'\nu'} &= A^{\mu\nu\lambda;\mu'\nu'\lambda'}, &(5.17)
\end{aligned}$$

where S are symmetried products of g_T and A are any-symmetrized products of g_T. Explicitly they read

$$\begin{aligned}
S^{\mu\nu;\alpha\beta} &= \frac{g_T^{\mu\alpha} g_T^{\nu\beta} + g_T^{\nu\alpha} g_T^{\mu\beta}}{2}, &(5.18)\\
A^{\mu\nu;\alpha\beta} &= \frac{g_T^{\mu\alpha} g_T^{\nu\beta} - g_T^{\nu\alpha} g_T^{\mu\beta}}{2}, &(5.19)\\
S^{\mu\nu\lambda;\alpha\beta\gamma} &= \frac{g_T^{\mu\alpha} g_T^{\nu\beta} g_T^{\lambda\gamma} + ...}{6}, &(5.20)\\
A^{\mu\nu\lambda;\alpha\beta\gamma} &= \frac{g_T^{\mu\alpha} g_T^{\nu\beta} g_T^{\lambda\gamma} - ...}{6}. &(5.21)
\end{aligned}$$

The projectors are normalized and orthogonal

$$P_i^{\mu\nu\lambda;\alpha\beta\gamma} P_j^{\alpha\beta\gamma;\mu'\nu'\lambda'} = \delta_{ij} P_i^{\mu\nu\lambda;\mu'\nu'\lambda'}. \tag{5.22}$$

The dimension of corresponding irreducible sub-spaces are

$$\dim_i = P_i^{\mu\nu\lambda;\mu\nu\lambda} = \Big\{ \frac{(d-1)d(d+4)}{6}, d, \frac{d(d^2-4)}{3}, d, \frac{d(d^2-4)}{3}, d, \frac{d(d-1)(d-2)}{6} \Big\}. \tag{5.23}$$

So,

$$\sum_{i=1}^{7} \dim_i = d^3. \tag{5.24}$$

In $d = 2$ only the components $3, 5, 7$ has zero dimension.

To parameterize the matrix element in 2-dimensions we can use a single vector s^μ, an odd number of $\epsilon_T^{\mu\nu}$, and $g_T^{\mu\nu}$. In d-dimensions we use analogs s^μ, $a^{\mu\nu}$ and $g_T^{\mu\nu}$ where $a^{\mu\nu} \to \epsilon_T^{\mu\nu}$ as $d \to 2$. We introduce the following tensors defined in d-dimensions

$$\begin{aligned}
t_2^{\mu\nu\lambda} &= s^\alpha a^{\mu\alpha} g_T^{\nu\lambda} + s^\alpha a^{\nu\alpha} g_T^{\lambda\mu} + s^\alpha a^{\lambda\alpha} g^{\mu\nu}, &(5.25)\\
t_3^{\mu\nu\lambda} &= s^\alpha a^{\mu\alpha} g_T^{\nu\lambda} - 2 s^\alpha a^{\nu\alpha} g_T^{\lambda\mu} + s^\alpha a^{\lambda\alpha} g_T^{\mu\nu} + (d-1)(s^\mu a^{\nu\lambda} - s^\lambda a^{\mu\nu}), &(5.26)\\
t_4^{\mu\nu\lambda} &= -s^\alpha a^{\mu\alpha} g_T^{\nu\lambda} + 2 s^\alpha a^{\nu\alpha} g_T^{\lambda\mu} - s^\alpha a^{\lambda\alpha} g_T^{\mu\nu}, &(5.27)\\
t_5^{\mu\nu\lambda} &= 3 s^\alpha a^{\mu\alpha} g_T^{\nu\lambda} - 3 s^\alpha a^{\lambda\alpha} g_T^{\mu\nu} + (d-1)(-s^\mu a^{\nu\lambda} + 2 s^\nu a^{\lambda\mu} - s^\lambda a^{\mu\nu}), &(5.28)\\
t_6^{\mu\nu\lambda} &= s^\alpha a^{\mu\alpha} g_T^{\nu\lambda} - s^\alpha a^{\lambda\alpha} g_T^{\mu\nu}, &(5.29)\\
t_7^{\mu\nu\lambda} &= -s^\mu a^{\nu\lambda} - s^\nu a^{\lambda\mu} - s^\lambda a^{\mu\nu}. &(5.30)
\end{aligned}$$

These tensors satisfy

$$P^{\mu\nu\lambda;\alpha\beta\gamma}t_j^{\alpha\beta\gamma} = \delta_{ij}t_i^{\mu\nu\alpha}. \tag{5.31}$$

The tensor $t_1 = 0$ since it is not possible to build a traceless-symmetric tensor with only single entry of a vector. At $d \to 2$ the subspaces $3, 5, 7$ vanishes, and the only remaining tensors are $t_{2,4,6}$ which reads

$$d \to 2, \qquad t_2^{\mu\nu\lambda} = \tilde{s}^\mu g_T^{\nu\lambda} + \tilde{s}^\nu g_T^{\lambda\mu} + \tilde{s}^\lambda g_T^{\mu\nu}, \tag{5.32}$$
$$d \to 2, \qquad t_4^{\mu\nu\lambda} = -\tilde{s}^\mu g_T^{\nu\lambda} + 2\tilde{s}^\nu g_T^{\lambda\mu} - \tilde{s}^\lambda g_T^{\mu\nu}, \tag{5.33}$$
$$d \to 2, \qquad t_6^{\mu\nu\lambda} = \tilde{s}^\mu g_T^{\nu\lambda} - \tilde{s}^\lambda g_T^{\mu\nu}, \tag{5.34}$$
$$d \to 2, \qquad t_{3,5,7}^{\mu\nu\lambda} = 0. \tag{5.35}$$

Consequently we parameterize

$$\langle P, S|\mathcal{T}_\pm^{\mu\nu\lambda}(z_1, z_2, z_3)|P, S\rangle = P_+^3 M\{t_2^{\mu\nu\lambda}F_2^\pm(z_1, z_2, z_3) + t_4^{\mu\nu\lambda}F_4^\pm(z_1, z_2, z_3) + t_6^{\mu\nu\lambda}F_6^\pm(z_1, z_2, z_3)\} \tag{5.36}$$

For future convenience we introduce functions

$$F_2^\pm(z_1, z_2, z_3) = \frac{G^\pm(z_1, z_2, z_3)}{d+2}, \qquad F_4^\pm(z_1, z_2, z_3) = \frac{Y^\pm(z_1, z_2, z_3)}{2(d-1)}. \tag{5.37}$$

C. Symmetries of parametrization

The operator $\mathcal{T}^{\mu\nu\lambda}$ has two entries

$$\mathcal{T}_+^{\mu\nu\lambda}(z_1, z_2, z_3) = if^{ABC}F^{\mu+}(z_1)F^{\nu+}(z_2)F^{\lambda+}(z_3), \tag{5.38}$$
$$\mathcal{T}_-^{\mu\nu\lambda}(z_1, z_2, z_3) = d^{ABC}F^{\mu+}(z_1)F^{\nu+}(z_2)F^{\lambda+}(z_3). \tag{5.39}$$

Therefore, they are

$$\mathcal{T}_+^{\mu\nu\lambda}(z_1, z_2, z_3) = -\mathcal{T}_+^{\mu\lambda\nu}(z_1, z_3, z_2) = -\mathcal{T}_+^{\nu\mu\lambda}(z_2, z_1, z_3) = \mathcal{T}_+^{\nu\lambda\mu}(z_2, z_3, z_1) = \mathcal{T}_+^{\lambda\mu\nu}(z_3, z_1, z_2) = -\mathcal{T}_+^{\lambda\nu\mu}(z_3, z_2, z_1), \tag{5.40}$$
$$\mathcal{T}_-^{\mu\nu\lambda}(z_1, z_2, z_3) = \mathcal{T}_-^{\mu\lambda\nu}(z_1, z_3, z_2) = \mathcal{T}_-^{\nu\mu\lambda}(z_2, z_1, z_3) = \mathcal{T}_-^{\nu\lambda\mu}(z_2, z_3, z_1) = \mathcal{T}_-^{\lambda\mu\nu}(z_3, z_1, z_2) = \mathcal{T}_-^{\lambda\nu\mu}(z_3, z_2, z_1). \tag{5.41}$$

I.e. $\mathcal{T}_{-(+)}$ is (anti-)symmetric with respect to permutation of any coordinate-index-pair.

Under the exchange of indices the tensors t_i mixes. The interesting to us sector of $t_{2,4,6}$ mixes as following

$$t_2 = P_{12}t_2 = P_{23}t_2 = P_{13}t_2 = P_{12}P_{23}t_2 = P_{23}P_{12}t_2 \tag{5.42}$$
$$P_{12}t_4 = \frac{-t_4 + 3t_6}{2}, \qquad P_{12}t_6 = \frac{t_4 + t_6}{2}, \tag{5.43}$$
$$P_{13}t_4 = t_4, \qquad P_{13}t_6 = -t_6, \tag{5.44}$$
$$P_{23}t_4 = \frac{-t_4 - 3t_6}{2}, \qquad P_{12}t_6 = \frac{-t_4 + t_6}{2}, \tag{5.45}$$
$$P_{23}P_{12}t_4 = \frac{-t_4 - 3t_6}{2}, \qquad P_{23}P_{12}t_6 = \frac{t_4 - t_6}{2}, \tag{5.46}$$
$$P_{12}P_{23}t_4 = \frac{-t_4 + 3t_6}{2}, \qquad P_{12}P_{23}t_6 = \frac{-t_4 - t_6}{2}, \tag{5.47}$$

where $t_i = t_i^{\mu_1\mu_2\mu_3}$ and P_{ij} is operator of permutation of μ_i and μ_j, i.e.

$$P_{12}t^{\mu_1\mu_2\mu_3} = t^{\mu_2\mu_1\mu_3}.$$

These relations implies the following symmetries

$$F_2^+(z_1, z_2, z_3) = -F_2^+(z_1, z_3, z_2) = -F_2^+(z_2, z_1, z_3) = F_2^+(z_2, z_3, z_1) = F_2^+(z_3, z_1, z_2) = -F_2^+(z_3, z_2, z_1), \tag{5.48}$$
$$F_2^-(z_1, z_2, z_3) = F_2^-(z_1, z_3, z_2) = F_2^-(z_2, z_1, z_3) = F_2^-(z_2, z_3, z_1) = F_2^-(z_3, z_1, z_2) = F_2^-(z_3, z_2, z_1). \tag{5.49}$$

I.e. function F_1^+ is anti-symmetric for permutation of any pair of arguments, function F_1^- is symmetric for permutation of any pair of arguments.

The permutation rule P_{13} implies

$$F_4^{\pm}(z_1,z_2,z_3) = \mp F_4^{\pm}(z_3,z_2,z_1), \qquad F_6^{\pm}(z_1,z_2,z_3) = \pm F_6^{\pm}(z_3,z_2,z_1). \tag{5.50}$$

The other permutations mix functions $F_{4,6}$ with each other. We have the following set of relations

$$F_4^{\pm}(z_2,z_1,z_3) = \pm\frac{F_4^{\pm}(z_1,z_2,z_3) - F_6^{\pm}(z_1,z_2,z_3)}{2}, \quad F_6^{\pm}(z_2,z_1,z_3) = \pm\frac{-3F_4^{\pm}(z_1,z_2,z_3) - F_6^{\pm}(z_1,z_2,z_3)}{2}, \tag{5.51}$$

$$F_4^{\pm}(z_1,z_3,z_2) = \pm\frac{F_4^{\pm}(z_1,z_2,z_3) + F_6^{\pm}(z_1,z_2,z_3)}{2}, \quad F_6^{\pm}(z_1,z_3,z_2) = \pm\frac{3F_4^{\pm}(z_1,z_2,z_3) - F_6^{\pm}(z_1,z_2,z_3)}{2}. \tag{5.52}$$

Here we show only for a single permutation since double-permutation gives equivalent equations.

The pairs of these equations have the following solutions

$$F_6^{\pm}(z_1,z_2,z_3) = F_4^{\pm}(z_1,z_2,z_3) \mp 2F_4^{+}(z_2,z_1,z_3) = -F_4^{\pm}(z_1,z_2,z_3) \pm 2F_4^{\pm}(z_1,z_3,z_2). \tag{5.53}$$

It is possible only if the function F_4 satisfies:

$$F_4^{\pm}(z_1,z_2,z_3) \mp F_4^{\pm}(z_2,z_1,z_3) \mp F_4^{\pm}(z_1,z_3,z_2) = 0, \tag{5.54}$$

or equivalently

$$F_4^{\pm}(z_1,z_2,z_3) + F_4^{\pm}(z_2,z_3,z_1) + F_4^{\pm}(z_3,z_1,z_2) = 0. \tag{5.55}$$

The solution is

$$F_6^{\pm}(z_1,z_2,z_3) = \pm\left(F_4^{\pm}(z_1,z_3,z_2) - F_4^{\pm}(z_2,z_1,z_3)\right), \tag{5.56}$$

It could be also solved with respect to F_4 with the result

$$F_4^{\pm}(z_1,z_2,z_3) = \pm\frac{1}{3}\left(F_6^{\pm}(z_1,z_3,z_2) - F_6^{\pm}(z_2,z_1,z_3)\right). \tag{5.57}$$

and

$$F_6^{\pm}(z_1,z_2,z_3) \pm F_6^{\pm}(z_2,z_1,z_3) \pm F_6^{\pm}(z_1,z_3,z_2) = 0, \tag{5.58}$$

or

$$F_6^{\pm}(z_1,z_2,z_3) + F_6^{\pm}(z_3,z_1,z_2) + F_6^{\pm}(z_2,z_3,z_1) = 0. \tag{5.59}$$

Concluding

$$F_6^{\pm}(z_1,z_2,z_3) = \pm\left(F_4^{\pm}(z_1,z_3,z_2) - F_4^{\pm}(z_2,z_1,z_3)\right), \tag{5.60}$$

with

$$F_4^{\pm}(z_1,z_2,z_3) = \mp F_4^{\pm}(z_3,z_2,z_1), \qquad F_4^{\pm}(z_1,z_2,z_3) + F_4^{\pm}(z_2,z_3,z_1) + F_4^{\pm}(z_3,z_1,z_2) = 0. \tag{5.61}$$

The reverse solution

$$F_4^{\pm}(z_1,z_2,z_3) = \frac{\pm 1}{3}\left(F_6^{\pm}(z_1,z_3,z_2) - F_6^{\pm}(z_2,z_1,z_3)\right), \tag{5.62}$$

with

$$F_6^{\pm}(z_1,z_2,z_3) = \pm F_6^{\pm}(z_3,z_2,z_1), \qquad F_6^{\pm}(z_1,z_2,z_3) + F_6^{\pm}(z_2,z_3,z_1) + F_6^{\pm}(z_3,z_1,z_2) = 0. \tag{5.63}$$

One can check that these solutions are consistent with each other.

Let us compare to [BMP]. They define (check (27))

$$\mathcal{F}^{\pm}(z_1,z_2,z_3) \sim \mathcal{T}^{\mu\nu\mu}(z_1,z_2,z_3) \mp \mathcal{T}^{\mu\nu\mu}(z_1,z_3,z_2) \pm \mathcal{T}^{\mu\nu\mu}(z_2,z_1,z_3). \tag{5.64}$$

According to our decomposition we find

$$\mathcal{F}^{\pm}(z_1,z_2,z_3) \simeq F_2^{\pm}(z_1,z_2,z_3) \pm F_4^{\pm}(z_2,z_1,z_3). \tag{5.65}$$

These combinations indeed satisfy $\mathcal{F}^{\pm}(z_1,z_2,z_3) = \mp\mathcal{F}^{\pm}(z_1,z_3,z_2)$ (check equation before (31)).

D. Application of T and hermiticity

It is convinient to pass to the distributions. We denote

$$F_i^{\pm}(z_1,z_2,z_3) = p_+^3 M \int [dx] e^{-ip_+\sum z_i x_i} F_i^{\pm}(x_1,x_2,x_3). \tag{5.66}$$

The application of T implies the complex conjugation of the operator and change $z_i \to -z_i$, which happens due to $n \to \bar{n}$m that can be rotated back to $z_i \to -z_i$. I.e.

$$F_i^{\pm}(x_1,x_2,x_3) = \mp F_i^{\pm}(-x_1,-x_2,-x_3). \tag{5.67}$$

The application of hermiticity requires the change of variables $x_i \to -x_i$ in order to gain the same Fourier exponents. We obtain

$$\left[F_i^{\pm}(x_1,x_2,x_3)\right]^* = \mp F_i^{\pm}(-x_1,-x_2,-x_3). \tag{5.68}$$

Therefore, function $F_i^{\pm}$ are real.

Taking into account permutation properties we get

$$F_{2,4}^{\pm}(x_1,x_2,x_3) = F_i^{\pm}(-x_3,-x_2,-x_1). \tag{5.69}$$

In the article I have changed the sign of G and Y, in order to have the same definition is [BMP] and [Kang].

E. The LO matching for the gluon operator

In our calculation we also need the gluon operator. In analogy to the quark operator (2.1) we write

$$\mathcal{U}^{\mu\nu}_{\rm DY}(z,\boldsymbol{b}) \;=\; F^{\mu+}(zn+\boldsymbol{b})[zn+\boldsymbol{b},-\infty n+\boldsymbol{b}]\Gamma[-\infty n-\boldsymbol{b},-zn-\boldsymbol{b}]F^{\nu+}(-zn-\boldsymbol{b}), \tag{5.70}$$

where Wilson lines are in the adjoint representation. The formulas for quark case are practically repeated with the only difference that the derivative is in the adjoint representation. For convenience we introduce the generic notation for two- and three-point operators

$$\mathcal{O}_{\mu\nu}(z) \;=\; F^{\mu+}(zn)[zn,-zn]F^{\nu+}(-zn), \tag{5.71}$$

$$\mathcal{T}^{\mu\rho\nu}(z_1,z_2,z_3) \;=\; gF^{\mu+}(z_1n)[z_1n,z_2n]_A(if^{ABC})F^{\rho+}_B(z_2n)\,_C[z_2n,z_3n]F^{\nu+}(z_3n). \tag{5.72}$$

The expression for the first terms of small-b expansion for TMD operator reads (at leading order in α_s)

$$\mathcal{U}^{\mu\nu}_{\rm DY}(z,\boldsymbol{b}) \;=\; \mathcal{O}^{\mu\nu}(z)+b_\rho\Big\{\lim_{y\to zn}\frac{\partial}{\partial y_\rho}\mathcal{O}^{\mu\nu}(y)-i\int_{-1}^{1}dv\,vz\,\mathcal{T}^{\mu\rho\nu}(z,vz,-z) +i\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\tau\mathcal{T}^{\mu\rho\nu}(z,\tau,-z)\Big\}+O(\boldsymbol{b}^2), \tag{5.73}$$

$$\mathcal{U}^{\mu\nu}_{\rm DY}(z,\boldsymbol{b}) \;=\; \mathcal{O}^{\mu\nu}(z)+b_\rho\Big\{\lim_{y\to zn}\frac{\partial}{\partial y_\rho}\mathcal{O}^{\mu\nu}(y)-i\int_{-1}^{1}dv\,vz\,\mathcal{T}^{\mu\rho\nu}(z,vz,-z) -i\left(\int_{z}^{\infty}+\int_{-z}^{\infty}\right)d\tau\mathcal{T}^{\mu\rho\nu}(z,\tau,-z)\Big\}+O(\boldsymbol{b}^2). \tag{5.74}$$

The limit $y\to zn$ implies $y^2\to 0$ such that the light-like separation between fields is z.

F. Vanishing structures

In our calculation we have the following operator

$$\begin{aligned}\mathcal{T}^{\mu(\rho)\nu}(z,-z) \;&=\; F^{\mu+}(z)\overleftrightarrow{\partial_\rho}F^{\nu+}(-z) \\ &=\; \lim_{y\to zn}\frac{\partial}{\partial y_\rho}\mathcal{O}^{\mu\nu}(y)-i\int_{-1}^{1}dv\,vz\,\mathcal{T}^{\mu\rho\nu}(z,vz,-z)+i\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\tau\mathcal{T}^{\mu\rho\nu}(z,\tau,-z).\end{aligned} \tag{5.75}$$

However, it enters only in particular contractions. Let us demonstrate which part of this operator is non-vanishing.

This part is to be updated. Now it is very rudimentary.

In order to apply the formula, we have to derive the representation for the gluon distribution at the general y. It can be done using (5) from [0009343], where $k\to y$. Then we need to differentiate it, over y. The long expression can be shortly presented as

$$\lim_{y\to zn}\frac{\partial}{\partial y_\rho}\mathcal{O}^{\mu\nu}(y)\sim F_1\left(\tilde s^\mu g_T^{\nu\rho}-\tilde s^\nu g_T^{\mu\rho}\right)+F_2\left(s^\mu\epsilon_T^{\nu\rho}-s^\nu\epsilon_T^{\mu\rho}\right)+F_3s^\rho\epsilon_T^{\mu\nu}. \tag{5.76}$$

During our evaluation we meet the following convolutions

$$\mathcal{T}^{\mu(\rho)\mu}\to F_1(\tilde s^\rho-\tilde s^\rho)-F_2(\tilde s^\rho-\tilde s^\rho)+F_3s^\rho\epsilon_T^{\mu\mu}=0, \tag{5.77}$$

$$\begin{aligned}\mathcal{T}^{\mu(\mu)\rho}+\mathcal{T}^{\rho(\mu)\mu} \;&\to\; F_1\left(\tilde s^\rho-2\tilde s^\rho\right)+F_2\tilde s^\rho+F_3(-\tilde s^\rho) \\ &\quad +F_1\left(2\tilde s^\rho-\tilde s^\rho\right)+F_2\left(-\tilde s^\rho\right)+F_3\tilde s^\rho=0.\end{aligned} \tag{5.78}$$

Thus, there is no contribution from that terms.

The term proportional to the integral in $[-1,1]$ also vanish due to the time-reversal. Indeed, since $\mathcal{T}^{\mu\rho\nu}(z_1,z_2,z_3)=-\mathcal{T}^{\nu\rho\mu}(z_3,z_2,z_1)$ we get

$$\int_{-1}^{1} dv\, vz\, \mathcal{T}^{\mu\rho\nu}(z,vz,-z) \underbrace{=}_{\mu\leftrightarrow\nu} -\int_{-1}^{1} dv\, vz\, \mathcal{T}^{\nu\rho\mu}(-z,vz,z)$$
$$\underbrace{=}_{T} +\int_{-1}^{1} dv\, vz\, \mathcal{T}^{\nu\rho\mu}(z,-vz,-z) \underbrace{=}_{v\to -v} -\int_{-1}^{1} dv\, vz\, \mathcal{T}^{\nu\rho\mu}(z,vz,-z). \tag{5.79}$$

I.e. this tensor is antisymmetric with respect to $\mu\nu$. Thus

$$\int_{-1}^{1} dv\, vz\, \mathcal{T}^{\mu\rho\mu}(z,vz,-z) = \int_{-1}^{1} dv\, vz\, \left(\mathcal{T}^{\mu\mu\rho}(z,vz,-z)+\mathcal{T}^{\rho\mu\mu}(z,vz,-z)\right)=0. \tag{5.80}$$

Therefore, only the "QS"-like term survives in the matrix elements of $\mathcal{T}^{\mu(\rho)\mu}$ and $\mathcal{T}^{\mu(\mu)\rho}+\mathcal{T}^{\rho(\mu)\mu}$.

G. The structure that appear in calculation

In the calculation we meet particular combinations of indices. Let us find out these combinations.

First of all we have $\mathcal{T}^{\mu\rho\mu}$ that is

$$g_T^{\mu\lambda}\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = 2\tilde s^{\nu}p_+^3M\int [dx]e^{-ip_+x_iz_i}\left((2-\epsilon)F_2^{\pm}(z_1,z_2,z_3)+(1-2\epsilon)F_4^{\pm}(z_1,z_2,z_3)\right). \tag{5.81}$$

In the terms of functions

$$G^{\pm}(z_1,z_2,z_3)=2(2-\epsilon)F_2^{\pm}(z_1,z_2,z_3),\qquad Y^{\pm}(z_1,z_2,z_3)=2(1-2\epsilon)F_4^{\pm}(z_1,z_2,z_3), \tag{5.82}$$

we have

$$g_T^{\mu\lambda}\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = \tilde s^{\nu}p_+^3M\int [dx]e^{-ip_+x_iz_i}\left(G^{\pm}(x_1,x_2,x_3)+Y^{\pm}(x_1,x_2,x_3)\right). \tag{5.83}$$

Note, that functions G and Y satisfy all properties of F_2 and F_4 correspondingly. Compare to [BMP] (5) we get

$$G^{\pm}(x_1,x_2,x_3)+Y^{\pm}(x_1,x_2,x_3)=-T_{3F}^{\pm}(x_1,x_2,x_3). \tag{5.84}$$

There could be relative minus, since $\tilde s$ is defined differently.

Another two structures

$$g_T^{\mu\nu}\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = 2\tilde s^{\lambda}p_+^3M\int [dx]e^{-ip_+x_iz_i}\left((2-\epsilon)F_2^{\pm}(x_1,x_2,x_3)-(1-2\epsilon)(F_4^{\pm}(x_1,x_2,x_3)+F_6^{\pm}(x_1,x_2,x_3))\right) \tag{5.85}$$

$$g_T^{\nu\lambda}\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = 2\tilde s^{\mu}p_+^3M\int [dx]e^{-ip_+x_iz_i}\left((2-\epsilon)F_2^{\pm}(x_1,x_2,x_3)-(1-2\epsilon)(F_4^{\pm}(x_1,x_2,x_3)-F_6^{\pm}(x_1,x_2,x_3))\right) \tag{5.86}$$

In the terms of function H and Y we have

$$g_T^{\mu\nu}\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = \tilde s^{\lambda}p_+^3M\int [dx]e^{-ip_+x_iz_i}\left(G^{\pm}(x_1,x_2,x_3)\mp Y^{\pm}(x_1,x_3,x_2)\right), \tag{5.87}$$

$$g_T^{\nu\lambda}\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = \tilde s^{\mu}p_+^3M\int [dx]e^{-ip_+x_iz_i}\left(G^{\pm}(x_1,x_2,x_3)\mp Y^{\pm}(x_2,x_1,x_3)\right). \tag{5.88}$$

Note, that in the terms of precious calculation these structures are $\mp T_{3F}^{\pm}(x_1,x_3,x_2)$ and $\mp T_{3F}^{\pm}(x_2,x_1,x_3)$, which is correct.

Finally, we have

$$b_\mu b_\nu b_\lambda\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = 3b^2(b_\mu\tilde s^{\mu})p_+^3M\int [dx]e^{-ip_+x_iz_i}F_2^{\pm}(x_1,x_2,x_3), \tag{5.89}$$

or

$$b_\mu b_\nu b_\lambda\langle P,S|\mathcal{T}_{\pm}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = \frac{3}{2(2-\epsilon)}b^2(b_\mu\tilde s^{\mu})p_+^3M\int [dx]e^{-ip_+x_iz_i}H^{\pm}(x_1,x_2,x_3). \tag{5.90}$$

In the terms of previous structure it coulf be written as $\sim T_{3F}^{\pm}(x_1,x_2,x_3)+T_{3F}^{\pm}(x_3,x_1,x_2)+T_{3F}^{\pm}(x_2,x_3,x_1)$.

H. QS-type structures

In our calculation the QS-type of structure plays a special role. It is then $x_2 = 0$. We will reduce everything to a case $x_1 = -\xi$ and $x_3 = \xi$. We have the following relations for the function G

$$G^+(-\xi,0,\xi) = -G^+(\xi,0,-\xi) = -G^+(-\xi,\xi,0) = G^+(\xi,-\xi,0) = -G^+(0,-\xi,\xi) = G^+(0,\xi,-\xi), \quad (5.91)$$
$$G^-(-\xi,0,\xi) = G^-(\xi,0,-\xi) = G^-(-\xi,\xi,0) = G^-(\xi,-\xi,0) = G^-(0,-\xi,\xi) = G^-(0,\xi,-\xi). \quad (5.92)$$

For the functions Y we have

$$Y^+(-\xi,0,\xi) = -Y^+(\xi,0,-\xi), \qquad Y^+(-\xi,\xi,0) = -Y^+(\xi,-\xi,0) = -Y^+(0,\xi,-\xi) = Y^+(0,-\xi,\xi), \quad (5.93)$$
$$Y^-(-\xi,0,\xi) = Y^-(\xi,0,-\xi), \qquad Y^-(-\xi,\xi,0) = Y^-(\xi,-\xi,0) = Y^-(0,\xi,-\xi) = Y^-(0,-\xi,\xi). \quad (5.94)$$

The cyclic rule gives

$$Y^{\pm}(\xi,-\xi,0) = -\frac{Y^{\pm}(-\xi,0,\xi)}{2}. \quad (5.95)$$

If I compare to [Qui,Kang,,0811.3101] (14) I get

$$\mathcal{T}_{G,F} = \pm(G+Y). \quad (5.96)$$

where + is the definition of $\tilde{s}$ coincides, and - if not (it is not).

I. Parametrization of $T^{\mu\nu\rho}$ OLD

This section has been written before observation that the functions has extra symmetry.
NOTE

$$d=2: \qquad s^\mu \epsilon_{\nu\rho} = \tilde s^\nu g_T^{\mu\rho} - \tilde s^\rho g_T^{\mu\nu}. \tag{5.97}$$

The operator $\mathcal{T}$ has the following obvious symmetries

$$\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3) = -\mathcal{T}^{\mu\lambda\nu}(z_1,z_3,z_2) = -\mathcal{T}^{\nu\mu\lambda}(z_2,z_1,z_3) = \mathcal{T}^{\nu\lambda\mu}(z_2,z_3,z_1) = \mathcal{T}^{\lambda\mu\nu}(z_3,z_1,z_2) = -\mathcal{T}^{\lambda\nu\mu}(z_3,z_2,z_1), \tag{5.98}$$

i.e. it is antisymmetric with respect to permutation of pairs of (index-argument).
We can build the following general parameterization of $\mathcal{T}^{\mu\rho\nu}(z_1,z_2,z_3)$

$$\langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = P_+^3 M\int [dx] e^{-ip^+x_iz_i}\Big\{\tilde s^\mu g_T^{\nu\lambda}g_1 + \tilde s^\nu g_T^{\lambda\mu}g_2 + \tilde s^\lambda g_T^{\mu\nu}g_3 + s^\mu\epsilon_T^{\nu\lambda}g_4 + s^\nu\epsilon_T^{\lambda\mu}g_5 + s^\lambda\epsilon_T^{\mu\nu}g_6\Big\} \tag{5.99}$$

This parameterization has a proper P-parity transformation (as a P-even tensor). The functions g_i are functions of (x_1,x_2,x_3). The change of arguments $z_{123}\to z_{ijk}$ can be compensated by the change of integration variables $x_{123}\to x_{ijk}$, such that the Fourier exponent remains $\exp(-i(z_1x_1+z_2x_2+z_3x_3))$. Thus, using the symmetry (5.98) we obtain the set of expressions

$$\begin{aligned}
\langle\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)\rangle &\sim \Big\{\tilde s^\mu g_T^{\nu\lambda}g_1 + \tilde s^\nu g_T^{\lambda\mu}g_2 + \tilde s^\lambda g_T^{\mu\nu}g_3 + s^\mu\epsilon_T^{\nu\lambda}g_4 + s^\nu\epsilon_T^{\lambda\mu}g_5 + s^\lambda\epsilon_T^{\mu\nu}g_6\Big\}(x_1,x_2,x_3), && (5.100)\\
=-\langle\mathcal{T}^{\mu\lambda\nu}(z_1,z_3,z_2)\rangle &\sim -\Big\{\tilde s^\mu g_T^{\lambda\nu}g_1 + \tilde s^\lambda g_T^{\nu\mu}g_2 + \tilde s^\nu g_T^{\mu\lambda}g_3 + s^\mu\epsilon_T^{\lambda\nu}g_4 + s^\lambda\epsilon_T^{\nu\mu}g_5 + s^\nu\epsilon_T^{\mu\lambda}g_6\Big\}(x_1,x_3,x_2), && (5.101)\\
=-\langle\mathcal{T}^{\nu\mu\lambda}(z_2,z_1,z_3)\rangle &\sim -\Big\{\tilde s^\nu g_T^{\mu\lambda}g_1 + \tilde s^\mu g_T^{\lambda\nu}g_2 + \tilde s^\lambda g_T^{\nu\mu}g_3 + s^\nu\epsilon_T^{\mu\lambda}g_4 + s^\mu\epsilon_T^{\lambda\nu}g_5 + s^\lambda\epsilon_T^{\nu\mu}g_6\Big\}(x_2,x_1,x_3), && (5.102)\\
=\langle\mathcal{T}^{\nu\lambda\mu}(z_2,z_3,z_1)\rangle &\sim \Big\{\tilde s^\nu g_T^{\lambda\mu}g_1 + \tilde s^\lambda g_T^{\lambda\mu}g_2 + \tilde s^\mu g_T^{\nu\lambda}g_3 + s^\nu\epsilon_T^{\lambda\mu}g_4 + s^\lambda\epsilon_T^{\mu\nu}g_5 + s^\mu\epsilon_T^{\nu\lambda}g_6\Big\}(x_2,x_3,x_1), && (5.103)\\
=\langle\mathcal{T}^{\lambda\mu\nu}(z_3,z_1,z_2)\rangle &\sim \Big\{\tilde s^\lambda g_T^{\mu\nu}g_1 + \tilde s^\mu g_T^{\nu\lambda}g_2 + \tilde s^\nu g_T^{\lambda\mu}g_3 + s^\lambda\epsilon_T^{\mu\nu}g_4 + s^\mu\epsilon_T^{\nu\lambda}g_5 + s^\nu\epsilon_T^{\lambda\mu}g_6\Big\}(x_3,x_1,x_2), && (5.104)\\
=-\langle\mathcal{T}^{\lambda\nu\mu}(z_3,z_2,z_1)\rangle &\sim -\Big\{\tilde s^\lambda g_T^{\nu\mu}g_1 + \tilde s^\nu g_T^{\mu\lambda}g_2 + \tilde s^\mu g_T^{\lambda\nu}g_3 + s^\lambda\epsilon_T^{\nu\mu}g_4 + s^\nu\epsilon_T^{\mu\lambda}g_5 + s^\mu\epsilon_T^{\lambda\nu}g_6\Big\}(x_3,x_2,x_1) && (5.105)
\end{aligned}$$

All these lines have the same Fourier exponent. Such a system has more equations then needed, but I keep it for cross-check. Comparing the same structure I get

$$\begin{aligned}
\tilde s^\mu g_T^{\nu\lambda}: &\quad g_1(x_{123}) = -g_1(x_{132}) = -g_2(x_{213}) = g_3(x_{231}) = g_2(x_{312}) = -g_3(x_{321}), && (5.106)\\
\tilde s^\nu g_T^{\mu\lambda}: &\quad g_2(x_{123}) = -g_3(x_{132}) = -g_1(x_{213}) = g_1(x_{231}) = g_3(x_{312}) = -g_2(x_{321}), && (5.107)\\
\tilde s^\lambda g_T^{\mu\nu}: &\quad g_3(x_{123}) = -g_2(x_{132}) = -g_3(x_{213}) = g_2(x_{231}) = g_3(x_{312}) = -g_1(x_{321}), && (5.108)\\
\tilde s^\mu \epsilon_T^{\nu\lambda}: &\quad g_4(x_{123}) = g_4(x_{132}) = g_5(x_{213}) = g_6(x_{231}) = g_5(x_{312}) = g_6(x_{321}), && (5.109)\\
\tilde s^\nu \epsilon_T^{\mu\lambda}: &\quad g_5(x_{123}) = g_6(x_{132}) = g_4(x_{213}) = g_4(x_{231}) = g_6(x_{312}) = g_5(x_{321}), && (5.110)\\
\tilde s^\lambda \epsilon_T^{\mu\nu}: &\quad g_6(x_{123}) = g_5(x_{132}) = g_6(x_{213}) = g_5(x_{231}) = g_4(x_{312}) = g_4(x_{321}), && (5.111)
\end{aligned}$$

where $(x_{ijk})=(x_i,x_j,x_k)$. It implies that we can rewrite this expression in the terms of two functions. For definiteness we select $g_2=G$ and $g_5=\Delta G$.

We have

$$\begin{aligned}
\langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = P_+^3 M\int [dx] e^{-ip^+x_iz_i}\Big\{ & \qquad (5.112)\\
\tilde s^\nu g_T^{\lambda\mu} G(x_1,x_2,x_3) - \tilde s^\mu g_T^{\nu\lambda} G(x_2,x_1,x_3) - \tilde s^\lambda g_T^{\mu\nu} G(x_1,x_3,x_2) &\\
+ s^\nu\epsilon_T^{\lambda\mu}\Delta G(x_1,x_2,x_3) + s^\mu\epsilon_T^{\nu\lambda}\Delta G(x_2,x_1,x_3) + s^\lambda\epsilon_T^{\mu\nu}\Delta G(x_1,x_3,x_2)\Big\}. &
\end{aligned}$$

The functions G and ΔG are subjects of symmetry

$$G(x_1,x_2,x_3) = -G(x_3,x_2,x_1), \qquad \Delta G(x_1,x_2,x_3) = \Delta G(x_3,x_2,x_1). \tag{5.113}$$

The T-reversal and hermiticity implies

$$T: \qquad g_i(x_1,x_2,x_3) = -g_i(-x_1,-x_2,-x_3), \tag{5.114}$$
$$H: \qquad g_i^*(x_1,x_2,x_3) = -g_i(-x_1,-x_2,-x_3). \tag{5.115}$$

It gives that g_i is a real function. For function G and ΔG we have

$$G(x_1,x_2,x_3) = G(-x_3,-x_2,-x_1), \qquad \Delta G(x_1,x_2,x_3) = -\Delta G(-x_3,-x_2,-x_1). \tag{5.116}$$

Together with the permutation property (5.113) we get the rule for reflection of arguments

$$G(x_1,x_2,x_3) = -G(-x_1,-x_2,-x_3), \qquad \Delta G(x_1,x_2,x_3) = -\Delta G(-x_1,-x_2,-x_3). \tag{5.117}$$

Let me consider the following convolutions

$$\tilde{s}_\nu g_{\mu\lambda} \quad \langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = P_+^3 M \int [dx] e^{-ip^+x_iz_i}\Big\{ \tag{5.118}$$
$$-2G(x_1,x_2,x_3) + G(x_2,x_1,x_3) + G(x_1,x_3,x_2) - \Delta G(x_2,x_1,x_3) + \Delta G(x_1,x_3,x_2)\Big\},$$
$$\tilde{s}_\nu \epsilon_{\mu\lambda} \quad \langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = 0, \tag{5.119}$$
$$s_\nu \epsilon_{\mu\lambda} \quad \langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle = P_+^3 M \int [dx] e^{-ip^+x_iz_i}\Big\{ \tag{5.120}$$
$$G(x_2,x_1,x_3) - G(x_1,x_3,x_2) + 2\Delta G(x_1,x_2,x_3) - \Delta G(x_2,x_1,x_3) - \Delta G(x_1,x_3,x_2)\Big\}.$$

Following [BMP] we define

$$T_{3F}^+(x_1,x_2,x_3) = -2G(x_1,x_2,x_3) + G(x_2,x_1,x_3) + G(x_1,x_3,x_2) - \Delta G(x_2,x_1,x_3) + \Delta G(x_1,x_3,x_2), \tag{5.121}$$
$$\Delta T_{3F}^+(x_1,x_2,x_3) = G(x_2,x_1,x_3) - G(x_1,x_3,x_2) + 2\Delta G(x_1,x_2,x_3) - \Delta G(x_2,x_1,x_3) - \Delta G(x_1,x_3,x_2). \tag{5.122}$$

These functions are not independent. Straightforward to check that

$$\Delta T_{3F}^+(x_1,x_2,x_3) = T_{3F}^+(x_1,x_3,x_2) - T_{3F}^+(x_2,x_1,x_3), \tag{5.123}$$

compare to (32) of [BMP]. Using symmetries (5.116,5.117) we obtain

$$T_{3F}^+(x_1,x_2,x_3) = T_{3F}^+(-x_3,-x_2,-x_1), \qquad \Delta T_{3F}^+(x_1,x_2,x_3) = -\Delta T_{3F}^+(-x_3,-x_2,-x_1), \tag{5.124}$$
$$T_{3F}^+(x_1,x_2,x_3) = -T_{3F}^+(-x_1,-x_2,-x_3), \qquad \Delta T_{3F}^+(x_1,x_2,x_3) = -\Delta T_{3F}^+(-x_1,-x_2,-x_3). \tag{5.125}$$

That also implies the permutation relation

$$T_{3F}^+(x_1,x_2,x_3) = -T_{3F}^+(x_3,x_2,x_1), \qquad \Delta T_{3F}^+(x_1,x_2,x_3) = \Delta T_{3F}^+(x_3,x_2,x_1). \tag{5.126}$$

We note that these relations are more restrictive if one of the arguments x_i is zero. In this case the rest two arguments are $x_j = -x_k$ (since $x_i + x_j + x_k = 0$).

Thus we have

$$T_{3F}^+(-\xi,0,\xi) = -T_{3F}^+(\xi,0,-\xi), \tag{5.127}$$
$$T_{3F}^+(0,\xi,-\xi) = -T_{3F}^+(0,-\xi,\xi) = T_{3F}^+(\xi,-\xi,0) = -T_{3F}^+(-\xi,\xi,0). \tag{5.128}$$

J. The matrix elements of T

In the calculation we face combinations $\mathcal{T}^{\mu(\rho)\mu}$ and $\mathcal{T}^{\mu(\mu)\rho}+\mathcal{T}^{\rho(\mu)\mu}$. As it was shown only QS-element contributes. Let us find the relevant structure. We have

$$g_{\mu\lambda}\langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle=\tilde{s}^\nu P_+^3 M\int[dx]e^{-ip_+x_iz_i}\Big\{ \tag{5.129}$$
$$2G(x_1,x_2,x_3)-G(x_2,x_1,x_3)-G(x_1,x_3,x_2)+\Delta G(x_2,x_1,x_3)-\Delta G(x_1,x_3,x_2)\Big\}$$
$$=-\tilde{s}^\nu P_+^3 M\int[dx]e^{-ip_+x_iz_i}T_{3F}^+(x_1,x_2,x_3),$$
$$g_{\mu\nu}\langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle=\tilde{s}^\lambda P_+^3 M\int[dx]e^{-ip_+x_iz_i}T_{3F}^+(x_1,x_3,x_2)$$
$$g_{\nu\lambda}\langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle=\tilde{s}^\lambda P_+^3 M\int[dx]e^{-ip_+x_iz_i}T_{3F}^+(x_2,x_1,x_3)$$

$$(g_{\rho\lambda}g_{\mu\nu}+g_{\rho\mu}g_{\nu\lambda})\langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle=\tilde{s}^\rho P_+^3 M\int[dx]e^{-ip_+x_iz_i}\Big\{ \tag{5.130}$$
$$2G(x_1,x_2,x_3)-3G(x_2,x_1,x_3)-3G(x_1,x_3,x_2)-\Delta G(x_2,x_1,x_3)+\Delta G(x_1,x_3,x_2)\Big\}$$
$$=\tilde{s}^\rho P_+^3 M\int[dx]e^{-ip_+x_iz_i}\left(T_{3F}^+(x_1,x_3,x_2)+T_{3F}^+(x_2,x_1,x_3)\right).$$

We also need

$$b^\mu b^\nu b^\lambda\langle P,S|\mathcal{T}^{\mu\nu\lambda}(z_1,z_2,z_3)|P,S\rangle=(\tilde{s}^\nu b_\nu)b^2P_+^3 M\int[dx]e^{-ip_+x_iz_i}\Big\{ \tag{5.131}$$
$$G(x_1,x_2,x_3)-G(x_2,x_1,x_3)-G(x_1,x_3,x_2)\Big\}$$
$$=(\tilde{s}^\nu b_\nu)b^2P_+^3 M\int[dx]e^{-ip_+x_iz_i}\frac{-T_{3F}^+(x_1,x_2,x_3)+T_{3F}^+(x_2,x_1,x_3)+T_{3F}^+(x_1,x_3,x_2)}{4}.$$

VI. EVALUATION IN THE BACKGROUND FIELD

A. QCD in the background field

To perform the calculation in the background technique we first need the Lagrangian. The QCD Lagrangian reads

$$\mathcal{L}_{QCD} = \bar{q}(i\not{D})q + \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a. \tag{6.1}$$

We remind

$$\overrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - igA_\mu, \qquad \overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + igA_\mu. \tag{6.2}$$

$$F_{\mu\nu} = \frac{i}{g}\left[D_\mu, D_\nu\right] = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \tag{6.3}$$
$$F^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + gf^{ABC}A^B_\mu A^C_\nu,$$

We introduce the notion of background and dynamical modes of fields and denote these fields as

$$\text{background} \;:\; q,\; A,$$
$$\text{quantum} \;:\; \psi,\; B.$$

I.e. in the original Lagrangian we make the substitution

$$A_\mu \to A_\mu + B_\mu, \qquad q \to q + \psi, \qquad \bar{q} \to \bar{q} + \bar{\psi}. \tag{6.4}$$

The quark sector of the Lagrangian turns into

$$\mathcal{L}_{\bar{q}Aq} \to \mathcal{L}_{\bar{q}Aq} + \mathcal{L}_{\bar{\psi}B\psi} + g\Big[\bar{q}\not{B}\psi + \bar{\psi}\not{B}q + \bar{\psi}\not{A}\psi\Big] + \Big\{\bar{q}i\not{D}_A\psi + \bar{\psi}i\not{D}_A q + g\bar{q}\not{B}q\Big\}, \tag{6.5}$$

where subscripts denote the field content of the structure (i.e. $\not{D}_A = \not{\partial} - ig\not{A}$). Here the terms in square brackets are quadratic in dynamical fields. The terms in curly brackets are linear in dynamical fields.

To deal with the gauge part is trickier, since we have to take care on the gauge fixation condition. We follow refs.[31, 32]. In the background field we have

$$F^A_{\mu\nu}[A+B] = F^A_{\mu\nu}[A] + F^A_{\mu\nu}[B] + gf^{ABC}A^B_\mu B^C_\nu + gf^{ABC}B^B_\mu A^C_\nu \tag{6.6}$$

We are going to use the background gauge fixation condition for the quantum field.

$$G^A = D^\mu B^A_\mu = \partial_\mu B^A_\mu + gf^{ABC}A^B_\mu B^C_\mu. \tag{6.7}$$

Making the gauge transformation (it incorporates the background field to keep Lagrangian invariant)

$$B^A_\mu \to B^A_\mu - f^{ABC}\omega^B(A^C_\mu + B^C_\mu) + \frac{1}{g}\partial_\mu\omega^A + \mathcal{O}(\omega^2),$$

we derive

$$G^A \;\to\; G^A + \frac{1}{g}\Big[\partial^2\delta^{AB} + f^{ACB}\overrightarrow{\partial_\mu}(A^C_\mu + B^C_\mu) + f^{ACB}A^c_\mu\partial_\mu + gf^{AC\alpha}f^{\alpha DB}A^C_\mu(A^D_\mu + B^D_\mu)\Big]\omega^B + \mathcal{O}(\omega^2). \tag{6.8}$$

Therefore, the ghost Lagrangian reads

$$\mathcal{L}_{ghost} = -\eta^\dagger_A\Big[\partial^2\delta^{AB} - f^{ACB}\overleftarrow{\partial_\mu}(A^C_\mu + B^C_\mu) + f^{ACB}A^c_\mu\partial_\mu + gf^{AC\alpha}f^{\alpha DB}A^C_\mu(A^D_\mu + B^D_\mu)\Big]\eta_B. \tag{6.9}$$

The gauge fixation term reads

$$\mathcal{L}_{fix} = -\frac{1}{2\alpha}G_A G^A = -\frac{1}{2\alpha}B^A_\mu\Big(-\partial_\mu\partial_\nu + g\overleftarrow{\partial_\mu}f^{ACB}A^C_\nu - gf^{ACB}A^C_\mu\partial_\nu - g^2 f^{AC\alpha}f^{\alpha DB}A^C_\mu A^D_\nu\Big)B^B_\nu \tag{6.10}$$

The Lagrangian of pure Yang-Mils reads

$$\mathcal{L}_{YM} \;\to\; \mathcal{L}_A+\mathcal{L}_B+\Big\{2F^A_{\mu\nu}[A](\partial_\mu B^A_\nu-\partial_\nu B^A_\mu)+4gF^A_{\mu\nu}[A]f^{ABC}A^B_\mu B^C_\nu\Big\} \tag{6.11}$$
$$+\frac{g}{2}\Big[F^A_{\mu\nu}[A]f^{ABC}B^B_\mu B^C_\nu+2F^A_{\mu\nu}[B]f^{ABC}A^B_\mu B^C_\nu-gf^{AB\alpha}f^{\alpha CD}A^A_\mu B^B_\nu(A^C_\mu B^D_\nu+B^C_\mu A^D_\nu)\Big].$$

Here again the terms in curly brackets are linear in the background field.

The terms linear in the backgroud field collects into the EOMS of QCD, $\delta\mathcal{L}/\delta\psi+\delta\mathcal{L}/\delta\bar\psi+\delta\mathcal{L}/\delta B=0$, and therefore, could be dropped from the consideration. The final expression for the Lagrangian reads

$$\mathcal{L} \;=\; \mathcal{L}[q,A]+\mathcal{L}[\psi,B]+\delta\mathcal{L}, \tag{6.12}$$

where $\delta\mathcal{L}$ is the iteration between dynamical and background fields. It reads

$$\delta\mathcal{L} \;=\; g\Big[\bar q\,\not{B}\psi+\bar\psi\,\not{B}q+\bar\psi\,\not{A}\psi\Big]+\mathcal{L}_{ABB}+\mathcal{L}_{AABB}+\mathcal{L}_{ABBB}, \tag{6.13}$$

where $\mathcal{L}_F$ represent the pure gluon interaction terms with the field content F. In particular,

$$\mathcal{L}_{ABB} \;=\; gf^{ABC}\Big\{(\partial_\nu A^A_\mu)B^B_\nu B^C_\mu+A^A_\mu\Big[(\partial_\nu B^B_\mu)B^C_\nu-(\partial_\mu B^B_\nu)B^C_\nu-\frac{1}{\alpha}(\partial_\nu B^B_\nu)B^C_\mu\Big]\Big\} \tag{6.14}$$
$$\;=\; gf^{ABC}A^A_\mu\Big\{2(\partial_\nu B^B_\mu)B^C_\nu-(\partial_\mu B^B_\nu)B^C_\nu-\frac{1+\alpha}{\alpha}(\partial_\nu B^B_\nu)B^C_\mu\Big\}, \tag{6.15}$$

where in the second line we have extracted the total derivative. The gauge is fixed with $\alpha=1$. The Feynman rules derived from this expression seems to coincide with the one given in [31]. The convenient form to write this expression reads

$$\mathcal{L}_{ABB} \;=\; gf^{ABC}A^A_\mu(\partial_\alpha B^B_\beta)B^C_\gamma\left(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\right). \tag{6.16}$$

B. Gauge conditions for the background field

The convenient side of the background technique is that the gauge condition for the background field is independent in the condition of the dynamical field. We use the light-cone gauge

$$A^+(x)=0 \tag{6.17}$$

with boundary condition different for different processes

$$\text{DY:}\qquad A(-n\infty)=0,\qquad\qquad \text{SIDIS:}\qquad A(+n\infty)=0. \tag{6.18}$$

Consequently,

$$\text{DY:}\qquad A^\mu(x)=-\int_{-\infty}^0 d\lambda\;F^{\mu+}(\lambda n+x),\qquad\qquad \text{SIDIS:}\qquad A^\mu(x)=\int_0^\infty d\lambda\;F^{\mu+}(\lambda n+x), \tag{6.19}$$
$$\text{DY:}\qquad \partial_+A^\mu(x)=-F^{\mu+}(x),\qquad\qquad \text{SIDIS:}\qquad A^\mu(x)=-F^{\mu+}(x). \tag{6.20}$$

We note that the expansion of Wilson lines in the light-cone background reads

$$\begin{aligned}
[y,-\infty n] &\to [y,-\infty n]_{A+B} \qquad (6.21)\\
&= \underbrace{[y,-\infty n]_A}_{=1} + ig\int_{-\infty}^{0} d\sigma n^\mu \underbrace{[y,y+n\sigma]_A}_{=1} B_\mu(y+n\sigma)\underbrace{[y+n\sigma,-n\infty]_A}_{=1} + \mathcal{O}(B^2)\\
&= 1+ig\int_{-\infty}^{0} d\sigma n^\mu B_\mu(y+n\sigma)+\mathcal{O}(B^2),\\
[-\infty n,y] &\to [-\infty n,y]_{A+B} \qquad (6.22)\\
&= \underbrace{[-\infty n,y]_A}_{=1} + ig\int_{0}^{-\infty} d\sigma n^\mu \underbrace{[-\infty n,y+n\sigma]_A}_{=1} B_\mu(y+n\sigma)\underbrace{[y+n\sigma,y]_A}_{=1} + \mathcal{O}(B^2)\\
&= 1+ig\int_{0}^{-\infty} d\sigma n^\mu B_\mu(y+n\sigma)+\mathcal{O}(B^2).
\end{aligned}$$

The propagators are

$$\overbrace{\psi_i(x)\bar\psi_j}(0) = \int \frac{d^dk}{(2\pi)^d}e^{-ikx}\frac{i\not k_{ij}}{k^2+i0} = \frac{\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\not x_{ij}}{(-x^2+i0)^{2-\epsilon}} \qquad (6.23)$$

$$\overbrace{B^a_\mu(x)B^b_\nu}(0) = \int \frac{d^dk}{(2\pi)^d}e^{-ikx}\frac{-i}{k^2+i0} = \frac{\Gamma(1-\epsilon)}{4\pi^{d/2}}\frac{-g_{\mu\nu}\delta^{ab}}{(-x^2+i0)^{1-\epsilon}}, \qquad (6.24)$$

$$(\partial_\alpha \overbrace{B^a_\mu(x))B^b_\nu}(0) = \int \frac{d^dk}{(2\pi)^d}e^{-ikx}\frac{-k_\alpha}{k^2+i0} = \frac{\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-g_{\mu\nu}\delta^{ab}x_\alpha}{(-x^2+i0)^{1-\epsilon}}. \qquad (6.25)$$

Also every action term comes with factor i. I.e. the vertex is $i\int d^dx\mathcal{L}(x)$.

With the arbitrary gauge fixing parameter one has

$$\begin{aligned}
\overbrace{B^a_\mu(x)B^b_\nu}(0) &= \int \frac{d^dk}{(2\pi)^d}e^{-ikx}\frac{-i}{k^2+i0}\left(g^{\mu\nu}-\xi\frac{k^\mu k^\nu}{k^2+i0}\right) \qquad (6.26)\\
&= \frac{-\delta^{ab}}{4\pi^{d/2}}\left[\left(1-\frac{\xi}{2}\right)\frac{g^{\mu\nu}\Gamma(1-\epsilon)}{(-x^2+i0)^{1-\epsilon}}-\xi\frac{x^\mu x^\nu\Gamma(2-\epsilon)}{(-x^2+i0)^{2-\epsilon}}\right].
\end{aligned}$$

One can check that $\partial_\mu\Delta^{\mu\nu}\sim(1-\xi)$ and that $\Delta^{\mu\nu}\sim\Delta^{\mu\nu}(\xi=0)$.

C. OPE in the background field: general

We consider the distribution

$$\Phi(x,\boldsymbol{b}) = \langle p|\mathcal{U}(x,\boldsymbol{b})|p\rangle = \int[d\bar q dq dA_\mu]\Psi_p^\dagger\mathcal{U}(x,\boldsymbol{b})\Psi_p e^{i\int\mathcal{L}}, \qquad (6.27)$$

where Ψ_p represents the field configuration which is a hadron state. In the background field we expect that the states are built of only "slow" field, i.e. from the background fields. Thus we can integrate the dynamical modes. Making the slip $q\to q+\psi$ and $A\to A+B$ we get

$$\Phi(x,\boldsymbol{b}) = \int[d\bar q dq dA]_\mu[d\bar\psi d\psi dB]_\mu\Psi_p^\dagger[q,A]\mathcal{U}[q+\psi,A+B](x,\boldsymbol{b})\Psi_p[q,A]e^{i\int\mathcal{L}[q,A]+\mathcal{L}[\psi,B]+\delta\mathcal{L}}, \qquad (6.28)$$

where the integration measures are complimentary to each other, and properly normalized. Here we expect that the infrequences of background fields are less then μ, $\omega_A<\mu$, while frequencies of dynamical field are large, $\omega_B>\mu$. It can be rewritten as

$$\Phi(x,\boldsymbol{b}) = \langle p|\mathcal{U}^{eff}(x,\boldsymbol{b})|p\rangle, \qquad (6.29)$$

where

$$\mathcal{U}^{eff}(x,\boldsymbol{b}) \;=\; \int [d\bar{\psi}d\psi dB]_\mu \; \mathcal{U}[q+\psi,A+B](x,\boldsymbol{b}) \; e^{\mathcal{L}[\psi,B]+\delta\mathcal{L}}. \tag{6.30}$$

Here we can extend the integration to the full range of integration [check the Balitskii ref], with μ being the renomalization scale.

D. OPE in the background: structure of LO and NLO

Our operator has a particular form as $\sim \bar{q}[..]_A q$, where $[..]$ is a Wilson lines. Moreover since we work in the light-cone gauge for field A and our Wilson lines are on the light-cone, $[..]_{A+B}=[..]_B$. Therefore, we have 4 term with different background content. Schematically they are

$$\mathcal{U}[q+\psi,A+B] = \bar{q}[..]_B q + \bar{q}[..]_B\psi + \bar{\psi}[..]_B q + \bar{\psi}[..]_B\psi. \tag{6.31}$$

We remind that in the perturbation theory the integration over field B in the Wilson lines happens order by order in expansion, i.e. $[..]_B \sim 1+gB+g^2B^2+...$ (see (6.21,6.22)). Also the action exponent is to be expanded

$$e^{\int \mathcal{L}[\psi,B]+\delta\mathcal{L}} = e^{iS_0}(1+\int d^dx\mathcal{L}[\psi,B]+\int d^dx\delta\mathcal{L}+...),$$

where S_0 is the free action that defines the propagator.

Let us consider (6.31) term-by-term due to their contribution into the $\mathcal{U}^{eff}$, at LO and NLO' ($\sim g^3$) in the orders of coupling constant.

The first term: The expansion up to NLO'

$$\bar{q}[..]_B q = \underbrace{\bar{q}..q}_{\text{tree}} + \underbrace{g\bar{q}..B..q}_{=0} + \underbrace{g^2\bar{q}..BB..q}_{=0} + \underbrace{g^3\bar{q}..BBB..q}_{=0}... \; .$$

The second term is equal to zero due to fact that it is connected to the tadpole part, and cannot be connected to background fields (at NLO). The third and the forth terms are zero since it has B_+B_+, which vanish under the contraction by propagator. Thus this term has only tree part. Moreover, it is clear that it could not contribute at higher pertrubative orders (except possible high-twist multi-parton operators, which is also unlikely).

The second term: It expansion

$$\bar{q}[..]_B\psi = \underbrace{\bar{q}..\psi}_{=0} + \underbrace{g\bar{q}..B..\psi}_{(1)} + \underbrace{g^2\bar{q}..BB..\psi}_{NNLO} + \underbrace{g^3\bar{q}..BBB..\psi}_{NNLO} + ... \; .$$

The fist term is zero due to the single field coupling. The last two terms contribute to NNLO since at least 2 vertices are required to couple together $BB\psi$. The term (1) can be contracted at NLO in three ways

$$\bar{q}[..]_B\psi = \underbrace{g\bar{q}..B..\psi\delta\mathcal{L}_{\bar{\psi}Bq}}_{=\mathbf{A}\sim g^2} + \underbrace{g\bar{q}..B..\psi\delta\mathcal{L}_{ABB}\delta\mathcal{L}_{\bar{\psi}Bq}}_{=\mathbf{C}\sim g^3} + \underbrace{g\bar{q}..B..\psi\delta\mathcal{L}_{\bar{\psi}A\psi}\delta\mathcal{L}_{\bar{\psi}Bq}}_{=\mathbf{D}\sim g^3} + O(g^4)\;, \tag{6.32}$$

where bold letters denote the diagrams names.

The third term: It is literally reproduces the structure of the second term. Thus only the final result

$$\bar{\psi}[..]_B q = \underbrace{g\bar{\psi}..B..q\delta\mathcal{L}_{\bar{q}B\psi}}_{=\mathbf{A}^*\sim g^2} + \underbrace{g\bar{\psi}..B..q\delta\mathcal{L}_{ABB}\delta\mathcal{L}_{\bar{q}B\psi}}_{=\mathbf{C}^*\sim g^3} + \underbrace{g\bar{\psi}..B..q\delta\mathcal{L}_{\bar{\psi}A\psi}\delta\mathcal{L}_{\bar{q}B\psi}}_{=\mathbf{D}^*\sim g^3} + O(g^4)\;, \tag{6.33}$$

where bold letters denote the diagrams names.

The fourth term: It expansions

$$\bar{\psi}[..]_B\psi = \underbrace{\bar{\psi}..\psi}_{(1)} + \underbrace{g\bar{\psi}..B..\psi}_{NNLO} + \underbrace{g^2\bar{\psi}..BB..\psi}_{NNLO} + ... \; .$$

The second term is NNLO, since at least two couplings are required to couple B to ψ's. The same for higher terms. The term (1) can be contracted at NLO in three ways

$$\bar{p}si[..]_B\psi = \underbrace{g\bar{q}..\psi\delta\mathcal{L}_{\bar{\psi}Bq}\delta\mathcal{L}_{\bar{q}B\psi}}_{=\mathbf{B}\sim g^2} + \underbrace{g\bar{\psi}..\psi\delta\mathcal{L}_{ABB}\delta\mathcal{L}_{\bar{\psi}Bq}\delta\mathcal{L}_{\bar{q}B\psi}}_{=\mathbf{E}\sim g^3} + \underbrace{g\bar{\psi}..\psi\delta\mathcal{L}_{\bar{\psi}A\psi}\delta\mathcal{L}_{\bar{\psi}Bq}\delta\mathcal{L}_{\bar{q}B\psi}}_{=\mathbf{F}\sim g^3} + O(g^4)\;, \tag{6.34}$$

where bold letters denote the diagrams names.

The explicit expression for these diagrams and their evaluation is given in the section VIII.

VII. COMPENDIUM OF DEFINITIONS

A. Operators

For convenience we generalize the TMD operators (2.1,2.3)

$$\mathcal{U}^{\Gamma}_{\rm DY}(z_1,z_2;\boldsymbol{b}) = \bar q(z_1n+\boldsymbol{b})[z_1n+\boldsymbol{b},-\infty n+\boldsymbol{b}]\Gamma[-\infty n-\boldsymbol{b},z_2n-\boldsymbol{b}]q(z_2n-\boldsymbol{b}), \tag{7.1}$$

$$\mathcal{U}^{\Gamma}_{\rm DIS}(z_1,z_2;\boldsymbol{b}) = \bar q(z_1n+\boldsymbol{b})[z_1n+\boldsymbol{b},+\infty n+\boldsymbol{b}]\Gamma[+\infty n-\boldsymbol{b},z_2n-\boldsymbol{b}]q(z_2n-\boldsymbol{b}). \tag{7.2}$$

Also we have collinear operators

$$\mathcal{O}_\Gamma(z_1,z_2) = \bar q(z_1n)[z_1n,z_2n]\Gamma q(z_2n), \tag{7.3}$$

$$\mathcal{T}^{\mu}_{\Gamma}(z_1,z_2,z_3) = g\bar q(z_1n)[z_1n,z_2n]\Gamma F^{\mu+}(z_2n)[z_2n,z_3n]q(z_3n). \tag{7.4}$$

The relation to (2.1,2.3,2.17,2.18) is trivial

$$\mathcal{U}(z,\boldsymbol{b})=\mathcal{U}(z,-z;\boldsymbol{b}),\qquad O_\Gamma(z)=\mathcal{O}_\Gamma(z,-z). \tag{7.5}$$

We also introduce (see also (2.13,2.14))

$$\mathcal{O}^{\mu}_{\Gamma;DY}(z_1,z_2) = \frac{\partial}{\partial b_\mu}\mathcal{U}^{\Gamma}_{\rm DY}(z_1,z_2;\boldsymbol{b})\Big|_{\boldsymbol{b}=0} \tag{7.6}$$

$$\mathcal{O}^{\mu}_{\Gamma;DIS}(z_1,z_2) = \frac{\partial}{\partial b_\mu}\mathcal{U}^{\Gamma}_{\rm DIS}(z_1,z_2;\boldsymbol{b})\Big|_{\boldsymbol{b}=0} \tag{7.7}$$

Thus

$$\mathcal{U}^{\Gamma} = \mathcal{O}_\Gamma(z_1,z_2)+b_\mu\mathcal{O}^{\mu}_{\Gamma}(z_1,z_2)+\mathcal{O}(\boldsymbol{b}^2) \tag{7.8}$$

B. Loop integrals

The integral over y appears in the diagrams with 2 propagators. It reads (**take care on the general sign of the sign due to** $+i0$)

$$\checkmark\int\frac{y^{\mu_1}...y^{\mu_{2n}}}{(-y^2+X+i0)^{3-2\epsilon}} = -i\pi^{d/2}\frac{\Gamma(1-\epsilon-n)}{\Gamma(3-2\epsilon)}\frac{(-1)^n g_s^{\mu_1...\mu_{2n}}}{2^nX^{1-\epsilon-n}}, \tag{7.9}$$

$$\checkmark\int\frac{\overbrace{y^{\mu_1}...y^{\mu_{2n+1}}}^{\text{odd }\#}}{(-y^2+X+i0)^{3-2\epsilon}} = 0. \tag{7.10}$$

In particular,

$$\int\frac{y^\mu y^\nu}{(-y^2+X+i0)^{3-2\epsilon}} = i\pi^{d/2}\frac{\Gamma(-\epsilon)}{\Gamma(3-2\epsilon)}\frac{g^{\mu\nu}}{2}X^{\epsilon}. \tag{7.11}$$

The integrals over x and y reads

$$\int d^dxd^dy\frac{\{x^\mu y^\nu,x^\mu x^\nu,y^\mu y^\nu,1\}}{(-\bar\beta x^2-\bar\alpha y^2+2\gamma(xy)+X+i0)^{5-3\epsilon}} = \frac{\pi^d\Gamma(-\epsilon)}{\Gamma(5-3\epsilon)}\frac{X^\epsilon}{\lambda^{2-\epsilon}}\{\frac{\gamma}{\lambda}\frac{g^{\mu\nu}}{2},\frac{\bar\alpha}{\lambda}\frac{g^{\mu\nu}}{2},\frac{\bar\beta}{\lambda}\frac{g^{\mu\nu}}{2},\frac{\epsilon}{X}\} \tag{7.12}$$

$$\int d^dxd^dy\frac{\overbrace{\{x^\mu..y^\nu\}}^{\text{odd }\#}}{(-\bar\beta x^2-\bar\alpha y^2+2\gamma(xy)+X+i0)^{5-3\epsilon}} = 0 \tag{7.13}$$

The integrals over x and y and u reads

✓ $$\int d^dx d^dy d^du \frac{1}{[-(\gamma+\beta)x^2-(\rho+\gamma)y^2-(\alpha+\rho)u^2+2\gamma(xy)+2\rho(yu)+X]^{7-4\epsilon}} = \frac{-i\pi^{3d/2}\Gamma(-\epsilon)}{\Gamma(7-4\epsilon)}\frac{\epsilon\lambda^{\epsilon-2}}{X^{1-\epsilon}} \quad (7.14)$$

✓ $$\int d^dx d^dy d^du \frac{\{x^\mu x^\nu, y^\mu y^\nu, u^\mu u^\nu, x^\mu y^\nu x^\mu u^\nu, y^\mu u^\nu\}}{[-(\gamma+\beta)x^2-(\rho+\gamma)y^2-(\alpha+\rho)u^2+2\gamma(xy)+2\rho(yu)+X]^{7-4\epsilon}}$$
$$= \frac{-i\pi^{3d/2}\Gamma(-\epsilon)}{\Gamma(7-4\epsilon)}\frac{\lambda^{\epsilon-3}}{X^{-\epsilon}}\frac{g^{\mu\nu}}{2}\{\alpha\rho+\alpha\gamma+\rho\gamma,\ (\alpha+\rho)(\gamma+\beta),\ \beta\rho+\rho\gamma+\gamma\beta,\ (\alpha+\rho)\gamma,\ \rho\gamma,\ \rho(\gamma+\beta)\}$$

$$\int d^dx d^dy d^du \frac{\overbrace{\{x^\mu..y^\nu\}}^{\text{odd }\#}}{[-(\gamma+\beta)x^2-(\rho+\gamma)y^2-(\alpha+\rho)u^2+2\gamma(xy)+2\rho(yu)+4\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2]^{7-4\epsilon}} = 0 \quad (7.15)$$

The integral over y reads (**take care on the general sign of the sign due to** $+i0$)

$$\int d^dy \frac{y^{\mu_1}...y^{\mu_{2n}}}{(-y^2+X+i0)^{4-2\epsilon}} = -i\pi^{d/2}\frac{\Gamma(2-\epsilon-n)}{\Gamma(4-2\epsilon)}\frac{(-1)^n g_s^{\mu_1\cdots\mu_{2n}}}{2^n X^{2-\epsilon-n}}. \quad (7.16)$$

VIII. EVALUATION OF DIAGRAMS

We use the standard notation

$$\mathbf{B} = \boldsymbol{b}^2 = -b^2 > 0, \qquad a_s = \frac{g^2}{(4\pi)^{d/2}}. \tag{8.1}$$

$$z_{ij}^{\alpha} = \bar{\alpha} z_i + \alpha z_j. \tag{8.2}$$

Moreover if, z_i or related variable is used as a vector, i.e. $\not{z}_i$, it implies $z_i n^\mu$.
In the following we do not drop the total derivatives. However, we do drop them in the next section.

A. Virtual diagrams

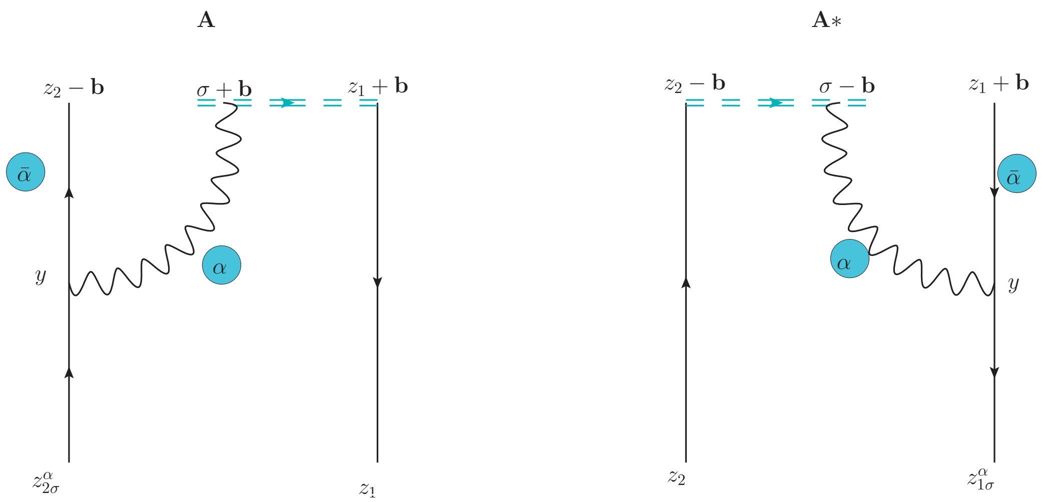
There are virtual-like parts of the diagrams **A**, **C**, **D**. Here we imply the contraction with the Wilson line that start from the dynamical field. These diagrams are zero due to the simple fact that there is no Lorentz invariant scale. So, the loop integral comes out proportional to $0^\epsilon = 0$.

IX. DIAGRAMS A

A. Diagram A

The diagram reads

$$\mathbf{A} \;=\; \Big\{\bar{q}(z_1 n + \boldsymbol{b})\Big[ig\int_{-\infty}^{z_1} d\sigma n^\mu t^A B_\mu^A(n\sigma + \boldsymbol{b})\Big]\gamma^+\psi(z_2 n - \boldsymbol{b})\Big\}\Big(ig\int d^d y \bar{\psi}(y)\ \not{B}(y) q(y)\Big) \tag{9.1}$$



Step I: Substituting the expressions

$$\mathbf{A} \;=\; (ig)^2 \frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}} \frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}} \int_{-\infty}^{z_1} d\sigma \int d^d y \bar{q}(z_1 n + \boldsymbol{b}) t^A t^A \frac{n_\mu \gamma^+(\gamma^+ z_2 - \not{b} - \not{y})\gamma^\mu}{(-(y - nz_2 + \boldsymbol{b})^2 + i0)^{2-\epsilon}(-(y - n\sigma - \boldsymbol{b})^2 + i0)^{1-\epsilon}} q(y) \tag{9.2}$$

Step II: Simplify a bit

$$\begin{aligned}\mathbf{A} &= -ig^2C_F\frac{\Gamma(2-\epsilon)\Gamma(1-\epsilon)}{8\pi^d} \\ &\int_{-\infty}^{z_1}d\sigma\int d^dy\bar q(z_1n+\boldsymbol{b})\frac{2\gamma^+y^+}{(-(y-nz_2+\boldsymbol{b})^2+i0)^{2-\epsilon}(-(y-n\sigma-\boldsymbol{b})^2+i0)^{1-\epsilon}}q(y),\end{aligned} \tag{9.3}$$

Here, we have used that $\gamma^+(\gamma^+z_2-\not{b}-\not{y})\gamma^+=-\gamma^+\not{y}\gamma^+=-2y^+\gamma^+$.

Step III:

We introduce Feynman parameters

$$\begin{aligned}(y-n\sigma-\boldsymbol{b})^2+i0 &\to \alpha\\ (y-nz_2+\boldsymbol{b})^2+i0 &\to \bar\alpha\end{aligned}$$

And make a shift

$$y\to y+n(\alpha\sigma+\bar\alpha z_2)-(1-2\alpha)\boldsymbol{b}=y+n(z^\alpha_{2\sigma})-(1-2\alpha)\boldsymbol{b}.$$

We obtain ($\boldsymbol{b}^2=-b^2>0$)

$$\mathbf{A} = -ig^2C_F\frac{\Gamma(3-2\epsilon)}{4\pi^d}\int_{-\infty}^{z_1}d\sigma\int d^dy\int_0^1d\alpha\bar q(z_1n+\boldsymbol{b})\frac{\gamma^+y^+\alpha^{-\epsilon}\bar\alpha^{1-\epsilon}}{(-y^2+4\alpha\bar\alpha\boldsymbol{b}^2+i0)^{3-2\epsilon}}q(y+nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}). \tag{9.4}$$

Step IV:

Fields are to be expanded up to the first order in y^μ. The higher orders will result into higher twist-contributions since will contain ∂^2's. So we restrict our self to

$$\begin{aligned}\mathbf{A} &= -ig^2C_F\frac{\Gamma(3-2\epsilon)}{4\pi^d}\\ &\int_{-\infty}^{z_1}d\sigma\int d^dy\int_0^1d\alpha\bar q(z_1n+\boldsymbol{b})\frac{\gamma^+y^+\alpha^{-\epsilon}\bar\alpha^{1-\epsilon}}{(-y^2+4\alpha\bar\alpha\boldsymbol{b}^2+i0)^{3-2\epsilon}}(1+y^\mu\overrightarrow{\partial_\mu})q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}).\end{aligned} \tag{9.5}$$

Step V: The term $\sim y^\mu y^\nu$ is not zero, while the $\sim y^\mu$ is zero. Therefore, only the term with the derivative survives.

$$\mathbf{A} = -ig^2C_F\frac{\Gamma(3-2\epsilon)}{4\pi^d}\int_{-\infty}^{z_1}d\sigma\int d^dy\int_0^1d\alpha\bar q(z_1n+\boldsymbol{b})\frac{\gamma^+y^+y^\mu\alpha^{-\epsilon}\bar\alpha^{1-\epsilon}}{(-y^2+4\alpha\bar\alpha\boldsymbol{b}^2+i0)^{3-2\epsilon}}\overrightarrow{\partial_\mu}q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}). \tag{9.6}$$

Note, that next non-zero term has three derivatives, it corresponds to at least $\boldsymbol{b}^2\partial^2q$ contribution.

Step VI: Evaluating integral with (7.11)

$$\begin{aligned}\mathbf{A} &= -ig^2C_F\frac{i\pi^{d/2}4^\epsilon}{4\pi^d}\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{z_1}d\sigma\int_0^1d\alpha\bar q(nz_1+\boldsymbol{b})\frac{\gamma^+n^\mu}{2}(\alpha\bar\alpha)^\epsilon\alpha^{-\epsilon}\bar\alpha^{1-\epsilon}\overrightarrow{\partial_\mu}q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}) \quad (9.7)\\ &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{z_1}d\sigma\int_0^1d\alpha\,\bar\alpha\,\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_+}q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}).\end{aligned}$$

Step VII: We rewrite the ∂_+ as the derivative with respect to σ. Note, the the SIDIS case is similar but with different integral over σ.

Thus we have

$$\text{DY:}\qquad \mathbf{A} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{z_1}d\sigma\int_0^1d\alpha\,\frac{\bar\alpha}{\alpha}\,\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_\sigma}q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}), \tag{9.8}$$

$$\text{SIDIS:}\qquad \mathbf{A} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{+\infty}^{z_1}d\sigma\int_0^1d\alpha\,\frac{\bar\alpha}{\alpha}\,\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_\sigma}q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}). \tag{9.9}$$

B. Diagram A^*

The evaluation of this diagram nearly identical to the diagram **A**. The diagram reads

$$\mathbf{A}^* = \Big(ig\int d^dy\bar q(y)\,\not{B}(y)\psi(y)\Big)\Big(\bar\psi(z_1+\boldsymbol{b})\Big[-ig\int_{-\infty}^{z_2}d\sigma n^\mu t^A B_\mu^A(n\sigma-\boldsymbol{b})\Big]\gamma^+q(z_2-\boldsymbol{b})\Big) \tag{9.10}$$

In the following we repeat steps without comments **Step I:**

$$\begin{aligned}\mathbf{A}^* &= i(-i)g^2C_F\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}}\\ &\int_{-\infty}^{z_2}d\sigma\int d^dy\bar q(y)t^At^A\frac{\gamma^\mu(\not{y}-\not{z}_1-\not{b})\gamma^+n_\mu}{(-(y-z_1n-\boldsymbol{b})^2+i0)^{2-\epsilon}(-(y-n\sigma+\boldsymbol{b})^2+i0)^{1-\epsilon}}q(z_2-\boldsymbol{b})\end{aligned} \tag{9.11}$$

✓**Step II:**

$$\begin{aligned}\mathbf{A}^* &= -ig^2C_F\frac{\Gamma(2-\epsilon)\Gamma(1-\epsilon)}{8\pi^d}\\ &\int_{-\infty}^{z_2}d\sigma\int d^dy\bar q(y)\frac{2\gamma^+y^+}{(-(y-z_1-\boldsymbol{b})^2+i0)^{2-\epsilon}(-(y-\sigma+\boldsymbol{b})^2+i0)^{1-\epsilon}}q(z_2-\boldsymbol{b}).\end{aligned} \tag{9.12}$$

Step III:
We introduce Feynman parameters

$$\begin{aligned}(y-nz_1-\boldsymbol{b})^2+i0 &\to \bar\alpha\\ (y-n\sigma+\boldsymbol{b})^2+i0 &\to \alpha\end{aligned}$$

And make a shift

$$y\to y+n(\alpha\sigma+\bar\alpha z_1)+(1-2\alpha)\boldsymbol{b}=y+nz_{1\sigma}^\alpha+(1-2\alpha)\boldsymbol{b}.$$

We obtain

$$\mathbf{A}^* = -ig^2C_F\frac{\Gamma(3-2\epsilon)}{4\pi^d}\int_{-\infty}^{z_2}d\sigma\int d^dy\int_0^1 d\alpha\bar q(y+nz_{1\sigma}^\alpha+(1-2\alpha)\boldsymbol{b})\frac{\gamma^+y^+\alpha^{-\epsilon}\bar\alpha^{1-\epsilon}}{(-y^2+4\alpha\bar\alpha\boldsymbol{b}^2+i0)^{3-2\epsilon}}q(z_2-\boldsymbol{b}). \tag{9.13}$$

Step V:

$$\mathbf{A}^* = -ig^2C_F\frac{\Gamma(3-2\epsilon)}{4\pi^d}\int_{-\infty}^{z_2}d\sigma\int d^dy\int_0^1 d\alpha\bar q(nz_{1\sigma}^\alpha+(1-2\alpha)\boldsymbol{b})\overleftarrow{\partial}_\mu\frac{\gamma^+y^+y^\mu\alpha^{-\epsilon}\bar\alpha^{1-\epsilon}}{(-y^2+4\alpha\bar\alpha\boldsymbol{b}^2+i0)^{3-2\epsilon}}q(z_2-\boldsymbol{b}). \tag{9.14}$$

Step VI:

$$\mathbf{A}^* = 2a_sC_F\mathbf{B}^\epsilon\Gamma(-\epsilon)\int_{-\infty}^{z_2}d\sigma\int_0^1 d\alpha\,\bar\alpha\;\bar q(nz_{1\sigma}^\alpha+(1-2\alpha)\boldsymbol{b})\overleftarrow{\partial}_+\gamma^+q(z_2-\boldsymbol{b}). \tag{9.15}$$

✓**Step VII:**

DY $$\mathbf{A}^* = 2a_sC_F\mathbf{B}^\epsilon\Gamma(-\epsilon)\int_{-\infty}^{z_2}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\;\bar q(z_{1\sigma}^\alpha+(1-2\alpha)\boldsymbol{b})\overleftarrow{\partial}_\sigma\gamma^+q(z_2-\boldsymbol{b}), \tag{9.16}$$

SIDIS $$\mathbf{A}^* = 2a_sC_F\mathbf{B}^\epsilon\Gamma(-\epsilon)\int_{+\infty}^{z_2}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\;\bar q(z_{1\sigma}^\alpha+(1-2\alpha)\boldsymbol{b})\overleftarrow{\partial}_\sigma\gamma^+q(z_2-\boldsymbol{b}). \tag{9.17}$$

C. Rapidity divergences renormalization and diagrams A

The diagrams $\mathbf{A}$ and $\mathbf{A}^*$ contain rapidity divergences. Let us elaborate it. We start form the DY case

$$\mathbf{A} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{z_1}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\;\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_\sigma}q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}), \tag{9.18}$$

$$\mathbf{A}^* = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{z_2}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\;\bar q(nz^\alpha_{1\sigma}+(1-2\alpha)\boldsymbol{b})\overleftarrow{\partial_\sigma}\gamma^+q(nz_2-\boldsymbol{b}). \tag{9.19}$$

The limits $\sigma\to\infty$ and $\alpha\to 0$ to not commute. Therefore, this integral is ambiguous. In fact, it is rapidity divergent.
We simplify the extraction of the rapidity divergences, we split the integrals over σ as

$$\int_{-\infty}^{z_1}d\sigma=\int_{-\infty}^{z_2}d\sigma+\int_{z_2}^{z_1}d\sigma,\qquad \int_{-\infty}^{z_2}d\sigma=\int_{-\infty}^{z_1}d\sigma+\int_{z_1}^{z_2}d\sigma.$$

The integrals over the finite domain could be taken without any tricks, using integration by parts. We have

$$\mathbf{A}_{\text{reg}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big[\bar q(nz_1+\boldsymbol{b})\gamma^+q(nz^\alpha_{21}-(1-2\alpha)\boldsymbol{b})-\bar q(nz_1+\boldsymbol{b})\gamma^+q(nz_2-(1-2\alpha)\boldsymbol{b})\Big], \tag{9.20}$$

$$\mathbf{A}^*{}_{\text{reg}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big[\bar q(nz^\alpha_{12}+(1-2\alpha)\boldsymbol{b})\gamma^+q(nz_2-\boldsymbol{b})-\bar q(nz_1+(1-2\alpha)\boldsymbol{b})\gamma^+q(nz_2-\boldsymbol{b})\Big]. \tag{9.21}$$

To evaluate the singular part we use the δ-regularization, as

$$\mathbf{A}_{\text{sing}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{z_2}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\sigma}\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_\sigma}q(nz^\alpha_{2\sigma}-(1-2\alpha)\boldsymbol{b}), \tag{9.22}$$

$$\mathbf{A}^*{}_{\text{sing}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{z_1}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\sigma}\bar q(nz^\alpha_{1\sigma}+(1-2\alpha)\boldsymbol{b})\overleftarrow{\partial_\sigma}\gamma^+q(nz_2-\boldsymbol{b}). \tag{9.23}$$

We make the change of variables

$$\text{for }\mathbf{A}\quad \tau=(\sigma-z_2)\alpha,\qquad\qquad \text{for }\mathbf{A}^*\quad \tau=(\sigma-z_1)\alpha. \tag{9.24}$$

We obtain

$$\mathbf{A}_{\text{sing}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{0}d\tau\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\frac{\tau}{\alpha}}e^{\delta z_2}\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_\tau}q(nz_2+n\tau-(1-2\alpha)\boldsymbol{b}), \tag{9.25}$$

$$\mathbf{A}^*{}_{\text{sing}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_{-\infty}^{0}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\frac{\tau}{\alpha}}e^{\delta z_1}\bar q(nz_1+n\tau+(1-2\alpha)\boldsymbol{b})\overleftarrow{\partial_\tau}\gamma^+q(nz_2-\boldsymbol{b}). \tag{9.26}$$

Here the exponents $e^{\delta z_{1,2}}$ could be dropped, since they produce power corrections in δ. We extract the singular part at $\alpha\to 0$

$$\begin{aligned}\mathbf{A}_{\text{sing}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Bigg\{&\int_{-\infty}^{0}d\tau\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\frac{\tau}{\alpha}}\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_\tau}\Big[q(nz_2+n\tau-(1-2\alpha)\boldsymbol{b})-q(nz_2+n\tau-\boldsymbol{b})\Big]\\ &+\int_{-\infty}^{0}d\tau\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\frac{\tau}{\alpha}}\bar q(nz_1+\boldsymbol{b})\gamma^+\overrightarrow{\partial_\tau}q(nz_2+n\tau-\boldsymbol{b})\Bigg\},\end{aligned} \tag{9.27}$$

$$\begin{aligned}\mathbf{A}^*{}_{\text{sing}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Bigg\{&\int_{-\infty}^{0}d\sigma\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\frac{\tau}{\alpha}}\Big[\bar q(nz_1+n\tau+(1-2\alpha)\boldsymbol{b})-\bar q(nz_1+n\tau+\boldsymbol{b})\Big]\overleftarrow{\partial_\tau}\gamma^+q(nz_2-\boldsymbol{b})\\ &+\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}e^{\delta\frac{\tau}{\alpha}}\bar q(nz_1+n\tau+\boldsymbol{b})\overleftarrow{\partial_\tau}\gamma^+q(nz_2-\boldsymbol{b})\Bigg\}.\end{aligned} \tag{9.28}$$

The first terms are regular at $\alpha\to 0$ and therefore, integral over τ can be taken immediately with $\delta=0$. In the last terms we integrate over α. We have

$$\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}e^{\frac{\delta}{\alpha}\tau}\simeq(-1-\gamma_E-\ln(\tau\delta))+O(\delta). \tag{9.29}$$

Then

$$\int_{-\infty}^{0} d\tau \left(-1-\gamma_E-\ln(\tau\delta)\right)\partial_\tau q(\tau) = \left(-1-\ln\frac{\delta}{\nu^+}\right)q(0), \tag{9.30}$$

where ν^+ is parameters that defines the rapidity renormalization. In particular, in the standard definition of rapidity renormalization scheme $\nu^+ = p^+$. Thus we have

$$\begin{aligned}\mathbf{A}_{\text{sing}} &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big[\bar q(nz_1+\boldsymbol{b})\gamma^+q(nz_2-(1-2\alpha)\boldsymbol{b})-\bar q(nz_1+\boldsymbol{b})\gamma^+q(nz_2-\boldsymbol{b})\Big]\\&\qquad+\left(-1-\ln\frac{\delta}{\nu^+}\right)\bar q(nz_1+\boldsymbol{b})\gamma^+q(nz_2-\boldsymbol{b})\Big\}, \qquad (9.31)\\ \mathbf{A}^*{}_{\text{sing}} &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{\int_0^1 d\alpha\,\frac{\bar\alpha}{\alpha}\Big[\bar q(nz_1+(1-2\alpha)\boldsymbol{b})\gamma^+q(nz_2-\boldsymbol{b})-\bar q(nz_1+\boldsymbol{b})\gamma^+q(nz_2-\boldsymbol{b})\Big]\\&\qquad+\left(-1-\ln\frac{\delta}{\nu^+}\right)\bar q(nz_1+\boldsymbol{b})\gamma^+q(nz_2-\boldsymbol{b})\Big\}. \qquad (9.32)\end{aligned}$$

Important that SIDIS kinematics gives the same structure.

We add the total shifts in $\boldsymbol{b}$-directions and write the final result

$$\begin{aligned}\mathbf{A} &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\mathcal{U}^{\gamma^+}(z_1,z_{21}^\alpha;\bar\alpha\boldsymbol{b})-\mathcal{U}^{\gamma^+}(z_1,z_2;\boldsymbol{b})\Big]-(1+\boldsymbol{\lambda}_\delta)\mathcal{U}^{\gamma^+}(z_1,z_2;\boldsymbol{b})\Big\}, \qquad (9.33)\\ \mathbf{A}^* &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\mathcal{U}^{\gamma^+}(z_{12}^\alpha,z_2;\bar\alpha\boldsymbol{b})-\mathcal{U}^{\gamma^+}(z_1,z_2;\boldsymbol{b})\Big]-(1+\boldsymbol{\lambda}_\delta)\mathcal{U}^{\gamma^+}(z_1,z_2;\boldsymbol{b})\Big\}. \qquad (9.34)\end{aligned}$$

It is the same for SIDIS and DY kinematics. Also it is simple to check that for any Γ it is the same.

Using the definition (7.8) we obtain at $O(\boldsymbol{b}^2)$

$$\begin{aligned}\mathbf{A} &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\mathcal{O}_{\gamma^+}(z_1,z_{21}^\alpha)-\mathcal{O}_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}_{\gamma^+}(z_1,z_2) \qquad (9.35)\\&\quad+b_\mu\Big(\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\bar\alpha\mathcal{O}^\mu_{\gamma^+}(z_1,z_{21}^\alpha)-\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big)+O(\boldsymbol{b}^2)\Big\},\\ \mathbf{A}^* &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\mathcal{O}_{\gamma^+}(z_{12}^\alpha,z_2)-\mathcal{O}_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}_{\gamma^+}(z_1,z_2) \qquad (9.36)\\&\quad+b_\mu\Big(\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\bar\alpha\mathcal{O}^\mu_{\gamma^+}(z_{12}^\alpha,z_2)-\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big)+O(\boldsymbol{b}^2)\Big\},\end{aligned}$$

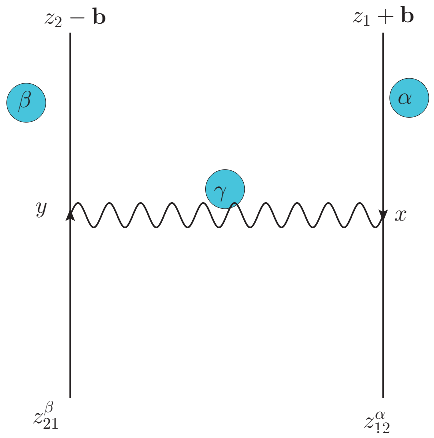
X. DIAGRAM B

A. Diagram B

The diagram has the form

$$\mathbf{B} = \left(ig\int d^dx\bar q(x)\,\not{B}(x)\psi(x)\right)\left(\bar\psi(z_1n+\boldsymbol{b})\gamma^+\psi(z_2n-\boldsymbol{b})\right)\left(ig\int d^dy\bar\psi(y)\,\not{B}(y)q(y)\right) \tag{10.1}$$

B



Step I: Substituting the propagators we obtain

$$\mathbf{B} = (ig)^2 \frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}} \frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}} \frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}} \int d^dx d^dy \tag{10.2}$$
$$\bar{q}(x) \frac{t^A \gamma^\mu (\not{x} - \gamma^+ z_1 - \not{b})\gamma^+(\gamma^+ z_2 - \not{b} - \not{y})\gamma^\mu t^A}{[-(x-z_1n-b)^2+i0]^{2-\epsilon}[-(z_2n-b-y)^2+i0]^{2-\epsilon}[-(x-y)^2+i0]^{1-\epsilon}} q(y)$$

Step II: Simplifying

$$\mathbf{B} = g^2 C_F \frac{\Gamma^2(2-\epsilon)\Gamma(1-\epsilon)}{16\pi^{3d/2}} \int d^dx d^dy \tag{10.3}$$
$$\bar{q}(x) \frac{\gamma^\mu (\not{x} - \not{b})\gamma^+(\not{y} + \not{b})\gamma^\mu}{[-(x-z_1n-b)^2+i0]^{2-\epsilon}[-(z_2n-b-y)^2+i0]^{2-\epsilon}[-(x-y)^2+i0]^{1-\epsilon}} q(y)$$

Step III: We introduce Feynman parameters according

$$\begin{aligned} (x-z_1n-b)^2 &\to \alpha \\ (z_2n-b-y)^2 &\to \beta \\ (x-y)^2 &\to 1-\alpha-\beta=\gamma \end{aligned}$$

Next we shift

$$\begin{aligned} x &\to x + \frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1 - 2\frac{\beta\gamma}{\lambda}\right), \\ y &\to y + \frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma}{\lambda}\right), \\ \lambda &= \alpha\beta+\beta\gamma+\gamma\alpha. \end{aligned} \tag{10.4}$$

$$\mathbf{B} = g^2 C_F \frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}} \int d^dx d^dy \int [d\alpha d\beta d\gamma] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon} \tag{10.5}$$
$$\bar{q}(x + \frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1 - 2\frac{\beta\gamma}{\lambda}\right)) \frac{\gamma^\mu(\not{x} - 2\frac{\gamma\beta}{\lambda}\not{b})\gamma^+(\not{y} + 2\frac{\alpha\gamma}{\lambda}\not{b})\gamma^\mu}{[-\bar{\beta}x^2 - \bar{\alpha}y^2 + 2\gamma(xy) + \frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2 + i0]^{5-3\epsilon}}$$
$$q(y + \frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma}{\lambda}\right))$$

Step IV: We need to expand up to one derivative. It is simple to check that 2-derivative term contributes to $\sim \boldsymbol{b}^2$. To make the calculation clearer we split diagram to two parts with and without derivatives of fields.

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1, \tag{10.6}$$

where

$$\mathbf{B}_0 = g^2 C_F \frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}} \int d^d x d^d y \int [d\alpha d\beta d\gamma] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \quad (10.7)$$
$$\frac{\gamma^\mu(\not{x} - 2\frac{\gamma\beta}{\lambda}\ \not{b})\gamma^+(\not{y} + 2\frac{\alpha\gamma}{\lambda}\ \not{b})\gamma_\mu}{[-\bar{\beta}x^2 - \bar{\alpha}y^2 + 2\gamma(xy) + \frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2 + i0]^{5-3\epsilon}} q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)),$$

$$\mathbf{B}_1 = g^2 C_F \frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}} \int d^d x d^d y \int [d\alpha d\beta d\gamma] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \quad (10.8)$$
$$\frac{\gamma^\mu(\not{x} - 2\frac{\gamma\beta}{\lambda}\ \not{b})\gamma^+(\not{y} + 2\frac{\alpha\gamma}{\lambda}\ \not{b})\gamma_\mu}{[-\bar{\beta}x^2 - \bar{\alpha}y^2 + 2\gamma(xy) + \frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2 + i0]^{5-3\epsilon}} (x^\nu \overleftarrow{\partial_\nu} + y^\nu \overrightarrow{\partial_\nu}) q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).$$

Step V: We leave only even powers of xy

$$\mathbf{B}_0 = g^2 C_F \frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}} \int d^d x d^d y \int [d\alpha d\beta d\gamma] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \quad (10.9)$$
$$\frac{\gamma^\mu[\not{x}\gamma^+\ \not{y} - 4\frac{\alpha\beta\gamma^2}{\lambda^2}\ \not{b}\gamma^+\ \not{b}]\gamma_\mu}{[-\bar{\beta}x^2 - \bar{\alpha}y^2 + 2\gamma(xy) + \frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2 + i0]^{5-3\epsilon}} q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)),$$

$$\mathbf{B}_1 = g^2 C_F \frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}} \int d^d x d^d y \int [d\alpha d\beta d\gamma] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \quad (10.10)$$
$$\frac{\gamma^\mu[2\frac{\alpha\gamma}{\lambda}\ \not{x}\gamma^+\ \not{b} - 2\frac{\gamma\beta}{\lambda}\ \not{b}\gamma^+\ \not{y}]\gamma_\mu}{[-\bar{\beta}x^2 - \bar{\alpha}y^2 + 2\gamma(xy) + \frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2 + i0]^{5-3\epsilon}} (x^\nu \overleftarrow{\partial_\nu} + y^\nu \overrightarrow{\partial_\nu}) q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).$$

Step VI: We integrate using (7.12)

$$\begin{aligned}
\mathbf{B}_0 &= g^2 C_F \frac{\pi^d \Gamma(-\epsilon)}{16\pi^{3d/2}} \int [d\alpha d\beta d\gamma] \frac{\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}}{\lambda^{2-\epsilon}} \frac{4^\epsilon(\alpha\beta\gamma)^\epsilon}{\lambda^\epsilon} \mathbf{B}^\epsilon\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \quad (10.11)\\
&\quad \gamma^\mu[\frac{\gamma}{2\lambda}\gamma^\nu\gamma^+\gamma_\nu - 4\epsilon\frac{\alpha\beta\gamma^2}{\lambda^2}\frac{\lambda}{4\alpha\beta\gamma\boldsymbol{b}^2}\ \not{b}\gamma^+\ \not{b}]\gamma_\mu\, q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)),\\
&= a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma] \frac{\alpha\beta\gamma}{\lambda^3}\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))\\
&\quad \gamma^\mu[\frac{1}{2}\gamma^\nu\gamma^+\gamma_\nu - \epsilon\frac{1}{\boldsymbol{b}^2}\ \not{b}\gamma^+\ \not{b}]\gamma_\mu\, q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)),
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_1 &= g^2 C_F \frac{\Gamma(-\epsilon)}{16\pi^{3d/2}} \int [d\alpha d\beta d\gamma] \frac{\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}}{\lambda^{2-\epsilon}} \frac{4^\epsilon(\alpha\beta\gamma)^\epsilon}{\lambda^\epsilon} \mathbf{B}^\epsilon\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \quad (10.12)\\
&\quad \gamma^\mu[2\frac{\alpha\gamma}{\lambda}\frac{\bar{\alpha}}{2\lambda}\overleftarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} + 2\frac{\alpha\gamma}{\lambda}\frac{\gamma}{2\lambda}\overrightarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} - 2\frac{\gamma\beta}{\lambda}\frac{\gamma}{2\lambda}\overleftarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu - 2\frac{\gamma\beta}{\lambda}\frac{\bar{\beta}}{2\lambda}\overrightarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu]\gamma_\mu\\
&\qquad q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))\\
&= a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma] \frac{\alpha\beta\gamma}{\lambda^3}\ \bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda} z_1 n + \frac{\beta\gamma}{\lambda} z_2 n + \boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))\\
&\quad \gamma^\mu[\frac{\alpha\bar{\alpha}}{\lambda}\overleftarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} + \frac{\alpha\gamma}{\lambda}\overrightarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} - \frac{\beta\gamma}{\lambda}\overleftarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu - \frac{\beta\bar{\beta}}{\lambda}\overrightarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu]\gamma_\mu\, q(\frac{\alpha\beta+\beta\gamma}{\lambda} z_2 n + \frac{\alpha\gamma}{\lambda} z_1 n - \boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).
\end{aligned}$$

Step VII:

The convenient change to the dual variables. They defined as

$$\frac{\beta\gamma}{\lambda} = \alpha', \qquad \frac{\alpha\gamma}{\lambda} = \beta', \qquad 0 < \beta' < \bar{\alpha}', \qquad 0 < \alpha' < 1. \tag{10.13}$$

$$\frac{\alpha\beta}{\lambda} = \gamma', \qquad \alpha + \beta + \gamma = 1 \tag{10.14}$$

$$J = \frac{d\alpha d\beta}{d\alpha' d\beta'} = \frac{\lambda^3}{\alpha\beta\gamma} = \frac{\alpha'\beta'\gamma'}{\lambda'^3}. \tag{10.15}$$

It also implies

$$\frac{\alpha\beta + \alpha\gamma}{\lambda} = \frac{\alpha\bar{\alpha}}{\lambda} = \bar{\alpha}', \qquad \frac{\alpha\beta + \beta\gamma}{\lambda} = \frac{\beta\bar{\beta}}{\lambda} = \bar{\beta}'. \tag{10.16}$$

We change the variables and drop the primes.

$$\mathbf{B}_0 = a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\ \bar{q}(z_{12}^\alpha n + \boldsymbol{b}\,(1-2\alpha))\gamma^\mu \Big[\frac{1}{2}\gamma^\nu\gamma^+\gamma_\nu - \epsilon\frac{1}{\boldsymbol{b}^2}\ \not{b}\gamma^+\ \not{b}\Big]\gamma_\mu\, q(z_{21}^\beta n - \boldsymbol{b}\,(1-2\beta)), \tag{10.17}$$

$$\begin{aligned}\mathbf{B}_1 = {}& a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\ \bar{q}(z_{12}^\alpha n + \boldsymbol{b}\,(1-2\alpha)) \\ & \gamma^\mu \Big[\bar{\alpha}\overleftarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} + \beta\overrightarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} - \alpha\overleftarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu - \bar{\beta}\overrightarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu\Big]\gamma_\mu\, q(z_{21}^\beta n - \boldsymbol{b}\,(1-2\beta)).\end{aligned} \tag{10.18}$$

Step VIII: We evaluate Dirac algebra

$$\mathbf{B}_0 = 2(1-\epsilon)a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\ \bar{q}(z_{12}^\alpha n + \boldsymbol{b}\,(1-2\alpha))\gamma^+\, q(z_{21}^\beta n - \boldsymbol{b}\,(1-2\beta)), \tag{10.19}$$

$$\begin{aligned}\mathbf{B}_1 = {}& 2a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\ \bar{q}(z_{12}^\alpha n + \boldsymbol{b}\,(1-2\alpha)) \\ & \Big[(\alpha+\epsilon\bar{\alpha})\overleftarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} + (\bar{\beta}+\epsilon\beta)\overrightarrow{\partial_\nu}\gamma^\nu\gamma^+\ \not{b} - (\bar{\alpha}+\epsilon\alpha)\overleftarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu - (\beta+\epsilon\bar{\beta})\overrightarrow{\partial_\nu}\ \not{b}\gamma^+\gamma^\nu\Big]\, q(z_{21}^\beta n - \boldsymbol{b}\,(1-2\beta)).\end{aligned} \tag{10.20}$$

B. Refining the diagram B

We start form the part $\mathbf{B}_0$. Dropping the total shift we get

$$\mathbf{B}_0 = 2(1-\epsilon)a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\mathcal{U}^{\gamma^+}(z_{12}^\alpha, z_{21}^\beta; \gamma\boldsymbol{b}). \tag{10.21}$$

Or

$$\mathbf{B}_0 = 2(1-\epsilon)a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\Big[\mathcal{O}_{\gamma^+}(z_{12}^\alpha, z_{21}^\beta) + \gamma b_\mu \mathcal{O}_{\gamma^+}^\mu(z_{12}^\alpha, z_{21}^\beta) + O(\boldsymbol{b}^2)\Big]. \tag{10.22}$$

To simplify the part $\mathbf{B}_1$ we mention that

$$(\alpha+\epsilon\bar{\alpha})\overleftarrow{\partial_\nu} + (\bar{\beta}+\epsilon\beta)\overrightarrow{\partial_\nu} = \frac{\epsilon-1}{2}\gamma\left(\overleftarrow{\partial_\nu} - \overrightarrow{\partial_\nu}\right) + \frac{1}{2}(1+\epsilon+\alpha(1-\epsilon)-\beta(1-\epsilon))\left(\overleftarrow{\partial_\nu} + \overrightarrow{\partial_\nu}\right) \tag{10.23}$$

$$-(\bar{\alpha}+\epsilon\alpha)\overleftarrow{\partial_\nu} + (\beta+\epsilon\bar{\beta})\overrightarrow{\partial_\nu} = \frac{\epsilon-1}{2}\gamma\left(\overleftarrow{\partial_\nu} - \overrightarrow{\partial_\nu}\right) - \frac{1}{2}(1+\epsilon-\alpha(1-\epsilon)+\beta(1-\epsilon))\left(\overleftarrow{\partial_\nu} + \overrightarrow{\partial_\nu}\right). \tag{10.24}$$

Therefore, dropping the total derivatives and corrections in $\boldsymbol{b}$ in the arguments of fields we get

$$\mathbf{B}_1 = -(1-\epsilon)a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\, \gamma\, \bar{q}(z_{12}^\alpha n)(\overleftarrow{\partial_\nu} - \overrightarrow{\partial_\nu})\Big[\gamma^\nu \gamma^+ \not{b} + \not{b}\gamma^+\gamma^\nu\Big]\, q(z_{21}^\beta n). \tag{10.25}$$

We simplify

$$\gamma^\nu \gamma^+ \not{b} + \not{b}\gamma^+\gamma^\nu = 2n^\nu \not{b} - 2b^\nu \gamma^+, \tag{10.26}$$

and get

$$\mathbf{B}_1 = 2(1-\epsilon)a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\, \gamma\, \bar{q}(z_{12}^\alpha n)(\overleftarrow{\partial_\nu} - \overrightarrow{\partial_\nu})\Big[b^\nu \gamma^+ - n^\nu \not{b}\Big]\, q(z_{21}^\beta n). \tag{10.27}$$

The transverse part is precisely $\mathcal{O}^\mu$. The light-cone derivatives can be simplified

$$\bar{q}(z_{12}^\alpha n)(\overleftarrow{\partial_+} - \overrightarrow{\partial_+})\not{b} q(z_{21}^\beta n) = \frac{\partial_\alpha + \partial_\beta}{z_2 - z_1}\bar{q}(z_{12}^\alpha n)\not{b} q(z_{21}^\beta n). \tag{10.28}$$

Integrals over α and β could be taken by integration by parts. We have

$$\begin{aligned}\int [d\alpha d\beta](1-\alpha-\beta)(\partial_\alpha + \partial_\beta)\bar{q}(z_{12}^\alpha n)\not{b} q(z_{21}^\beta n) &= 2\int [d\alpha d\beta]\bar{q}(z_{12}^\alpha n)\not{b} q(z_{21}^\beta n) \\ &\quad - \int_0^1 d\alpha \bar{\alpha}\left(\bar{q}(z_{12}^\alpha n)\not{b} q(z_2 n) + \bar{q}(z_1 n)\not{b} q(z_{21}^\alpha n)\right)\end{aligned} \tag{10.29}$$

Thus combining together we obtain

$$\mathbf{B}_1 = 2(1-\epsilon)a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta] b_\mu \left[\gamma\, \mathcal{O}^\mu_{\gamma^+}(z_{12}^\alpha, z_{21}^\beta) - \frac{2 - \bar{\alpha}\delta(\beta) - \bar{\beta}\delta(\alpha)}{z_2 - z_1}\mathcal{O}_{\gamma^\mu}(z_{12}^\alpha, z_{21}^\beta)\right] + O(\boldsymbol{b}^2) \tag{10.30}$$

The complete expression for the diagram $\mathbf{B}$ is

$$\begin{aligned}\mathbf{B} &= 2(1-\epsilon)a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta]\Bigg[\mathcal{O}_{\gamma^+}(z_{12}^\alpha, z_{21}^\beta) + 2\gamma b_\mu \mathcal{O}^\mu_{\gamma^+}(z_{12}^\alpha, z_{21}^\beta) \\ &\quad - b_\mu \frac{2 - \bar{\alpha}\delta(\beta) - \bar{\beta}\delta(\alpha)}{z_2 - z_1}\mathcal{O}_{\gamma^\mu}(z_{12}^\alpha, z_{21}^\beta)\Bigg] + O(\boldsymbol{b}^2).\end{aligned} \tag{10.31}$$

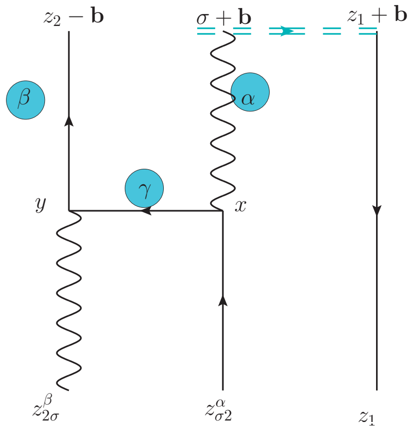
XI. DIAGRAM C

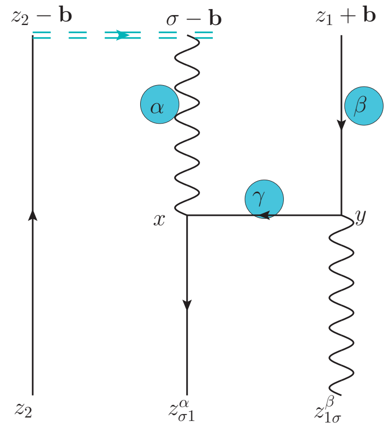
A. Diagram C

The diagram is

$$\mathbf{C} = \Big\{\bar{q}(z_1 n + \boldsymbol{b})[ig\int_{-\infty}^{z_1} d\sigma n^\mu t^A B_\mu^A(n\sigma + \boldsymbol{b})]\gamma^+ \psi(z_2 n - \boldsymbol{b})\Big\}\Big(ig\int d^d y \bar{\psi}(y)\not{A}(y)\psi(y)\Big)\Big(ig\int d^d x \bar{\psi}(x)\not{B}(x) q(x)\Big)$$

C **C**$*$





Step I: Substituting propagators

$$\begin{aligned}\mathbf{C} &= (ig)^3\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}}\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma \\ &\bar{q}(z_1+\boldsymbol{b})\left\{t^An_\mu\gamma^+\frac{\gamma^+z_2-\not{b}-\not{y}}{[-(z_2n-\boldsymbol{b}-y)^2]^{2-\epsilon}}\not{A}(y)\frac{\not{y}-\not{x}}{[-(y-x)^2]^{2-\epsilon}}\frac{t^A\gamma^\mu}{[-(\sigma n+\boldsymbol{b}-x)^2]^{1-\epsilon}}\right\}q(x).\end{aligned} \tag{11.1}$$

Step II: After minimal simplifications

$$\begin{aligned}\mathbf{C} &= ig^3\frac{\Gamma^2(2-\epsilon)\Gamma(1-\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(y)\frac{\gamma^+(\not{b}+\not{y})\gamma^\mu(\not{y}-\not{x})\gamma^+}{[-(z_2n-\boldsymbol{b}-y)^2]^{2-\epsilon}[-(\sigma n+\boldsymbol{b}-x)^2]^{1-\epsilon}[-(y-x)^2]^{2-\epsilon}}q(x).\end{aligned} \tag{11.2}$$

✓**Step III:** This loop integral is topologically similar to the diagram **B**. So, we make the similar change of variables

$$\begin{aligned}(x-\sigma n-b)^2 &= \alpha \\ (z_2n-b-y)^2 &= \beta \\ (x-y)^2 &= 1-\alpha-\beta=\gamma\end{aligned}$$

Next we shift

$$\begin{aligned}x &\to x+\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right), \\ y &\to y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right), \\ \lambda &= \alpha\beta+\beta\gamma+\gamma\alpha.\end{aligned} \tag{11.3}$$

$$\begin{aligned}\mathbf{C} &= ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon} \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)) \\ &\frac{\gamma^+(2\frac{\alpha\gamma}{\lambda}\not{b}+\not{y})\gamma^\mu(\not{y}-\not{x}-2\frac{\alpha\beta}{\lambda}\not{b})\gamma^+}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}}q(x+\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)).\end{aligned} \tag{11.4}$$

Step V: This diagram is to be expanded up to one-derivative terms. The nest expansion order gives contribution to $\boldsymbol{b}^2$ (the left derivative act on the gluon field)

$$\begin{aligned}\mathbf{C} &= ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon} \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))(1+y^\nu\overleftarrow{\partial_\nu}+x^\nu\overrightarrow{\partial_\nu}) \\ &\frac{\gamma^+(2\frac{\alpha\gamma}{\lambda}\not{b}+\not{y})\gamma^\mu(\not{y}-\not{x}-2\frac{\alpha\beta}{\lambda}\not{b})\gamma^+}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}}q(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)).\end{aligned} \tag{11.5}$$

Step V: We split this diagram into term with derivative and without derivative

$$\mathbf{C}=\mathbf{C}_0+\mathbf{C}_1 \tag{11.6}$$

The contribution with no derivative reads

$$\begin{aligned}\mathbf{C}_0 &= ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon} \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)) \\ &\left.\frac{\gamma^+\not{y}\gamma^\mu(\not{y}-\not{x})\gamma^+-4\frac{\alpha^2\beta\gamma}{\lambda^2}\gamma^+\not{b}\gamma^\mu\not{b}\gamma^+}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}}\right\}q(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)).\end{aligned} \tag{11.7}$$

This contribution is zero for the $A_+=0$. Indeed, the gamma structure gives

$$A_\mu\gamma^+\not{b}\gamma^\mu\not{b}\gamma^+=A_\mu\underbrace{\gamma^+\not{b}\gamma_T^\mu\not{b}\gamma^+}_{=\gamma^+\gamma^+=0}+\underbrace{A_+}_{=0}\gamma^+\not{b}\gamma^-\not{b}\gamma^++A_-\underbrace{\gamma^+\not{b}\gamma^+\not{b}\gamma^+}_{=\gamma^+\gamma^+=0}=0,$$

$$A_\mu\gamma^+\not{y}\gamma^\mu(\not{y}-\not{x})\gamma^+\xrightarrow{\text{loop}}A_\mu\gamma^+\gamma^\nu\gamma^\mu\gamma^\nu\gamma^+=(2-d)A_\mu\gamma^+\gamma^\mu\gamma^+=2(2-d)A_+\gamma^+=0.$$

Thus

$$\mathbf{C}_0=0,\qquad \mathbf{C}=\mathbf{C}_1.$$

Therefore, we drop this contribution and consider only the contribution with single derivative. We have

$$\begin{aligned}\mathbf{C} &= ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon} \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))(y^\nu\overleftarrow{\partial_\nu}+x^\nu\overrightarrow{\partial_\nu}) \\ &\frac{2\frac{\alpha\gamma}{\lambda}\gamma^+\not{b}\gamma^\mu(\not{y}-\not{x})\gamma^+-2\frac{\alpha\beta}{\lambda}\gamma^+\not{y}\gamma^\mu\not{b}\gamma^+}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}}q(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)).\end{aligned} \tag{11.8}$$

Step VI: We integrate with the help of (7.12)

$$\begin{aligned}\mathbf{C} &= ig^3\frac{4^\epsilon\Gamma(-\epsilon)}{16\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\frac{(\alpha\beta\gamma)^\epsilon}{2\lambda^3} \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)) \\ &\Big[2\frac{\alpha\gamma}{\lambda}(\alpha\overleftarrow{\partial_\nu}-\beta\overrightarrow{\partial_\nu})\gamma^+\not{b}\gamma^\mu\gamma^\nu\gamma^+-2\frac{\alpha\beta}{\lambda}(\bar{\beta}\overleftarrow{\partial_\nu}+\gamma\overrightarrow{\partial_\nu})\gamma^+\gamma^\nu\gamma^\mu\not{b}\gamma^+\Big]q(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \\ &= iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\frac{\alpha\beta\gamma}{\lambda^3}\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)) \\ &\Big[(\frac{\alpha\gamma}{\lambda}\overleftarrow{\partial_\nu}-\frac{\beta\gamma}{\lambda}\overrightarrow{\partial_\nu})\gamma^+\not{b}\gamma^\mu\gamma^\nu\gamma^+-(\frac{\beta\bar{\beta}}{\lambda}\overleftarrow{\partial_\nu}+\frac{\beta\gamma}{\lambda}\overrightarrow{\partial_\nu})\gamma^+\gamma^\nu\gamma^\mu\not{b}\gamma^+\Big]q(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)).\end{aligned} \tag{11.9}$$

✓**Step VII:** We change to dual variables (10.13) and drop the primes

$$\mathbf{C} = iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta]\bar{q}(z_1+\boldsymbol{b})A_\mu(nz_{2\sigma}^\beta-\boldsymbol{b}(1-2\beta)) \quad (11.10)$$
$$\Big[(\beta\overleftarrow{\partial_\nu}-\alpha\overrightarrow{\partial_\nu})\gamma^+\not{b}\gamma^\mu\gamma^\nu\gamma^+-(\bar\beta\overleftarrow{\partial_\nu}+\alpha\overrightarrow{\partial_\nu})\gamma^+\gamma^\nu\gamma^\mu\not{b}\gamma^+\Big]q(nz_{\sigma2}^\alpha+\boldsymbol{b}(1-2\alpha)).$$

Step VIII: We simplify the gamma-structure

$$\gamma^+\gamma^\nu\gamma^\mu\not{b}\gamma^+A_\mu=-\gamma^+\gamma^\nu\gamma^\mu\gamma^+\not{b}A_\mu=-\gamma^+\gamma^\nu(\underbrace{A_-\gamma^+}_{\gamma^+\gamma^+=0}+\underbrace{A_+\gamma^-}_{A_+=0}+A_\mu\gamma_\perp^\mu)\gamma^+\not{b} \quad (11.11)$$
$$=\gamma^+\gamma^\nu\gamma^+\gamma_\perp^\mu\not{b}A_\mu=\gamma^+(\underbrace{\bar{n}^\nu\gamma^+}_{=0}+n^\nu\gamma^-+\underbrace{\gamma_\perp^\nu}_{=0})\gamma^+\gamma_\perp^\mu\not{b}A_\mu=\gamma^+\gamma^-\gamma^+\gamma_\perp^\mu\not{b}A_\mu n^\nu=2\gamma^+\gamma_\perp^\mu\not{b}A_\mu n^\nu,$$
$$\gamma^+\not{b}\gamma^\mu\gamma^\nu\gamma^+A_\mu=2\not{b}\gamma_\perp^\mu\gamma^+n^\nu=2\gamma^+\not{b}\gamma_\perp^\mu n^\nu. \quad (11.12)$$

And obtain

$$\mathbf{C} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta]\bar{q}(z_1+\boldsymbol{b})A_\mu(z_{2\sigma}^\beta-\boldsymbol{b}(1-2\beta)) \quad (11.13)$$
$$\Big[(\beta\overleftarrow{\partial_+}-\alpha\overrightarrow{\partial_+})\gamma^+\not{b}\gamma_\perp^\mu-(\bar\beta\overleftarrow{\partial_+}+\alpha\overrightarrow{\partial_+})\gamma^+\gamma_\perp^\mu\not{b}\Big]q(z_{\sigma2}^\alpha+\boldsymbol{b}(1-2\alpha)).$$

Step IX: The variables $\boldsymbol{b}$ in the arguments of the fields could be dropped

$$\text{DY}\quad \mathbf{C} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta]\bar{q}(z_1)A_\mu(z_{2\sigma}^\beta) \quad (11.14)$$
$$\Big[(\beta\overleftarrow{\partial_+}-\alpha\overrightarrow{\partial_+})\gamma^+\not{b}\gamma_\perp^\mu-(\bar\beta\overleftarrow{\partial_+}+\alpha\overrightarrow{\partial_+})\gamma^+\gamma_\perp^\mu\not{b}\Big]q(z_{\sigma2}^\alpha),$$
$$\text{SIDIS}\quad \mathbf{C} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int_{+\infty}^{z_1}d\sigma\int[d\alpha d\beta]\bar{q}(z_1)A_\mu(z_{2\sigma}^\beta) \quad (11.15)$$
$$\Big[(\beta\overleftarrow{\partial_+}-\alpha\overrightarrow{\partial_+})\gamma^+\not{b}\gamma_\perp^\mu-(\bar\beta\overleftarrow{\partial_+}+\alpha\overrightarrow{\partial_+})\gamma^+\gamma_\perp^\mu\not{b}\Big]q(z_{\sigma2}^\alpha).$$

B. Diagram C^*

The diagram is the mirror diagram to the diagram **C**. It reads

$$\mathbf{C}^* = \left(ig\int d^dx\bar{q}(x)\not{B}(x)\psi(x)\right)\left(ig\int d^dy\bar\psi(y)\not{A}(y)\psi(y)\right)\left\{\bar\psi(z_1+\boldsymbol{b})[-ig\int_{-\infty}^{z_2}d\sigma n^\mu t^A B_\mu^A(n\sigma-\boldsymbol{b})]\gamma^+q(z_2-\boldsymbol{b})\right\}$$

Step I:

$$\mathbf{C}^* = -(ig)^3\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}}\int d^dxd^dy\int_{-\infty}^{z_2}d\sigma \quad (11.16)$$
$$\bar{q}(x)\,t^At^Bt^A\,A_\mu^B(y)\frac{\gamma^\nu(\not{x}-\not{y})\gamma^\mu(\not{y}-\gamma^+z_1-\not{b})n_\nu\gamma^+}{[-(x-y)^2]^{2-\epsilon}[-(y-z_1-b)^2]^{2-\epsilon}[-(x-\sigma+b)^2]^{1-\epsilon}}q(z_2+\boldsymbol{b}).$$

Step II:

$$\mathbf{C}^* = ig^3\frac{\Gamma^2(2-\epsilon)\Gamma(1-\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_2}d\sigma \quad (11.17)$$
$$\bar{q}(x)A_\mu(y)\frac{\gamma^+(\not{x}-\not{y})\gamma^\mu(\not{y}-\not{b})\gamma^+}{[-(x-y)^2]^{2-\epsilon}[-(y-z_1-b)^2]^{2-\epsilon}[-(x-\sigma+b)^2]^{1-\epsilon}}q(z_2+\boldsymbol{b}).$$

Step III: This loop integral is topologically similar to the diagram B. So, we make the similar change of variables

$$\begin{aligned}(\sigma-b-x)^2 &= \alpha\\ (y-z_1-b)^2 &= \beta\\ (x-y)^2 &= 1-\alpha-\beta=\gamma\end{aligned}$$

Next we shift

$$\begin{aligned}x &\to x+\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right),\\ y &\to y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right),\\ \lambda &= \alpha\beta+\beta\gamma+\gamma\alpha.\end{aligned} \tag{11.18}$$

We obtain

$$\begin{aligned}\mathbf{C}^* &= ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\ \alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\\ &\bar{q}(x+\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))A_\mu(y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))\\ &\frac{\gamma^+(\not x-\not y-2\frac{\alpha\beta}{\lambda}\not b)\gamma^\mu(\not y-2\frac{\alpha\gamma}{\lambda}\not b)\gamma^+}{[-\bar\beta x^2-\bar\alpha y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}}q(z_2+\boldsymbol{b}).\end{aligned} \tag{11.19}$$

Step IV: Expand up to one-derivative (here left derivative acts on $\bar q$ and right derivative acts of A)

$$\begin{aligned}\mathbf{C}^* &= ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\ \alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\\ &\bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))(1+x^\nu\overleftarrow{\partial_\nu}+yx^\nu\overrightarrow{\partial_\nu})A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))\\ &\frac{\gamma^+(\not x-\not y-2\frac{\alpha\beta}{\lambda}\not b)\gamma^\mu(\not y-2\frac{\alpha\gamma}{\lambda}\not b)\gamma^+}{[-\bar\beta x^2-\bar\alpha y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}}q(z_2+\boldsymbol{b}).\end{aligned} \tag{11.20}$$

✓**Step V:**The term without derivatives vanishes, see diagram **C**.

$$\begin{aligned}\mathbf{C}^* &= ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\left(C_F-\frac{C_A}{2}\right)\int d^dxd^dy\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\ \alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\\ &\bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))(x^\nu\overleftarrow{\partial_\nu}+y^\nu\overrightarrow{\partial_\nu})A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))\\ &\frac{-2\frac{\alpha\gamma}{\lambda}\gamma^+(\not x-\not y)\gamma^\mu\not b\gamma^+-2\frac{\alpha\beta}{\lambda}\gamma^+\not b\gamma^\mu\not y\gamma^+}{[-\bar\beta x^2-\bar\alpha y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}}q(z_2+\boldsymbol{b}).\end{aligned} \tag{11.21}$$

Step VI: Integrating

$$\begin{aligned}\mathbf{C}^* &= iga_s\Gamma(-\epsilon)\left(C_F-\frac{C_A}{2}\right)\mathbf{B}^\epsilon\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\ \alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\frac{(\alpha\beta\gamma)^{-\epsilon}}{\lambda^3}\\ &\bar{q}(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))A_\mu(\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))\\ &\Big(-\frac{\alpha\gamma}{\lambda}(\beta\overleftarrow{\partial_\nu}-\alpha\overrightarrow{\partial_\nu})\gamma^+\gamma^\nu\gamma^\mu\not b\gamma^+-\frac{\alpha\beta}{\lambda}(\gamma\overleftarrow{\partial_\nu}+\bar\beta\overrightarrow{\partial_\nu})\gamma^+\not b\gamma^\mu\gamma^\nu\gamma^+\Big)q(z_2+\boldsymbol{b}).\end{aligned} \tag{11.22}$$

Step VII: We change the variables to dual and drop the primes.

$$\begin{aligned}\mathbf{C}^* &= iga_s\Gamma(-\epsilon)\left(C_F-\frac{C_A}{2}\right)\mathbf{B}^\epsilon\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\\ &\bar{q}(z_{\sigma1}^\alpha-\boldsymbol{b}(1-2\alpha))A_\mu(z_{1\sigma}^\beta+\boldsymbol{b}(1-2\beta))\\ &\Big(-(\alpha\overleftarrow{\partial_\nu}-\beta\overrightarrow{\partial_\nu})\gamma^+\gamma^\nu\gamma^\mu\not b\gamma^+-(\alpha\overleftarrow{\partial_\nu}+\bar\beta\overrightarrow{\partial_\nu})\gamma^+\not b\gamma^\mu\gamma^\nu\gamma^+\Big)q(z_2+\boldsymbol{b}).\end{aligned} \tag{11.23}$$

Step VIII: We make the same simplification as in step VIII for the diagram **C**

$$\mathbf{C}^* = 2iga_s\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\mathbf{B}^\epsilon \int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\bar{q}(z^\alpha_{\sigma 1} - \boldsymbol{b}(1-2\alpha)) \tag{11.24}$$
$$\Big(-(\alpha\overleftarrow{\partial_+} - \beta\overrightarrow{\partial_+})\gamma^+\gamma^\mu_\perp \not{b} - (\alpha\overleftarrow{\partial_+} + \bar\beta\overrightarrow{\partial_+})\gamma^+ \not{b}\gamma^\mu_\perp\Big) A_\mu(z^\beta_{1\sigma} + \boldsymbol{b}(1-2\beta))q(z_2 + \boldsymbol{b}).$$

Step IX:

$$\text{DY} \qquad \mathbf{C}^* = 2iga_s\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\mathbf{B}^\epsilon \int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\bar{q}(z^\alpha_{\sigma 1}) \tag{11.25}$$
$$\Big(-(\alpha\overleftarrow{\partial_+} - \beta\overrightarrow{\partial_+})\gamma^+\gamma^\mu_\perp \not{b} - (\alpha\overleftarrow{\partial_+} + \bar\beta\overrightarrow{\partial_+})\gamma^+ \not{b}\gamma^\mu_\perp\Big) A_\mu(z^\beta_{1\sigma})q(z_2),$$
$$\text{SIDIS} \qquad \mathbf{C}^* = 2iga_s\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\mathbf{B}^\epsilon \int_{+\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\bar{q}(z^\alpha_{\sigma 1}) \tag{11.26}$$
$$\Big(-(\alpha\overleftarrow{\partial_+} - \beta\overrightarrow{\partial_+})\gamma^+\gamma^\mu_\perp \not{b} - (\alpha\overleftarrow{\partial_+} + \bar\beta\overrightarrow{\partial_+})\gamma^+ \not{b}\gamma^\mu_\perp\Big) A_\mu(z^\beta_{1\sigma})q(z_2).$$

C. Test of regularity for diagrams C and C*

The diagrams are

$$\mathbf{C} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int_{-\infty}^{z_1} d\sigma \int [d\alpha d\beta d\gamma]\bar{q}(z_1)A_\mu(z^\beta_{2\sigma}) \tag{11.27}$$
$$\Big[(\beta\overleftarrow{\partial_+} - \alpha\overrightarrow{\partial_+})\gamma^+ \not{b}\gamma^\mu_\perp - (\bar\beta\overleftarrow{\partial_+} + \alpha\overrightarrow{\partial_+})\gamma^+\gamma^\mu_\perp \not{b}\Big] q(z^\alpha_{\sigma 2}) + O(\boldsymbol{b}^2)$$
$$\mathbf{C}^* = 2iga_s\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\mathbf{B}^\epsilon \int_{+\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\bar{q}(z^\alpha_{\sigma 1}) \tag{11.28}$$
$$\Big(-(\alpha\overleftarrow{\partial_+} - \beta\overrightarrow{\partial_+})\gamma^+\gamma^\mu_\perp \not{b} - (\alpha\overleftarrow{\partial_+} + \bar\beta\overrightarrow{\partial_+})\gamma^+ \not{b}\gamma^\mu_\perp\Big) A_\mu(z^\beta_{1\sigma})q(z_2) + (\boldsymbol{b}^2).$$

Note that derivatives acts only between quark and gluon.

We slightly recombine the gamma structure using

$$\not{b}\gamma^\mu_\perp = b^\mu - b_\nu\gamma^{\mu\nu}_T, \qquad \gamma^\mu_\perp \not{b} = b^\mu + b_\nu\gamma^{\mu\nu}_T, \tag{11.29}$$

where

$$\gamma^{\mu\nu}_T = \frac{\gamma^\mu_\perp\gamma^\nu_\perp - \gamma^\nu_\perp\gamma^\mu_\perp}{2}. \tag{11.30}$$

$$\mathbf{C} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right) \tag{11.31}$$
$$\int_{-\infty}^{z_1} d\sigma \int [d\alpha d\beta d\gamma]\bar{q}(z_1)A_\mu(z^\beta_{2\sigma})\gamma^+\Big[-b^\mu((1-2\beta)\overleftarrow{\partial_+} + 2\alpha\overrightarrow{\partial_+}) - \overleftarrow{\partial_+}b_\nu\gamma^{\mu\nu}_T\Big]q(z^\alpha_{\sigma 2})$$
$$\mathbf{C}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right) \tag{11.32}$$
$$\int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\bar{q}(z^\alpha_{\sigma 1})\gamma^+\Big[-((1-2\beta)\overrightarrow{\partial_+} + 2\alpha\overleftarrow{\partial_+})b^\mu + \overrightarrow{\partial_+}b_\nu\gamma^{\mu\nu}_T\Big]A_\mu(z^\beta_{1\sigma})q(z_2).$$

These expressions contains the ambiguity of the same type as the rapidity divergence, presented in the diagrams A. It should be resolved accurately. Let me consider the case

$$\int_{-\infty}^{z_1} d\sigma \int [d\alpha d\beta d\gamma](1-2\beta)A_\mu(z^\beta_{2\sigma})\overleftarrow{\partial_+}q(z^\alpha_{\sigma 2}) = \Big\{\int_{z_2}^{z_1} d\sigma + \int_{-\infty}^{z_2} d\sigma\Big\}\int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \frac{(1-2\beta)}{\beta}\partial_\sigma A_\mu(z^\beta_{2\sigma})q(z^\alpha_{\sigma 2}) \tag{11.33}$$

We going to show that this integral is regular and does not contain ambiguity. It is obviously the case of the first integral in the curly brackets. For the second one we make a change of variables

$$\sigma = \beta(\tau - z_2), \qquad \alpha = 1 - \beta\omega, \tag{11.34}$$

and obtain

$$\int_{-\infty}^{z_2} d\sigma \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \frac{(1-2\beta)}{\beta} \partial_\sigma A_\mu(z_{2\sigma}^\beta) q(z_{\sigma 2}^\alpha) = \int_{-\infty}^0 d\tau \int_1^\infty d\omega \int_0^{1/\omega} d\beta\beta \frac{1-2\beta}{\beta} \partial_\tau A_\mu(\tau + z_2) q(\omega\tau + z_2)$$
$$= \int_{-\infty}^0 d\tau \int_1^\infty d\omega \frac{\omega - 1}{\omega^2} \partial_\tau A_\mu(\tau + z_2) q(\omega\tau + z_2), \tag{11.35}$$

where in the second line we have evaluated the integral over β explicitly. This expression is obviously regular at all points of integration domain. Similarly we show

$$\int_{-\infty}^{z_2} d\sigma \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \frac{1}{\beta} \partial_\sigma A_\mu(z_{2\sigma}^\beta) q(z_{\sigma 2}^\alpha) = \int_{-\infty}^0 d\tau \int_1^\infty d\omega \frac{1}{\omega} \partial_\tau A_\mu(\tau + z_2) q(\omega\tau + z_2),$$

which is also regular. Finally, we consider the expression with derivative over the quark field

$$\int_{-\infty}^{z_2} d\sigma \int_0^1 d\alpha \int_0^{\bar\alpha} d\beta \frac{2\alpha}{\bar\alpha} A_\mu(z_{2\sigma}^\beta) \partial_\sigma q(z_{\sigma 2}^\alpha) = \int_{-\infty}^0 d\tau \int_0^1 d\omega \int_0^1 d\alpha \bar\alpha \frac{2\alpha}{\bar\alpha} A_\mu(\omega\tau + z_2) \partial_\tau q(\tau + z_2)$$
$$= \int_{-\infty}^0 d\tau \int_0^1 d\omega A_\mu(\omega\tau + z_2) \partial_\tau q(\tau + z_2), \tag{11.36}$$

where we replaced $\bar\alpha(\sigma - z_2) = \tau$ and $\beta/\bar\alpha = \omega$. It is also regular.

Therefore, we can integrate by parts without any problems. All integrals are regular at would-be singular points in α and β.

D. Straightforward treatment for diagrams C and C^*

We consider the derivatives that acts on quarks. We rewrite then via derivative over σ, and integrate by parts. The derivative acting on A we rewrite again as ∂_+. I.e.

$$\int_{-\infty}^{z_1} d\sigma A_\mu(z_{2\sigma}^\beta) \alpha \overrightarrow{\partial_+} q(z_{\sigma 2}^\alpha) = \int_{-\infty}^{z_1} d\sigma A_\mu(z_{2\sigma}^\beta) \frac{\alpha}{\bar\alpha} \overrightarrow{\partial_\sigma} q(z_{\sigma 2}^\alpha) = A_\mu(z_{21}^\beta) \frac{\alpha}{\bar\alpha} q(z_{12}^\alpha) - \int_{-\infty}^{z_1} d\sigma (\partial_\sigma A_\mu(z_{2\sigma}^\beta)) \frac{\alpha}{\bar\alpha} q(z_{\sigma 2}^\alpha) \tag{11.37}$$
$$= A_\mu(z_{21}^\beta) \frac{\alpha}{\bar\alpha} q(z_{12}^\alpha) - \int_{-\infty}^{z_1} d\sigma (\partial_+ A_\mu(z_{2\sigma}^\beta)) \frac{\alpha\beta}{\bar\alpha} q(z_{\sigma 2}^\alpha)$$

$$\int_{-\infty}^{z_2} d\sigma (\partial_+ \bar q(z_{\sigma 1}^\alpha)) \alpha A_\mu(z_{1\sigma}^\beta) = \bar q(z_{21}^\alpha) \frac{\alpha}{\bar\alpha} A_\mu(z_{12}^\beta) - \int_{-\infty}^{z_2} d\sigma \bar q(z_{\sigma 1}^\alpha) \frac{\alpha\beta}{\bar\alpha} (\partial_+ A_\mu(z_{1\sigma}^\beta)). \tag{11.38}$$

It is important that the integrals over σ ans α are regular at $\alpha \to 1$.

Substituting it we obtain

$$\mathbf{C} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta] \Big\{ -2b^\mu \frac{\alpha}{\bar\alpha} \bar q(z_1) A_\mu(z_{21}^\beta) \gamma^+ q(z_{12}^\alpha) \tag{11.39}$$
$$+ \int_{-\infty}^{z_1} d\sigma \bar q(z_1) \left(\partial_+ A_\mu(z_{2\sigma}^\beta)\right) \gamma^+ \left[b^\mu \left(2\frac{\beta}{\bar\alpha} - 1\right) - b_\nu \gamma_T^{\mu\nu}\right] q(z_{\sigma 2}^\alpha) \Big\},$$

$$\mathbf{C}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta] \Big\{ -2b^\mu \frac{\alpha}{\bar\alpha} \bar q(z_{21}^\alpha) \gamma^+ A_\mu(z_{12}^\beta) q(z_2) \tag{11.40}$$
$$+ \int_{-\infty}^{z_2} d\sigma \bar q(z_{\sigma 1}^\alpha) \gamma^+ \left[b^\mu \left(2\frac{\beta}{\bar\alpha} - 1\right) + b_\nu \gamma_T^{\mu\nu}\right] \left(\partial_+ A_\mu(z_{1\sigma}^\beta)\right) q(z_2) \Big\}.$$

Next we use the relation between field A and the field strength tensor in the light-cone gauge (6.19,6.20). We have the relations

$$A_\mu(z_{21}^\beta n) = -\int_{-\infty}^0 d\lambda F^{\mu+}(\lambda n + z_{21}^\beta n) = -\beta\int_{-\infty}^{z_1} d\sigma F^{\mu+}(z_{2\sigma}^\beta n), \tag{11.41}$$

$$A_\mu(z_{12}^\beta n) = -\int_{-\infty}^0 d\lambda F^{\mu+}(\lambda n + z_{12}^\beta n) = -\beta\int_{-\infty}^{z_2} d\sigma F^{\mu+}(z_{1\sigma}^\beta n). \tag{11.42}$$

Substituting these relations (and $\partial_+ A_\mu(z) = -F^{\mu+}(z)$) we simply the expression

$$\begin{aligned}\mathbf{C} &= -2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta]\int_{-\infty}^{z_1} d\sigma\Big\{-2b^\mu\frac{\alpha\beta}{\bar\alpha}\bar q(z_1)F_{\mu+}(z_{2\sigma}^\beta)\gamma^+q(z_{12}^\alpha) \\ &\quad +\bar q(z_1)F_{\mu+}(z_{2\sigma}^\beta)\gamma^+\Big[b^\mu\left(2\frac{\beta}{\bar\alpha}-1\right)-b_\nu\gamma_T^{\mu\nu}\Big]q(z_{\sigma2}^\alpha)\Big\}, \end{aligned}\tag{11.43}$$

$$\begin{aligned}\mathbf{C}^* &= -2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta]\int_{-\infty}^{z_2} d\sigma\Big\{-2b^\mu\frac{\alpha\beta}{\bar\alpha}\bar q(z_{21}^\alpha)\gamma^+F_{\mu+}(z_{1\sigma}^\beta)q(z_2) \\ &\quad +\bar q(z_{\sigma1}^\alpha)\gamma^+\Big[b^\mu\left(2\frac{\beta}{\bar\alpha}-1\right)+b_\nu\gamma_T^{\mu\nu}\Big]F_{\mu+}(z_{1\sigma}^\beta)q(z_2)\Big\}. \end{aligned}\tag{11.44}$$

These expressions could possibly contain ambiguity at $\alpha\to1$. See the next section for its resolution.

Thus we write down the final expression for DY

$$\begin{aligned}\mathbf{C} &= 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\int_{-\infty}^{z_1} d\sigma\int[d\alpha d\beta d\gamma]\Big\{2\frac{\alpha\beta}{\bar\alpha}\mathcal{T}^\mu_{\gamma^+}(z_1,z_{2\sigma}^\beta,z_{12}^\alpha) \\ &\quad +\left(1-2\frac{\beta}{\bar\alpha}\right)\mathcal{T}^\mu_{\gamma^+}(z_1,z_{2\sigma}^\beta,z_{\sigma2}^\alpha)+\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_1,z_{2\sigma}^\beta,z_{\sigma2}^\alpha)\Big\}, \end{aligned}\tag{11.45}$$

$$\begin{aligned}\mathbf{C}^* &= 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\int_{-\infty}^{z_2} d\sigma\int[d\alpha d\beta]\Big\{2\frac{\alpha\beta}{\bar\alpha}\mathcal{T}^\mu_{\gamma^+}(z_{21}^\alpha,z_{1\sigma}^\beta,z_2) \\ &\quad +\left(1-2\frac{\beta}{\bar\alpha}\right)\mathcal{T}^\mu_{\gamma^+}(z_{\sigma1}^\alpha,z_{1\sigma}^\beta,z_2)-\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_{\sigma1}^\beta,z_{1\sigma}^\alpha,z_2)\Big\}. \end{aligned}\tag{11.46}$$

Thus we write down the final expression for SIDIS

$$\begin{aligned}\mathbf{C} &= -2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\int_{z_1}^{\infty} d\sigma\int[d\alpha d\beta d\gamma]\Big\{2\frac{\alpha\beta}{\bar\alpha}\mathcal{T}^\mu_{\gamma^+}(z_1,z_{2\sigma}^\beta,z_{12}^\alpha) \\ &\quad +\left(1-2\frac{\beta}{\bar\alpha}\right)\mathcal{T}^\mu_{\gamma^+}(z_1,z_{2\sigma}^\beta,z_{\sigma2}^\alpha)+\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_1,z_{2\sigma}^\beta,z_{\sigma2}^\alpha)\Big\}, \end{aligned}\tag{11.47}$$

$$\begin{aligned}\mathbf{C}^* &= -2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\int_{z_2}^{\infty} d\sigma\int[d\alpha d\beta]\Big\{2\frac{\alpha\beta}{\bar\alpha}\mathcal{T}^\mu_{\gamma^+}(z_{21}^\alpha,z_{1\sigma}^\beta,z_2) \\ &\quad +\left(1-2\frac{\beta}{\bar\alpha}\right)\mathcal{T}^\mu_{\gamma^+}(z_{\sigma1}^\alpha,z_{1\sigma}^\beta,z_2)-\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_{\sigma1}^\beta,z_{1\sigma}^\alpha,z_2)\Big\}. \end{aligned}\tag{11.48}$$

E. Elaboration of diagrams C and $C*$

We start from the expression obtained in the previous section.

$$\mathbf{C} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right) \tag{11.49}$$
$$\int_{-\infty}^{z_1} d\sigma\int[d\alpha d\beta d\gamma]\bar q(z_1)A_\mu(z^\beta_{2\sigma})\gamma^+\Big[-b^\mu((1-2\beta)\overleftarrow{\partial_+}+2\alpha\overrightarrow{\partial_+})-\overleftarrow{\partial_+}b_\nu\gamma_T^{\mu\nu}\Big]q(z^\alpha_{\sigma 2})$$
$$\mathbf{C}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right) \tag{11.50}$$
$$\int_{-\infty}^{z_2} d\sigma\int[d\alpha d\beta d\gamma]\bar q(z^\alpha_{\sigma 1})\gamma^+\Big[-((1-2\beta)\overrightarrow{\partial_+}+2\alpha\overleftarrow{\partial_+})b^\mu+\overrightarrow{\partial_+}b_\nu\gamma_T^{\mu\nu}\Big]A_\mu(z^\beta_{1\sigma})q(z_2).$$

It has been shown that it is regular and contains no rapidity divergences.

We split it into three terms with different behavior

$$\mathbf{CC}_1 = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b^\mu\int[d\alpha d\beta d\gamma](2\beta-1)\Bigg\{ \tag{11.51}$$
$$\int_{-\infty}^{z_1} d\sigma\bar q(z_1)\partial_+A_\mu(z^\beta_{2\sigma})\gamma^+q(z^\alpha_{\sigma 2})+\int_{-\infty}^{z_2} d\sigma\bar q(z^\alpha_{\sigma 1})\partial_+A_\mu(z^\beta_{1\sigma})\gamma^+q(z_2)\Bigg\}.$$

$$\mathbf{CC}_2 = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b^\mu\int[d\alpha d\beta d\gamma](-2\alpha)\Bigg\{ \tag{11.52}$$
$$\int_{-\infty}^{z_1} d\sigma\bar q(z_1)A_\mu(z^\beta_{2\sigma})\gamma^+\partial_+q(z^\alpha_{\sigma 2})+\int_{-\infty}^{z_2} d\sigma\partial_+\bar q(z^\alpha_{\sigma 1})A_\mu(z^\beta_{1\sigma})\gamma^+q(z_2)\Bigg\}.$$

$$\mathbf{CC}_3 = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b^\mu\int[d\alpha d\beta d\gamma]\Bigg\{ \tag{11.53}$$
$$\int_{-\infty}^{z_1} d\sigma\bar q(z_1)\partial_+A_\nu(z^\beta_{2\sigma})\gamma^+\gamma_T^{\mu\nu}q(z^\alpha_{\sigma 2})-\int_{-\infty}^{z_2} d\sigma\bar q(z^\alpha_{\sigma 1})\partial_+A_\nu(z^\beta_{1\sigma})\gamma^+\gamma_T^{\mu\nu}q(z_2)\Bigg\}.$$

We start from the evaluation of the $\mathbf{CC}_1$. Using that $\partial_+A_\mu=-F^{\mu+}$, and the definition of operators we get

$$\mathbf{CC}_1 = -2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b^\mu\int[d\alpha d\beta d\gamma](2\beta-1)\Bigg\{ \tag{11.54}$$
$$\int_{-\infty}^{z_1} d\sigma\mathcal{T}^\mu_{\gamma^+}(z_1,z^\beta_{2\sigma},z^\alpha_{\sigma 2})+\int_{-\infty}^{z_2} d\sigma\mathcal{T}^\mu_{\gamma^+}(z^\alpha_{\sigma 1},z^\beta_{1\sigma},z_2)\Bigg\}.$$

We consider the matrix element (and $b\to b/2$, we also set $p_+=1$ since it cancels by dimension)

$$\langle\mathbf{CC}_1\rangle = [b^\mu\epsilon_{\mu\nu}s^\nu M](-2i)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma](2\beta-1)\Bigg\{ \tag{11.55}$$
$$\int_{-\infty}^{z_1} d\sigma\tilde T(z_1,z^\beta_{2\sigma},z^\alpha_{\sigma 2})+\int_{-\infty}^{z_2} d\sigma\tilde T(z^\alpha_{\sigma 1},z^\beta_{1\sigma},z_2)\Bigg\}.$$

In the second term we change $\tilde{T}(z_1,z_2,z_3) \to T(-z_3,-z_2,-z_1)$ and get $(z_1 = -z_2 = z)$

$$\langle \mathbf{CC}_1 \rangle = [b^\mu \epsilon_{\mu\nu} s^\nu M](-2i) a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [d\alpha d\beta d\gamma](2\beta - 1) \Bigg\{ \int_{-\infty}^{z} d\sigma \tilde{T}(z, -\bar{\beta}z + \sigma\beta, \bar{\alpha}\sigma - \alpha z) + \int_{-\infty}^{-z} d\sigma \tilde{T}(z, -\bar{\beta}z - \beta\sigma, -\sigma\bar{\alpha} - z\alpha) \Bigg\}. \tag{11.56}$$

Changing in the second term $\sigma \to -\sigma$ we sum together the integrals

$$\langle \mathbf{CC}_1 \rangle = [b^\mu \epsilon_{\mu\nu} s^\nu M](-2i) a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [d\alpha d\beta d\gamma](2\beta - 1) \int_{-\infty}^{\infty} d\sigma \tilde{T}(z, -\bar{\beta}z + \sigma\beta, \bar{\alpha}\sigma - \alpha z). \tag{11.57}$$

Nest we perform the Fourier transformation, using the fraction momentum representation of T

$$\begin{aligned} \{\mathbf{CC}_1\} &= \int \frac{dz}{2\pi} e^{-2ixz} \langle \mathbf{CC}_1 \rangle \\ &= [b^\mu \epsilon_{\mu\nu} s^\nu M](-2i) a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [d\alpha d\beta d\gamma](2\beta - 1) \int \frac{dz}{2\pi} \int [dx] \\ &\quad \int_{-\infty}^{\infty} d\sigma e^{-2ixz} e^{-i(x_1 z + x_2(\bar{\beta}z + \sigma\beta) + x_2(\bar{\alpha}\sigma - \alpha z)} T(x_1, x_2, x_3) \\ &= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M](-2) a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [dx] \int [d\alpha d\beta d\gamma](2\beta - 1) \\ &\quad 2\delta(2x + x_1 - \bar{\beta}x_2 - x_3\alpha)\delta(x_2\beta + \bar{\alpha}x_3) T(x_1, x_2, x_3). \end{aligned} \tag{11.58}$$

Simplifying δ-functions we get

$$\{\mathbf{CC}_1\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M](-2) a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int dx_2 \int [d\alpha d\beta d\gamma](2\beta - 1)\delta(x_2\gamma - \bar{\alpha}x) T(-x, x_2, x - x_2). \tag{11.59}$$

We change variable

$$\xi = \frac{\gamma}{\bar{\alpha}}, \qquad \alpha \in (0,1), \qquad \beta \in (0,1), \qquad d\beta = \bar{\alpha} d\xi. \tag{11.60}$$

And get

$$\{\mathbf{CC}_1\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M](-2) a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int dx_2 \int_0^1 d\xi d\alpha (1 - 2\alpha - 2\bar{\alpha}\xi) \delta(x - \xi x_2) T(-x, x_2, x - x_2) \tag{11.61}$$

$$= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int dx_2 \int_0^1 d\xi \delta(x - \xi x_2) \xi T(-x, x_2, x - x_2) \tag{11.62}$$

We rewrite it renaming variables

$$\{\mathbf{CC}_1\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int d\xi \int_0^1 dy \delta(x - y\xi) y T(-x, \xi, x - \xi) \tag{11.63}$$

Next we consider the contribution $\mathbf{CC}_2$. Let us first rewrite the derivative acting on the quark as (we recall that it is a regular integral)

$$\begin{aligned} &-2\alpha \int_{-\infty}^{z_1} A_\mu(z_{2\sigma}^\beta) \partial_+ q(z_{\sigma 2}^\alpha) = \frac{-2\alpha}{\bar{\alpha}} \int_{-\infty}^{z_1} A_\mu(z_{2\sigma}^\beta) \partial_\sigma q(z_{\sigma 2}^\alpha) \\ &= \frac{-2\alpha}{\bar{\alpha}} A_\mu(z_{21}^\beta) q(z_{12}^\alpha) + \frac{2\alpha}{\bar{\alpha}} \int_{-\infty}^{z_1} \partial_\sigma A_\mu(z_{2\sigma}^\beta) q(z_{\sigma 2}^\alpha) = \frac{-2\alpha}{\bar{\alpha}} A_\mu(z_{21}^\beta) q(z_{12}^\alpha) + \frac{2\alpha\beta}{\bar{\alpha}} \int_{-\infty}^{z_1} \partial_+ A_\mu(z_{2\sigma}^\beta) q(z_{\sigma 2}^\alpha). \end{aligned} \tag{11.64}$$

We would like to make a $F^{\mu+}$ from the A_μ we do it as

$$A_\mu(z_{21}^\beta) = -\int_{-\infty}^{0} d\lambda F^{\mu+}(\lambda + z_2 + z_{12}\beta) = -\int_{-\infty}^{z_2} d\lambda F^{\mu+}(\lambda + z_{12}\beta). \tag{11.65}$$

We also rewrite

$$\beta\int_{-\infty}^{z_1} \partial_+ A_\mu(z_{2\sigma}^\beta) = -\beta\int_{-\infty}^{z_1} F^{\mu+}(z_{2\sigma}^\beta) = -\int_{-\infty}^{z_2} F^{\mu+}(\lambda + z_{12}\beta). \tag{11.66}$$

Such form is much better, since it does not leave a possibility to have ambiguity at $\beta \to 0$, where $\int_{-\infty} F^{\mu+}(z_{2\sigma}^\beta) \to \int_{-\infty} F^{\mu+}(z_2) \to \infty$. On the other hand one can use the usual variable σ if integral over it, is taken prior the integral over β. Thus we have

$$-2\alpha\int_{-\infty}^{z_1} A_\mu(z_{2\sigma}^\beta)\partial_+ q(z_{\sigma 2}^\alpha) = \frac{2\alpha}{\bar\alpha}\int_{-\infty}^{z_2} d\lambda\left(F^{\mu+}(\lambda + z_{12}\beta)q(z_{12}^\alpha) - F^{\mu+}(\lambda + z_{12}\beta)q(z_{\sigma 2}^\alpha)\right). \tag{11.67}$$

Note, that at $\alpha \to 1$ the bracket is 0, and thus the sum is well-defined. In order to preserve this property for each term (since we going to manipulate them separately) we introduce a "plus" distribution. It guaranty that $\alpha \neq 1$.

$$-2\alpha\int_{-\infty}^{z_1} A_\mu(z_{2\sigma}^\beta)\partial_+ q(z_{\sigma 2}^\alpha) = \left(\frac{2\alpha}{\bar\alpha}\right)_+\int_{-\infty}^{z_2} d\lambda\left(F^{\mu+}(\lambda + z_{12}\beta)q(z_{12}^\alpha) - F^{\mu+}(\lambda + z_{12}\beta)q\left((\lambda - z_2)\frac{\bar\alpha}{\beta} + z_{12}^\alpha\right)\right) \tag{11.68}$$

where "plus" distributions is understood as usual

$$(f(\alpha))_+ = f(\alpha) - \delta(\bar\alpha)\int_0^1 d\alpha' f(\alpha'). \tag{11.69}$$

The term with $\delta(\bar\alpha)$ is equal to zero, by definition. Similarly, we have for the diagram $\mathbf{C}^*$

$$-2\alpha\int_{-\infty}^{z_2} \partial_+\bar q(z_{\sigma 1}^\alpha)A_\mu(z_{1\sigma}^\beta) = \left(\frac{2\alpha}{\bar\alpha}\right)_+\int_{-\infty}^{z_1} d\lambda\left(\bar q(z_{21}^\alpha)F^{\mu+}(\lambda + z_{21}\beta) - \bar q\left((\lambda - z_1)\frac{\bar\alpha}{\beta} + z_{21}^\alpha\right)F^{\mu+}(\lambda + z_{21}\beta)\right) \tag{11.70}$$

We consider these contributions separately

$$\mathbf{CC}_{21} = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)b^\mu\int[d\alpha d\beta d\gamma]\left(\frac{2\alpha}{\bar\alpha}\right)_+\Bigg\{ \int_{-\infty}^{z_2} d\lambda\mathcal{T}^\mu_{\gamma+}(z_1, \lambda + z_{12}\beta, z_{12}^\alpha) + \int_{-\infty}^{z_1} d\sigma\mathcal{T}^\mu_{\gamma+}(z_{21}^\alpha, \lambda + z_{21}\beta, z_2)\Bigg\}. \tag{11.71}$$

$$\mathbf{CC}_{22} = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)b^\mu\int[d\alpha d\beta d\gamma]\left(\frac{-2\alpha}{\bar\alpha}\right)_+\Bigg\{ \int_{-\infty}^{z_2} d\lambda\mathcal{T}^\mu_{\gamma+}(z_1, \lambda + z_{12}\beta, (\lambda - z_2)\frac{\bar\alpha}{\beta} + z_{12}^\alpha) + \int_{-\infty}^{z_1} d\sigma\mathcal{T}^\mu_{\gamma+}((\lambda - z_1)\frac{\bar\alpha}{\beta} + z_{21}^\alpha, \lambda + z_{21}\beta, z_2)\Bigg\}. \tag{11.72}$$

Evaluating matrix element

$$\langle\mathbf{CC}_{21}\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma]\left(\frac{2\alpha}{\bar\alpha}\right)_+\Bigg\{ \int_{-\infty}^{z_2} d\lambda T(z_1, \lambda + z_{12}\beta, z_{12}^\alpha) + \int_{-\infty}^{z_1} d\sigma T(z_{21}^\alpha, \lambda + z_{21}\beta, z_2)\Bigg\}. \tag{11.73}$$

$$\langle\mathbf{CC}_{22}\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma]\left(\frac{-2\alpha}{\bar\alpha}\right)_+\Bigg\{ \int_{-\infty}^{z_2} d\lambda T(z_1, \lambda + z_{12}\beta, (\lambda - z_2)\frac{\bar\alpha}{\beta} + z_{12}^\alpha) + \int_{-\infty}^{z_1} d\sigma T((\lambda - z_1)\frac{\bar\alpha}{\beta} + z_{21}^\alpha, \lambda + z_{21}\beta, z_2)\Bigg\}. \tag{11.74}$$

Next, as usual, we take the second term revert the order, and change $\lambda \to -\lambda$. Then sum together the integrals

$$\langle \mathbf{CC}_{21} \rangle = [ib^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [d\alpha d\beta d\gamma] \left(\frac{2\alpha}{\bar{\alpha}} \right)_+ \int_{-\infty}^{\infty} d\lambda T(z, \lambda + 2z\beta, (1-2\alpha)z). \quad (11.75)$$

$$\langle \mathbf{CC}_{22} \rangle = [ib^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [d\alpha d\beta d\gamma] \left(\frac{-2\alpha}{\bar{\alpha}} \right)_+ \quad (11.76)$$
$$\int_{-\infty}^{\infty} d\lambda T(z, \lambda + 2z\beta, (\lambda + z)\frac{\bar{\alpha}}{\beta} + (1-2\alpha)z).$$

Evolving the Fourier we obtain

$$\{\mathbf{CC}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [dx] \int [d\alpha d\beta d\gamma] \left(\frac{2\alpha}{\bar{\alpha}} \right)_+ \quad (11.77)$$
$$2\delta(2x + x_1 + x_3(1-2\alpha) + 2x_2\beta)\delta(x_2) T(x_1, x_2, x_3).$$

$$\{\mathbf{CC}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [dx] \int [d\alpha d\beta d\gamma] \left(\frac{-2\alpha}{\bar{\alpha}} \right)_+ \quad (11.78)$$
$$2\delta\left(2x + x_1 + x_3(1-2\alpha) + x_3\frac{\bar{\alpha}}{\beta} + 2x_2\beta\right) \delta\left(\frac{x_3\bar{\alpha} + x_2\beta}{\beta}\right) T(x_1, x_2, x_3).$$

We simplify δ-functions

$$\{\mathbf{CC}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [dx] \int [d\alpha d\beta d\gamma] \left(\frac{2\alpha}{\bar{\alpha}} \right)_+ \quad (11.79)$$
$$2\delta(2x - 2\alpha x_3)\delta(x_2) T(x_1, x_2, x_3).$$

$$\{\mathbf{CC}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int [dx] \int [d\alpha d\beta d\gamma] \left(\frac{-2\alpha}{\bar{\alpha}} \right)_+ \quad (11.80)$$
$$2\delta\left(2x + 2x_1\right) \delta\left(x_3\frac{\bar{\alpha}}{\beta} + x_2\right) T(x_1, x_2, x_3).$$

And integrate them

$$\{\mathbf{CC}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int dx_3 \int [d\alpha d\beta d\gamma] \left(\frac{2\alpha}{\bar{\alpha}} \right)_+ \quad (11.81)$$
$$\delta(x - \alpha x_3) T(-x_3, 0, x_3).$$

$$\{\mathbf{CC}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int dx_2 \int [d\alpha d\beta d\gamma] \left(\frac{-2\alpha}{\bar{\alpha}} \right)_+ \quad (11.82)$$
$$\delta\left(x\frac{\bar{\alpha}}{\beta} - \frac{\gamma}{\beta}x_2\right) T(-x, x_2, x - x_2).$$

Now, we integrate over β in the first case and get

$$\{\mathbf{CC}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \int dx_3 \int_0^1 d\alpha \left(\frac{2\alpha}{\bar{\alpha}} \right)_+ \bar{\alpha}\delta(x - \alpha x_3) T(-x_3, 0, x_3). \quad (11.83)$$

For the second case we do the same change of variables as in (11.60), and obtain

$$\{\mathbf{CC}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2} \right) \quad (11.84)$$
$$\int dx_2 \int_0^1 d\alpha \int_0^1 d\xi \left(\frac{-2\alpha}{\bar{\alpha}} \right)_+ \bar{\alpha}(1-\xi)\delta\left(x - \xi x_2\right) T(-x, x_2, x - x_2).$$

We observe that in both cases, the "plus" distribution is multiplied by $\bar\alpha$ and thus it δ-part vanish. Renaming variables and integrate over α in the second case we obtain

$$\{\mathbf{CC}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int d\xi \int_0^1 dy\, 2y\, \delta(x-y\xi)T(-\xi,0,\xi), \quad (11.85)$$

$$\{\mathbf{CC}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int d\xi \int_0^1 dy\, (-\bar y)\delta\,(x-y\xi)\,T(-x,\xi,x-\xi). \quad (11.86)$$

Finally, we consider the third contribution

$$\mathbf{CC}_3 = -2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)b^\mu\int[d\alpha d\beta d\gamma]\left\{\int_{-\infty}^{z_1} d\sigma \mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z_1,z^\beta_{2\sigma},z^\alpha_{\sigma 2}) - \int_{-\infty}^{z_2} d\sigma \mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z^\alpha_{\sigma 1},z^\beta_{1\sigma},z_2)\right\} \quad (11.87)$$

In comparison to the previous cases it is almost trivial. We consider matrix element

$$\langle\mathbf{CC}_3\rangle = [ib^\mu \epsilon_{\mu\nu} s^\nu M](-2a_s)\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right) \quad (11.88)$$
$$\int[d\alpha d\beta d\gamma]\left\{\int_{-\infty}^{z_1} d\sigma \Delta\tilde T(z_1,z^\beta_{2\sigma},z^\alpha_{\sigma 2}) - \int_{-\infty}^{z_2} d\sigma \Delta\tilde T(z^\alpha_{\sigma 1},z^\beta_{1\sigma},z_2)\right\}.$$

We reverse the order of variables in the second term using that $\Delta T(z_1,z_2,z_3) = -\Delta T(-z_3,-z_2,-z_1)$. We also replace $\sigma\to-\sigma$ and join integrals

$$\langle\mathbf{CC}_3\rangle = [ib^\mu \epsilon_{\mu\nu} s^\nu M](-2a_s)\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma]\int_{-\infty}^{\infty} d\sigma\Delta\tilde T(z,\sigma\beta-\bar\beta z,\sigma\bar\alpha-\alpha z). \quad (11.89)$$

Note, that it is practically equivalent to the case of $\mathbf{CC}_1$ (11.57) up to common factor $(2\beta-1)$. Thus we just repeat the steps and obtain

$$\{\mathbf{CC}_3\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M](-2)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int dx_2\int[d\alpha d\beta d\gamma]\delta(x_2\gamma-\bar\alpha x)\Delta T(-x,x_2,x-x_2) \quad (11.90)$$

Using the change of variables (11.60) we obtain (we also rename variables)

$$\{\mathbf{CC}_3\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int d\xi\int_0^1 dy\delta(x-y\xi)(-1)\Delta T(-x,\xi,x-\xi) \quad (11.91)$$

F. Final expressions for the diagrams C

The expression of the diagrams C and C^* in the OPE is

$$\mathbf{C} = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)b_\mu\int_{-\infty}^{z_1} d\sigma\int[d\alpha d\beta d\gamma]\Big\{2\beta\left(\frac{\alpha}{\bar\alpha}\right)_+\mathcal{T}^\mu_{\gamma^+}(z_1,z^\beta_{2\sigma},z^\alpha_{12}) \quad (11.92)$$
$$+\left(1-2\beta-2\beta\left(\frac{\alpha}{\bar\alpha}\right)_+\right)\mathcal{T}^\mu_{\gamma^+}(z_1,z^\beta_{2\sigma},z^\alpha_{\sigma 2}) + \mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_1,z^\beta_{2\sigma},z^\alpha_{\sigma 2})\Big\},$$

$$\mathbf{C}^* = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)b_\mu\int_{-\infty}^{z_2} d\sigma\int[d\alpha d\beta d\gamma]\Big\{2\beta\left(\frac{\alpha}{\bar\alpha}\right)_+\mathcal{T}^\mu_{\gamma^+}(z^\alpha_{21},z^\beta_{1\sigma},z_2) \quad (11.93)$$
$$+\left(1-2\beta-2\beta\left(\frac{\alpha}{\bar\alpha}\right)_+\right)\mathcal{T}^\mu_{\gamma^+}(z^\alpha_{\sigma 1},z^\beta_{1\sigma},z_2) - \mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z^\beta_{\sigma 1},z^\alpha_{1\sigma},z_2)\Big\}.$$

Here, the integral over σ is to be taken prior to the integrals over α and β. For a more accurate (independent on the order of integration expression see the text). The $(..)_+$ distribution subtracts the $\delta(\bar\alpha)$.

Combining together results evaluated in the previous section we obtain the contribution of diagrams C

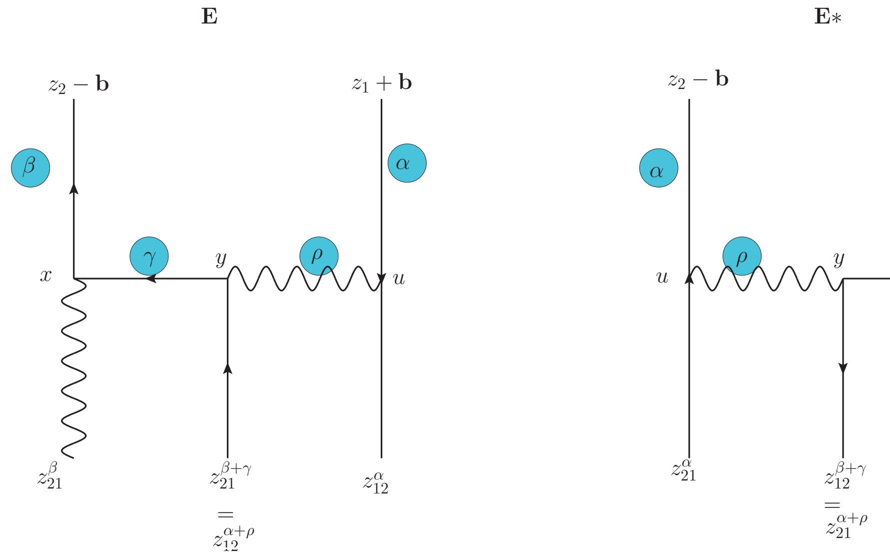
$$\{\mathbf{C}+\mathbf{C}^*\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int d\xi \int_0^1 dy \delta(x-y\xi)\Bigg\{2yT(-\xi,0,\xi)+(2y-1)T(-x,\xi,x-\xi)-\Delta T(-x,\xi,x-\xi)\Bigg\} \tag{11.94}$$

XII. DIAGRAMS E

A. Diagram E

The diagram **E** is given by the contraction

$$\mathbf{E} \;=\; \Big(ig\int d^d u\bar{q}(u)\ \not{B}(u)\psi(u)\Big)\Big\{\bar{\psi}(z_1+\boldsymbol{b})\gamma^+\psi(z_2-\boldsymbol{b})\Big\}\Big(ig\int d^d x\bar{\psi}(x)\ \not{A}(x)\psi(x)\Big)\Big(ig\int d^d y\bar{\psi}(y)\ \not{B}(y)q(y)\Big)$$



✓**Step I:** We substitute propagators

$$\mathbf{E} \;=\; (ig)^3\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}}\int d^d u d^d x d^d y \bar{q}(u)t^A t^B t^A A^B_\mu(x) \tag{12.1}$$
$$\gamma_\nu\frac{\not{u}-\gamma^+ z_1-\not{b}}{[-(u-z_1-b)^2+i0]^{2-\epsilon}}\gamma^+\frac{\gamma^+ z_2-\not{b}-\not{x}}{[-(x-z_2+b)^2+i0]^{2-\epsilon}}\gamma^\mu\frac{\not{x}-\not{y}}{[-(x-y)^2+i0]^{2-\epsilon}}\gamma^\nu\frac{1}{[-(u-y)^2+i0]^{1-\epsilon}}q(y).$$

✓**Step II:** Simplifying

$$\mathbf{E} \;=\; -g^3\frac{\Gamma^3(2-\epsilon)\Gamma(1-\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^d u d^d x d^d y \bar{q}(u)A_\mu(x) \tag{12.2}$$
$$\frac{\gamma_\nu(\not{u}-\not{b})\gamma^+(\not{x}+\not{b})\gamma^\mu(\not{x}-\not{y})\gamma^\nu}{[-(u-z_1-b)^2+i0]^{2-\epsilon}[-(x-z_2+b)^2+i0]^{2-\epsilon}[-(x-y)^2+i0]^{2-\epsilon}[-(u-y)^2+i0]^{1-\epsilon}}q(y).$$

✓**Step III:** We joining propagators according to

$$\begin{aligned}(u-z_1-b)^2 &\to \alpha\\ (x-z_2+b)^2 &\to \beta\\ (x-y)^2 &\to \gamma\\ (u-y)^2 &\to \rho\end{aligned}$$

We also make a shift

$$\begin{aligned}x &\to x+\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)\\ y &\to y+\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)\\ u &\to u+\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)\end{aligned}$$

$$\lambda = \alpha\gamma\rho + \alpha\beta\gamma + \alpha\beta\rho + \beta\gamma\rho.$$

The diagram turns into

$$\begin{aligned}
\mathbf{E} &= -g^3 \frac{\Gamma(7-4\epsilon)}{32\pi^{2d}} \left(C_F - \frac{C_A}{2}\right) \int d^d u d^d x d^d y \int [d\alpha d\beta d\gamma d\rho] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\quad \bar{q}(u + \frac{\alpha\gamma\rho + \alpha\beta\gamma + \alpha\beta\rho}{\lambda} z_1 + \frac{\beta\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&\quad A_\mu(x + \frac{\alpha\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \alpha\beta\rho + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\quad \frac{\gamma_\nu(\not{u} - 2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^+(\not{x} + 2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^\mu(\not{x} - \not{y} + \frac{\alpha\beta\rho}{\lambda}(-2\not{b} - \gamma^+ z_1 + \gamma^+ z_2))\gamma^\nu}{[-(\beta+\gamma)x^2 - (\gamma+\rho)y^2 - (\alpha+\rho)u^2 + 2\rho(uy) + 2\gamma(xy) + 4\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2 + i0]^{7-4\epsilon}} \\
&\quad q(y + \frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda}\right)).
\end{aligned} \tag{12.3}$$

Step IV: No expansion is required. Extra derivative bing extra x^μ b^μ, thus non-zero terms with derivatives are $\sim \boldsymbol{b}^2$. Therefore, we just drop x, y, u from the field arguments. **There is a contribution of derivatives. It is considered in the next section.**

$$\begin{aligned}
\mathbf{E} &= -g^3 \frac{\Gamma(7-4\epsilon)}{32\pi^{2d}} \left(C_F - \frac{C_A}{2}\right) \int d^d u d^d x d^d y \int [d\alpha d\beta d\gamma d\rho] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\quad \bar{q}(\frac{\alpha\gamma\rho + \alpha\beta\gamma + \alpha\beta\rho}{\lambda} z_1 + \frac{\beta\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&\quad A_\mu(\frac{\alpha\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \alpha\beta\rho + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\quad \frac{\gamma_\nu(\not{u} - 2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^+(\not{x} + 2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^\mu(\not{x} - \not{y} + \frac{\alpha\beta\rho}{\lambda}(-2\not{b} - \gamma^+ z_1 + \gamma^+ z_2))\gamma^\nu}{[-(\beta+\gamma)x^2 - (\gamma+\rho)y^2 - (\alpha+\rho)u^2 + 2\rho(uy) + 2\gamma(xy) + \frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2 + i0]^{7-4\epsilon}} \\
&\quad q(\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda}\right)).
\end{aligned} \tag{12.4}$$

✓**Step V:** We recombine numerator and drop odd powers of x, y, u

$$\begin{aligned}
\mathbf{E} &= -g^3 \frac{\Gamma(7-4\epsilon)}{32\pi^{2d}} \left(C_F - \frac{C_A}{2}\right) \int d^d u d^d x d^d y \int [d\alpha d\beta d\gamma d\rho] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\quad \bar{q}(\frac{\alpha\gamma\rho + \alpha\beta\gamma + \alpha\beta\rho}{\lambda} z_1 + \frac{\beta\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&\quad A_\mu(\frac{\alpha\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \alpha\beta\rho + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\quad \underline{\gamma_\nu\Big[-2\frac{\beta\gamma\rho}{\lambda}2\frac{\alpha\gamma\rho}{\lambda}\frac{\alpha\beta\rho}{\lambda}\not{b}\gamma^+\not{b}\gamma^\mu(-2\not{b} - \gamma^+ z_1 + \gamma^+ z_2) + \frac{\alpha\beta\rho}{\lambda}\not{u}\gamma^+\not{x}\gamma^\mu(-2\not{b} - \gamma^+ z_1 + \gamma^+ z_2)} \\
&\quad \frac{-2\frac{\beta\gamma\rho}{\lambda}\not{b}\gamma^+\not{x}\gamma^\mu(\not{x} - \not{y}) + 2\frac{\alpha\gamma\rho}{\lambda}\not{u}\gamma^+\not{b}\gamma^\mu(\not{x} - \not{y})\Big]\gamma^\nu}{[-(\beta+\gamma)x^2 - (\gamma+\rho)y^2 - (\alpha+\rho)u^2 + 2\rho(uy) + 2\gamma(xy) + \frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2 + i0]^{7-4\epsilon}} \\
&\quad q(\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda}\right)).
\end{aligned} \tag{12.5}$$

Step VI: Integrate with the help of (7.14)

$$
\checkmark\mathbf{E} = ig^3\frac{\Gamma(-\epsilon)}{32\pi^{d/2}}\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon}\frac{(4\alpha\beta\gamma\rho)^\epsilon\mathbf{B}^\epsilon}{\lambda^3}
\\
\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right))
\\
A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right))
\\
\gamma_\alpha\Big[-2\frac{\beta\gamma\rho}{\lambda}2\frac{\alpha\gamma\rho}{\lambda}\frac{\alpha\beta\rho}{\lambda}\frac{\epsilon\lambda^2}{4\alpha\beta\gamma\rho\boldsymbol{b}^2}\not{b}\gamma^+\not{b}\gamma^\mu(-2\not{b}-\gamma^+z_1+\gamma^+z_2)+\frac{\alpha\beta\rho}{\lambda}\gamma_\nu\gamma^+\gamma^\nu\gamma^\mu(-2\not{b}-\gamma^+z_1+\gamma^+z_2)\frac{\rho\gamma}{2}
\\
-2\frac{\beta\gamma\rho}{\lambda}\not{b}\gamma^+\gamma_\nu\gamma^\mu\gamma^\nu\frac{\alpha\rho}{2}+2\frac{\alpha\gamma\rho}{\lambda}\gamma_\nu\gamma^+\not{b}\gamma^\mu\gamma^\nu\frac{-\rho\beta}{2}\Big]\gamma^\alpha
\\
q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)). \tag{12.6}
$$

Step VII: We simplify the Dirac structure with Mathematica and get (we also use $A_+ = 0$), and obtain simple expression

$$
\checkmark\mathbf{E} = iga_s\mathbf{B}^\epsilon\frac{\Gamma(-\epsilon)}{2}\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\frac{\alpha\beta\gamma}{\lambda^3}
\\
\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right))A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right))
\\
\frac{4\alpha\beta\gamma\rho^2}{\lambda}\Big\{(1+2\epsilon)\gamma^+\gamma_\perp^\mu\not{b}-(2+\epsilon)\gamma^+\not{b}\gamma_\perp^\mu\Big\}q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)). \tag{12.7}
$$

Or

$$
\mathbf{E} = 2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\frac{(\alpha\beta\gamma\rho)^2}{\lambda^4}
\\
\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right))A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right))
\\
\Big\{(1+2\epsilon)\gamma^+\gamma_\perp^\mu\not{b}-(2+\epsilon)\gamma^+\not{b}\gamma_\perp^\mu\Big\}q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)). \tag{12.8}
$$

✓**Step VIII:** We pass to the dual variables

$$
\frac{\beta\gamma\rho}{\lambda} = \alpha', \qquad \frac{\alpha\gamma\rho}{\lambda}=\beta', \qquad \frac{\alpha\beta\rho}{\lambda}=\gamma' \qquad \frac{\alpha\beta\gamma}{\lambda}=\rho'.
$$

The Jacobian is

$$
\frac{[d\alpha' d\beta' d\gamma' d\rho']}{[d\alpha d\beta d\gamma d\rho]} = \frac{(\alpha\beta\gamma\rho)^2}{\lambda^4}. \tag{12.9}
$$

We substitute and drop primes.

$$
\mathbf{E} = 2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]
\\
\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))A_\mu(z_{21}^\beta-\boldsymbol{b}(1-2\beta))\Big\{(1+2\epsilon)\gamma^+\gamma_\perp^\mu\not{b}-(2+\epsilon)\gamma^+\not{b}\gamma_\perp^\mu\Big\}q(z_{21}^{\beta+\gamma}-\boldsymbol{b}(1-2\gamma-2\rho)). \tag{12.10}
$$

B. The derivative part of diagram E

The diagram **E** has terms which require encountering of the derivatives, that were missed in the previous section. In this section, we fix this issue. We start from **Step IV** expanding one derivative higher.

Step IV: We expand up to the first derivative

$$\begin{aligned}
\mathbf{E} &= -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\quad \bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right))\left(1+u^\mu\overleftarrow{\partial}_\mu\right) \\
&\quad (1+x^\mu\partial_\mu)A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\quad \frac{\gamma_\nu(\not u-2\frac{\beta\gamma\rho}{\lambda}\not b)\gamma^+(\not x+2\frac{\alpha\gamma\rho}{\lambda}\not b)\gamma^\mu(\not x-\not y+\frac{\alpha\beta\rho}{\lambda}(-2\not b-\gamma^+z_1+\gamma^+z_2))\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
&\quad (1+y^\mu\partial_\mu)q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)).
\end{aligned} \tag{12.11}$$

Step V: The main part of the diagram is evaluate in previous sections. Here we consider only derivative part, lets call it $\mathbf{E}'$. We have 3 contribution with derivative acting on different fields. The notation is straightforward

$$\begin{aligned}
\mathbf{E}'_{\bar{q}} &= -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\quad \partial_\alpha\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&\quad A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\quad u^\alpha\frac{\gamma_\nu[\not u\gamma^+\not x\gamma^\mu(\not x-\not y)-4\frac{\alpha\beta\gamma^2\rho^2}{\lambda^2}\not b\gamma^+\not b\gamma^\mu(\not x-\not y)}{\dots} \\
&\quad \frac{-2\frac{\alpha\beta^2\gamma\rho^2}{\lambda^2}\not b\gamma^+\not x\gamma^\mu(-2\not b-\gamma^+z_1+\gamma^+z_2)+2\frac{\alpha^2\beta\gamma\rho^2}{\lambda^2}\not u\gamma^+\not b\gamma^\mu(-2\not b-\gamma^+z_1+\gamma^+z_2)]\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
&\quad q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)),
\end{aligned} \tag{12.12}$$

and similar terms with derivative acting on A and q. Before we proceed with the integration let us count the powers. The integral will produce the $\mathbf{B}$ in the power $[-1+\epsilon+(\text{number of indices}/2)]$. Taking into account the factors of b that are presented in the numerator here we have three types of contributions:

$$\begin{aligned}
&u_\alpha\not u\gamma^+\not x\gamma^\mu(\not x-\not y)\to\mathbf{B}^{1+\epsilon}, \\
&u_\alpha\not b\gamma^+\not b\gamma^\mu(\not x-\not y)\to\not b\not b\mathbf{B}^\epsilon\to\mathbf{B}^{1+\epsilon}, \\
&u_\alpha\not u\gamma^+\not b\gamma^\mu(-\gamma^+z_1+\gamma^+z_2)\to\not b\mathbf{B}^\epsilon.
\end{aligned} \tag{12.13}$$

Thus, we conclude that only the term with z's could give contribution to twist-3. We continue only with this term:

$$\begin{aligned}
\mathbf{E}'_{\bar{q}} &= -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\quad \partial_\alpha\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&\quad A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\quad u^\alpha\frac{\gamma_\nu[-2\frac{\alpha\beta^2\gamma\rho^2}{\lambda^2}\not b\gamma^+\not x\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+2\frac{\alpha^2\beta\gamma\rho^2}{\lambda^2}\not u\gamma^+\not b\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
&\quad q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)),
\end{aligned} \tag{12.14}$$

$$\begin{aligned}
\mathbf{E'}_A &= -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&\partial_\alpha A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&x^\alpha\frac{\gamma_\nu[-2\frac{\alpha\beta^2\gamma\rho^2}{\lambda^2}\not{y}\gamma^+\not{x}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+2\frac{\alpha^2\beta\gamma\rho^2}{\lambda^2}\not{u}\gamma^+\not{y}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
&q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)),
\end{aligned} \tag{12.15}$$

$$\begin{aligned}
\mathbf{E'}_q &= -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&y^\alpha\frac{\gamma_\nu[-2\frac{\alpha\beta^2\gamma\rho^2}{\lambda^2}\not{y}\gamma^+\not{x}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+2\frac{\alpha^2\beta\gamma\rho^2}{\lambda^2}\not{u}\gamma^+\not{y}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
&\partial_\alpha q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)),
\end{aligned} \tag{12.16}$$

Step VI: Integrate with the help of (7.14)

$$\begin{aligned}
\mathbf{E'}_{\bar{q}} &= ig^3\frac{\Gamma(-\epsilon)4^\epsilon}{32\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon}\frac{(\alpha\beta\gamma\rho)^\epsilon}{\lambda^3} \\
&\partial_\alpha\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\gamma_\nu[-2\frac{\alpha\beta^2\gamma^2\rho^3}{2\lambda^2}\not{y}\gamma^+\gamma^\alpha\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+2\frac{\alpha^2\beta\gamma\rho^2}{2\lambda^2}(\beta\rho+\rho\gamma+\gamma\beta)\gamma^\alpha\gamma^+\not{y}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu \\
&q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)),
\end{aligned} \tag{12.17}$$

$$\begin{aligned}
\mathbf{E'}_A &= ig^3\frac{\Gamma(-\epsilon)4^\epsilon}{32\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon}\frac{(\alpha\beta\gamma\rho)^\epsilon}{\lambda^3} \\
&\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\
&\partial_\alpha A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\gamma_\nu[-2\frac{\alpha\beta^2\gamma\rho^2}{2\lambda^2}(\alpha\rho+\alpha\gamma+\rho\gamma)\not{y}\gamma^+\gamma^\alpha\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+2\frac{\alpha^2\beta\gamma^2\rho^3}{2\lambda^2}\gamma^\alpha\gamma^+\not{y}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu \\
&q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)),
\end{aligned} \tag{12.18}$$

$$\mathbf{E'}_q = ig^3\frac{\Gamma(-\epsilon)4^\epsilon}{32\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon}\frac{(\alpha\beta\gamma\rho)^\epsilon}{\lambda^3} \tag{12.19}$$
$$\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right))$$
$$A_\mu(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right))$$
$$\gamma_\nu[-2\frac{\alpha\beta^2\gamma\rho^2}{2\lambda^2}(\alpha+\rho)\gamma\ \not{b}\gamma^+\gamma^\alpha\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+2\frac{\alpha^2\beta\gamma\rho^2}{2\lambda^2}(\gamma+\beta)\rho\gamma^\alpha\gamma^+\ \not{b}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu$$
$$\partial_\alpha q(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)),$$

Step VII: We pass to the dual variables

$$\frac{\beta\gamma\rho}{\lambda} = \alpha', \qquad \frac{\alpha\gamma\rho}{\lambda}=\beta', \qquad \frac{\alpha\beta\rho}{\lambda}=\gamma' \qquad \frac{\alpha\beta\gamma}{\lambda}=\rho'.$$

The Jacobian is

$$\frac{[d\alpha' d\beta' d\gamma' d\rho']}{[d\alpha d\beta d\gamma d\rho]}=\frac{(\alpha\beta\gamma\rho)^2}{\lambda^4}. \tag{12.20}$$

We substitute and drop primes.

$$\mathbf{E'}_{\bar{q}} = ig^3\frac{\Gamma(-\epsilon)4^\epsilon}{32\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\partial_\alpha\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))A_\mu(z_{21}^\beta-\boldsymbol{b}(1-2\beta)) \tag{12.21}$$
$$\gamma_\nu[-\alpha\ \not{b}\gamma^+\gamma^\alpha\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+\bar{\alpha}\gamma^\alpha\gamma^+\ \not{b}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu$$
$$q(z_{21}^{\beta+\gamma}-\boldsymbol{b}(1-2(\beta+\gamma)),$$

$$\mathbf{E'}_A = ig^3\frac{\Gamma(-\epsilon)4^\epsilon}{32\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))\partial_\alpha A_\mu(z_{21}^\beta-\boldsymbol{b}(1-2\beta)) \tag{12.22}$$
$$\gamma_\nu[-\bar{\beta}\ \not{b}\gamma^+\gamma^\alpha\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+\beta\gamma^\alpha\gamma^+\ \not{b}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu$$
$$q(z_{21}^{\beta+\gamma}-\boldsymbol{b}(1-2(\beta+\gamma)),$$

$$\mathbf{E'}_q = ig^3\frac{\Gamma(-\epsilon)4^\epsilon}{32\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))A_\mu(z_{21}^\beta-\boldsymbol{b}(1-2\beta)) \tag{12.23}$$
$$\gamma_\nu[-(1-\beta-\gamma)\ \not{b}\gamma^+\gamma^\alpha\gamma^\mu(\gamma^+z_2-\gamma^+z_1)+(\beta+\gamma)\gamma^\alpha\gamma^+\ \not{b}\gamma^\mu(\gamma^+z_2-\gamma^+z_1)]\gamma^\nu$$
$$\partial_\alpha q(z_{21}^{\beta+\gamma}-\boldsymbol{b}(1-2(\beta+\gamma)),$$

Step VIII: We simplify γ-algebra. We have two structures

$$\gamma_\nu\gamma^\alpha\gamma^+\ \not{b}\gamma^\mu\gamma^+\gamma^\nu = 0, \tag{12.24}$$
$$\gamma_\nu\ \not{b}\gamma^+\gamma^\alpha\gamma^\mu\gamma^+\gamma^\nu = -2n^\alpha\gamma_\nu\ \not{b}\gamma^+\gamma^\mu\gamma^\nu=-4(1-\epsilon)n^\alpha b_\mu\gamma^+-4(1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b^\nu n_\alpha. \tag{12.25}$$

Thus the second terms in bracket vanish. We get

$$\mathbf{E'}_{\bar{q}} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\partial_+\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))A_\mu(z_{21}^\beta-\boldsymbol{b}(1-2\beta)) \tag{12.26}$$
$$\alpha z_{21}[(1-\epsilon)b^\mu\gamma^++(1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{21}^{\beta+\gamma}-\boldsymbol{b}(1-2(\beta+\gamma)),$$

$$\mathbf{E'}_A = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))\partial_+A_\mu(z_{21}^\beta-\boldsymbol{b}(1-2\beta)) \tag{12.27}$$
$$\bar{\beta}z_{21}[(1-\epsilon)b^\mu\gamma^++(1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{21}^{\beta+\gamma}-\boldsymbol{b}(1-2(\beta+\gamma)),$$

$$\mathbf{E'}_q = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))A_\mu(z_{21}^\beta-\boldsymbol{b}(1-2\beta))$$
$$(1-\beta-\gamma)z_{21}[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]\partial_+q(z_{21}^{\beta+\gamma}-\boldsymbol{b}(1-2(\beta+\gamma)),\tag{12.28}$$

Finally we can drop the $\boldsymbol{b}$ in the arguments of fields

$$\mathbf{E'}_{\bar{q}} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\partial_+\bar{q}(z_{12}^\alpha)A_\mu(z_{21}^\beta)$$
$$\alpha z_{21}[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{21}^{\beta+\gamma}),\tag{12.29}$$

$$\mathbf{E'}_A = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha)\partial_+A_\mu(z_{21}^\beta)$$
$$\bar{\beta}z_{21}[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{21}^{\beta+\gamma}),\tag{12.30}$$

$$\mathbf{E'}_q = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha)A_\mu(z_{21}^\beta)$$
$$(1-\beta-\gamma)z_{21}[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]\partial_+q(z_{21}^{\beta+\gamma}),\tag{12.31}$$

C. Diagram E*

The diagram reads

$$\mathbf{E}^* = \left(ig\int d^dy\bar{q}(y)\ \not{B}(y)\psi(y)\right)\left(ig\int d^dx\bar{\psi}(x)\ \not{A}(x)\psi(x)\right)\bar{\psi}(z_1+\boldsymbol{b})\gamma^+\psi(z_2-\boldsymbol{b})\Big\}\left(ig\int d^du\bar{\psi}(u)\ \not{B}(u)q(u)\right)$$

Step I: Substituting propagators

$$\mathbf{E}^* = (ig)^3\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}}\int d^dud^dxd^dy\bar{q}(y)t^At^Bt^AA_\mu^B(x)$$
$$\gamma_\nu\frac{\not{y}-\not{x}}{[-(y-x)^2+i0]^{2-\epsilon}}\gamma^\mu\frac{\not{x}-\not{z}_1-\not{b}}{[-(x-z_1-b)^2+i0]^{2-\epsilon}}\gamma^+\frac{\not{z}_2-\not{b}-\not{u}}{[-(u-z_2+b)^2+i0]^{2-\epsilon}}\gamma^\nu\frac{1}{[-(u-y)^2+i0]^{1-\epsilon}}q(u).\tag{12.32}$$

Step II: Simplifying

$$\mathbf{E}^* = -g^3\frac{\Gamma^3(2-\epsilon)\Gamma(1-\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\bar{q}(y)A_\mu(x)$$
$$\frac{\gamma_\nu(\not{y}-\not{x})\gamma^\mu(\not{x}-\not{b})\gamma^+(\not{u}+\not{b})\gamma^\nu}{[-(y-x)^2+i0]^{2-\epsilon}[-(x-z_1-b)^2+i0]^{2-\epsilon}[-(u-z_2+b)^2+i0]^{2-\epsilon}[-(u-y)^2+i0]^{1-\epsilon}}q(u).\tag{12.33}$$

Step III: We joining propagators according to

$$\begin{aligned}(x-z_1-b)^2 &\to \beta\\ (u-z_2+b)^2 &\to \alpha\\ (u-y)^2 &\to \rho\\ (x-y)^2 &\to \gamma\end{aligned}$$

We also make a shift

$$\begin{aligned}x &\to x+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)\\ y &\to y+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)\\ u &\to u+\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)\end{aligned}$$

The diagram turns into

$$\begin{aligned}
\mathbf{E}^* &= -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\bar{q}(y+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\
&A_\mu(x+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\frac{\gamma_\nu(\not{y}-\not{x}-\frac{\alpha\beta\rho}{\lambda}(2\not{b}+\gamma^+z_{12}))\gamma^\mu(\not{x}-2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^+(\not{u}+2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
&q(u+\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).
\end{aligned} \tag{12.34}$$

Step IV: Here we consider the part which has no derivatives. The part with derivative is considered in the next section. We drop integration variables from the fields and collect only even powers of it:

$$\begin{aligned}
\mathbf{E}^* &= -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F-\frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
&\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\
&A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\underline{\gamma_\nu[-\frac{\alpha\beta\rho}{\lambda}\frac{-2\alpha\gamma\rho}{\lambda}\frac{2\beta\gamma\rho}{\lambda}(2\not{b}+\gamma^+z_{12})\gamma^\mu\not{b}\gamma^+\not{b}+2\frac{\beta\gamma\rho}{\lambda}(\not{y}-\not{x})\gamma^\mu\not{x}\gamma^+\not{b}} \\
&\frac{-2\frac{\alpha\gamma\rho}{\lambda}(\not{y}-\not{x})\gamma^\mu\not{b}\gamma^+\not{u}-\frac{\alpha\beta\rho}{\lambda}(2\not{b}+\gamma^+z_{12})\gamma^\mu\not{x}\gamma^+\not{u}]\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
&q(\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).
\end{aligned} \tag{12.35}$$

Step V: Integrating

$$\begin{aligned}
\mathbf{E}^* &= ig^3\frac{4^\epsilon\Gamma(-\epsilon)}{32\pi^{d/2}}\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon}\frac{(\alpha\beta\gamma\rho)^\epsilon}{\lambda^3} \\
&\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\
&A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
&\gamma_\alpha\Bigg\{-\frac{\alpha\beta\rho}{\lambda}\frac{-2\alpha\gamma\rho}{\lambda}\frac{2\beta\gamma\rho}{\lambda}\frac{\epsilon\lambda^2}{-4\alpha\beta\gamma\rho b^2}(2\not{b}+\gamma^+z_{12})\gamma^\mu\not{b}\gamma^+\not{b}+2\frac{\beta\gamma\rho}{\lambda}\frac{-\alpha\rho}{2}\gamma_\nu\gamma^\mu\gamma^\nu\gamma^+\not{b} \\
&-2\frac{\alpha\gamma\rho}{\lambda}\frac{\rho\beta}{2}\gamma_\nu\gamma^\mu\not{b}\gamma^+\gamma^\nu-\frac{\alpha\beta\rho}{\lambda}\frac{\rho\gamma}{2}(2\not{b}+\gamma^+z_{12})\gamma^\mu\gamma_\nu\gamma^+\gamma^\nu\Bigg\}\gamma^\alpha \\
&q(\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).
\end{aligned} \tag{12.36}$$

We simplify the Dirac structure with Mathematica and get (we also use $A_+ = 0$), and obtain simple expression

$$\begin{aligned}\mathbf{E}^* \;=\; & 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\frac{(\alpha\beta\gamma\rho)^2}{\lambda^4} \\ & \bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2 - \boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\ & A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\gamma\rho}{\lambda}z_2 + \boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\ & \left\{(1+2\epsilon)\gamma^+ \not{b}\gamma^\mu - (2+\epsilon)\gamma^+\gamma^\mu \not{b}\right\}q(\frac{\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2 - \boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).\end{aligned} \tag{12.37}$$

✓**Step VI:** We substitute dual variables and drop primes.

$$\begin{aligned}\mathbf{E}^* \;=\; & 2iga_s\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma d\rho] \\ & \bar{q}(z_{21}^{\alpha+\rho} - \boldsymbol{b}\,(1-2\alpha-2\rho))A_\mu(z_{12}^{\beta} + \boldsymbol{b}\,(1-2\beta))\left[(1+2\epsilon)\gamma^+ \not{b}\gamma_\perp^\mu - (2+\epsilon)\gamma^+\gamma_\perp^\mu \not{b}\right]q(z_{21}^{\alpha} - \boldsymbol{b}\,(1-2\alpha)).\end{aligned} \tag{12.38}$$

D. Derivative part of diagram E*

We start from the **Step IV** and consider the term with derivative:

Step IV: Here we consider the part which has no derivatives. The part with derivative is considered in the next section. We drop integration variables from the fields and collect only even powers of it:

$$\begin{aligned}\mathbf{E}^* \;=\; & -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F - \frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\ & (1+y^\alpha\partial_\alpha)\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2 - \boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\ & \frac{\gamma_\nu(\not{y} - \not{x} - \frac{\alpha\beta\rho}{\lambda}(2\not{b} + \gamma^+ z_{12}))\gamma^\mu(\not{x} - 2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^+(\not{u} + 2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^\nu}{[-(\beta+\gamma)x^2 - (\gamma+\rho)y^2 - (\alpha+\rho)u^2 + 2\rho(uy) + 2\gamma(xy) + \frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2 + i0]^{7-4\epsilon}} \\ & (1+u^\alpha\partial_\alpha)q(\frac{\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2 - \boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).\end{aligned} \tag{12.39}$$

Step V: From the analysis made for the diagram **E** we know the only the part proportional to z_{12} contributes to twist-3. We obtain

$$\begin{aligned}\mathbf{E}^* \;=\; & -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F - \frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\ & (1+y^\alpha\partial_\alpha)\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2 - \boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\ & (1+x^\alpha\partial_\alpha)A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\gamma\rho}{\lambda}z_2 + \boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\ & \frac{-\frac{\alpha\beta\rho}{\lambda}z_{12}\gamma_\nu\gamma^+\gamma^\mu(\not{x} - 2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^+(\not{u} + 2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^\nu}{[-(\beta+\gamma)x^2 - (\gamma+\rho)y^2 - (\alpha+\rho)u^2 + 2\rho(uy) + 2\gamma(xy) + \frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2 + i0]^{7-4\epsilon}} \\ & (1+u^\alpha\partial_\alpha)q(\frac{\beta\gamma\rho}{\lambda}z_1 + \frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2 - \boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).\end{aligned} \tag{12.40}$$

We also know that $\gamma^+\gamma^\mu \not{b}\gamma^+ \sim 0$ since $A_+ = 0$. We simplify further

$$\begin{aligned}
\mathbf{E}^* \;=\; & -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F - \frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
& (1+y^\alpha\partial_\alpha)\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\
& (1+x^\alpha\partial_\alpha)A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
& \frac{-\frac{\alpha\beta\rho}{\lambda}2\frac{\beta\gamma\rho}{\lambda}z_{12}\gamma_\nu\gamma^+\gamma^\mu\not{x}\gamma^+\not{y}\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
& (1+u^\alpha\partial_\alpha)q(\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).
\end{aligned} \tag{12.41}$$

Step VI: The even parts are

$$\begin{aligned}
\mathbf{E}'^*_{\bar{\mathbf{q}}} \;=\; & -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F - \frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
& y^\alpha\partial_\alpha\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\
& A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
& \frac{-\frac{\alpha\beta\rho}{\lambda}2\frac{\beta\gamma\rho}{\lambda}z_{12}\gamma_\nu\gamma^+\gamma^\mu\not{x}\gamma^+\not{y}\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
& q(\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).
\end{aligned} \tag{12.42}$$

$$\begin{aligned}
\mathbf{E}'^*_{\mathbf{A}} \;=\; & -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F - \frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
& \bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\
& x^\alpha\partial_\alpha A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
& \frac{-\frac{\alpha\beta\rho}{\lambda}2\frac{\beta\gamma\rho}{\lambda}z_{12}\gamma_\nu\gamma^+\gamma^\mu\not{x}\gamma^+\not{y}\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
& q(\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).
\end{aligned} \tag{12.43}$$

$$\begin{aligned}
\mathbf{E}'^*_{\mathbf{q}} \;=\; & -g^3\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\left(C_F - \frac{C_A}{2}\right)\int d^dud^dxd^dy\int[d\alpha d\beta d\gamma d\rho]\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \\
& \bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right)) \\
& A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)) \\
& \frac{-\frac{\alpha\beta\rho}{\lambda}2\frac{\beta\gamma\rho}{\lambda}z_{12}\gamma_\nu\gamma^+\gamma^\mu\not{x}\gamma^+\not{y}\gamma^\nu}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(uy)+2\gamma(xy)+\frac{\alpha\beta\gamma\rho}{\lambda}\boldsymbol{b}^2+i0]^{7-4\epsilon}} \\
& u^\alpha\partial_\alpha q(\frac{\beta\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)).
\end{aligned} \tag{12.44}$$

Step VII: We integrate

$$\mathbf{E}'^{*}_{\bar{\mathbf{q}}} = ig^3 4^\epsilon \frac{\Gamma(-\epsilon)}{32\pi^{d/2}} \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma d\rho] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \frac{(\alpha\beta\gamma\rho)^\epsilon}{\lambda^3}$$
$$\partial_\alpha \bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right))$$
$$A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right))$$
$$\Big[-\frac{\alpha\gamma+\rho\gamma}{2}\frac{\alpha\beta\rho}{\lambda} 2\frac{\beta\gamma\rho}{\lambda} \not{b} z_{12}\gamma_\nu\gamma^+\gamma^\mu\gamma^\alpha\gamma^+\gamma^\nu\Big] q(\frac{\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)). \quad (12.45)$$

$$\mathbf{E}'^{*}_{\mathbf{A}} = ig^3 4^\epsilon \frac{\Gamma(-\epsilon)}{32\pi^{d/2}} \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma d\rho] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \frac{(\alpha\beta\gamma\rho)^\epsilon}{\lambda^3}$$
$$\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right))$$
$$\partial_\alpha A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right))$$
$$\Big[-\frac{\alpha\rho+\alpha\gamma+\rho\gamma}{2}\frac{\alpha\beta\rho}{\lambda} 2\frac{\beta\gamma\rho}{\lambda} \not{b} z_{12}\gamma_\nu\gamma^+\gamma^\mu\gamma^\alpha\gamma^+\gamma^\nu\Big] q(\frac{\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)). \quad (12.46)$$

$$\mathbf{E}'^{*}_{\mathbf{q}} = ig^3 4^\epsilon \frac{\Gamma(-\epsilon)}{32\pi^{d/2}} \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma d\rho] \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{1-\epsilon}\rho^{-\epsilon} \frac{(\alpha\beta\gamma\rho)^\epsilon}{\lambda^3}$$
$$\bar{q}(\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\rho+\alpha\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}\right))$$
$$A_\mu(\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right))$$
$$\Big[-\frac{\rho\gamma}{2}\frac{\alpha\beta\rho}{\lambda} 2\frac{\beta\gamma\rho}{\lambda} \not{b} z_{12}\gamma_\nu\gamma^+\gamma^\mu\gamma^\alpha\gamma^+\gamma^\nu\Big] \partial_\alpha q(\frac{\beta\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma+\alpha\beta\rho+\alpha\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)). \quad (12.47)$$

Step VIII: Passing to the dual variables

$$\mathbf{E}'^{*}_{\bar{\mathbf{q}}} = ig^3 4^\epsilon \frac{\Gamma(-\epsilon)}{32\pi^{d/2}} \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma d\rho] \partial_\alpha \bar{q}(z_{12}^{\beta+\gamma} + \boldsymbol{b}(1-2\beta-2\gamma)) A_\mu(z_{12}^{\beta} + \boldsymbol{b}(1-2\beta)) \quad (12.48)$$
$$\Big[-(\alpha+\rho) z_{12}\gamma_\nu\gamma^+\gamma^\mu\gamma^\alpha\gamma^+ \not{b}\gamma^\nu\Big] q(z_{21}^{\alpha} - \boldsymbol{b}(1-2\alpha)).$$

$$\mathbf{E}'^{*}_{\mathbf{A}} = ig^3 4^\epsilon \frac{\Gamma(-\epsilon)}{32\pi^{d/2}} \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma d\rho] \bar{q}(z_{12}^{\beta+\gamma} + \boldsymbol{b}(1-2\beta-2\gamma)) \partial_\alpha A_\mu(z_{12}^{\beta} + \boldsymbol{b}(1-2\beta)) \quad (12.49)$$
$$\Big[-\bar{\beta} z_{21}\gamma_\nu\gamma^+\gamma^\mu\gamma^\alpha\gamma^+ \not{b}\gamma^\nu\Big] q(z_{21}^{\alpha} - \boldsymbol{b}(1-2\alpha)).$$

$$\mathbf{E}'^{*}_{\mathbf{q}} = ig^3 4^\epsilon \frac{\Gamma(-\epsilon)}{32\pi^{d/2}} \mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma d\rho] \bar{q}(z_{12}^{\beta+\gamma} + \boldsymbol{b}(1-2\beta-2\gamma)) A_\mu(z_{12}^{\beta} + \boldsymbol{b}(1-2\beta)) \quad (12.50)$$
$$\Big[-\alpha z_{12}\gamma_\nu\gamma^+\gamma^\mu\gamma^\alpha\gamma^+ \not{b}\gamma^\nu\Big] \partial_\alpha q(z_{21}^{\alpha} - \boldsymbol{b}(1-2\alpha)).$$

Step IX: Simplifying

$$\mathbf{E}'^*_{\bar{\mathbf{q}}} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\partial_+\bar{q}(z_{12}^{\beta+\gamma} + \boldsymbol{b}\,(1-2\beta-2\gamma))A_\mu(z_{12}^\beta + \boldsymbol{b}\,(1-2\beta))(1-\beta-\gamma)z_{12}((1-\epsilon)\gamma^+b^\mu - (1+\epsilon)\gamma^+b_\nu\gamma^+\gamma_T^{\mu\nu})q(z_{21}^\alpha - \boldsymbol{b}\,(1-2\alpha)). \tag{12.51}$$

$$\mathbf{E}'^*_{\mathbf{A}} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\beta+\gamma} + \boldsymbol{b}\,(1-2\beta-2\gamma))\partial_+A_\mu(z_{12}^\beta + \boldsymbol{b}\,(1-2\beta))\bar{\beta}z_{12}((1-\epsilon)\gamma^+b^\mu - (1+\epsilon)\gamma^+b_\nu\gamma^+\gamma_T^{\mu\nu})q(z_{21}^\alpha - \boldsymbol{b}\,(1-2\alpha)). \tag{12.52}$$

$$\mathbf{E}'^*_{\mathbf{q}} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\beta+\gamma} + \boldsymbol{b}\,(1-2\beta-2\gamma))A_\mu(z_{12}^\beta + \boldsymbol{b}\,(1-2\beta))\alpha z_{12}((1-\epsilon)\gamma^+b^\mu - (1+\epsilon)\gamma^+b_\nu\gamma^+\gamma_T^{\mu\nu})\partial_+q(z_{21}^\alpha - \boldsymbol{b}\,(1-2\alpha)). \tag{12.53}$$

E. Combining the diagrams E and E*

The expressions for diagrams **E** and $\mathbf{E}^*$ are linear in $\boldsymbol{b}$. Therefore, we can set $\boldsymbol{b}$ to zero in the arguments of the fields

$$\mathbf{E} = 2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma]\bar{q}(z_{12}^\alpha)A_\mu(z_{21}^\beta)\Big\{(1+2\epsilon)\gamma^+\gamma_\perp^\mu\, \not{b} - (2+\epsilon)\gamma^+\, \not{b}\gamma_\perp^\mu\Big\}q(z_{21}^{\beta+\gamma}), \tag{12.54}$$

$$\mathbf{E}^* = 2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\beta+\gamma})A_\mu(z_{12}^\beta)\Big\{(1+2\epsilon)\gamma^+\, \not{b}\gamma_\perp^\mu - (2+\epsilon)\gamma^+\gamma_\perp^\mu\, \not{b}\Big\}q(z_{21}^\alpha) \tag{12.55}$$

We simplify the γ-algebra using (11.29)

$$\mathbf{E} = 2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma]\bar{q}(z_{12}^\alpha)A_\mu(z_{21}^\beta)\gamma^+\Big\{-(1-\epsilon)b^\mu + 3(1+\epsilon)b_\nu\gamma_T^{\mu\nu}\Big\}q(z_{21}^{\beta+\gamma}), \tag{12.56}$$

$$\mathbf{E}^* = 2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma]\bar{q}(z_{12}^{\beta+\gamma})A_\mu(z_{12}^\beta)\gamma^+\Big\{-(1-\epsilon)b^\mu - 3(1+\epsilon)b_\nu\gamma_T^{\mu\nu}\Big\}q(z_{21}^\alpha). \tag{12.57}$$

Next we apply (6.19) to translate the operators to the standard operators. In fact we use (11.41,11.42) we get

$$\mathbf{E} = -2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\int_{-\infty}^{z_1}\int[d\alpha d\beta d\gamma]\bar{q}(z_{12}^\alpha)F_{\mu+}(z_{2\sigma}^\beta)\gamma^+\Big\{-\beta(1-\epsilon)b^\mu + 3\beta(1+\epsilon)b_\nu\gamma_T^{\mu\nu}\Big\}q(z_{21}^{\beta+\gamma}), \tag{12.58}$$

$$\mathbf{E}^* = -2iga_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)\int_{-\infty}^{z_2}\int[d\alpha d\beta d\gamma]\bar{q}(z_{12}^{\beta+\gamma})A_\mu(z_{1\sigma}^\beta)\gamma^+\Big\{-\beta(1-\epsilon)b^\mu - 3\beta(1+\epsilon)b_\nu\gamma_T^{\mu\nu}\Big\}q(z_{21}^\alpha). \tag{12.59}$$

Thus the final expression is

$$\mathbf{E} = 2ia_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)b_\mu\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\Big\{\beta(1-\epsilon)\mathcal{T}^\mu_{\gamma^+}(z_{12}^\alpha, z_{2\sigma}^\beta, z_{21}^{\beta+\gamma}) + 3\beta(1+\epsilon)\mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z_{12}^\alpha, z_{2\sigma}^\beta, z_{21}^{\beta+\gamma})\Big\}, \tag{12.60}$$

$$\mathbf{E}^* = 2ia_s\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(C_F - \frac{C_A}{2}\right)b_\mu\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\Big\{\beta(1-\epsilon)\mathcal{T}^\mu_{\gamma^+}(z_{12}^{\beta+\gamma}, z_{1\sigma}^\beta, z_{21}^\alpha) - 3\beta(1+\epsilon)\mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z_{12}^{\beta+\gamma}, z_{1\sigma}^\beta, z_{21}^\alpha)\Big\}. \tag{12.61}$$

We also have $\mathbf{E}'$ part. It requires the integration by parts to get rid of derivatives. We demonstrate the procedure for $\mathbf{E}'_{\bar{q}}$ in details.

$$\int [d\alpha d\beta d\gamma d\rho]\partial_+\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})\alpha z_{21}q(z_{21}^{\beta+\gamma}) = \int [d\alpha d\beta d\gamma d\rho]\alpha\partial_\alpha\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma}) \tag{12.62}$$
$$= \int d\beta d\gamma \int_0^{1-\beta-\gamma} d\alpha\, \alpha\partial_\alpha\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma})$$
$$= \int d\beta d\gamma(1-\beta-\gamma)\bar{q}(z_{12}^{1-\beta-\gamma})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma}) - \int d\beta d\gamma \int_0^{1-\beta-\gamma} d\alpha\,\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma})$$
$$= \underbrace{\int d\beta d\gamma(1-\beta-\gamma)\bar{q}(z_{21}^{\beta+\gamma})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma})}_{A1} - \int [d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma}).$$

For other terms we get

$$\int [d\alpha d\beta d\gamma d\rho]\bar{\beta}\bar{q}(z_{12}^{\alpha})\partial_+A_\mu(z_{21}^{\beta})z_{21}q(z_{21}^{\beta+\gamma}) \tag{12.63}$$
$$= \underbrace{-\int d\alpha d\rho(\alpha+\rho)\bar{q}(z_{21}^{\alpha})A_\mu(z_{12}^{\alpha+\rho})q(z_{12}^{\alpha+\rho})}_{B1} + \int d\alpha d\rho\bar{q}(z_{12}^{\alpha})A_\mu(z_2)q(z_{12}^{\alpha+\rho}) - \int [d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma}).$$

$$\int [d\alpha d\beta d\gamma d\rho](1-\beta-\gamma)\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})z_{21}\partial_+q(z_{21}^{\beta+\gamma}) \tag{12.64}$$
$$= \underbrace{-\int d\alpha d\beta\,\alpha\,\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{12}^{\alpha})}_{A2} + \underbrace{\int d\alpha d\beta(1-\beta)\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta})}_{B2} - \int [d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma}).$$

Then we see that term $A1$ cancels the term $A2$ after replacement $1-\beta-\gamma \to \alpha$. The term $B1$ cancels the term $B2$ after replacement $1-\alpha-\rho \to \beta$. So from the sum of these terms we obtain

$$..+..+.. \;=\; \int d\alpha d\rho\bar{q}(z_{12}^{\alpha})A_\mu(z_2)q(z_{12}^{\alpha+\rho}) - 3\int [d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})q(z_{21}^{\beta+\gamma}). \tag{12.65}$$

Substituting it to the diagrams we get

$$\mathbf{E}'_{\bar{\mathbf{q}}} + \mathbf{E}'_{\mathbf{A}} + \mathbf{E}'_{\mathbf{q}} \;=\; 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\Bigg\{\int d\alpha d\rho\bar{q}(z_{12}^{\alpha})A_\mu(z_2)[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{12}^{\alpha+\rho}) \tag{12.66}$$
$$-3\int [d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\alpha})A_\mu(z_{21}^{\beta})[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{21}^{\beta+\gamma})\Bigg\}.$$

We transform A to $F^{+\mu}$ and get

$$\mathbf{E}' \;=\; 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)\Bigg\{-\int_{-\infty}^{z_2}\int d\alpha d\rho\bar{q}(z_{12}^{\alpha})F^{\mu+}(\sigma)[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{12}^{\alpha+\rho}) \tag{12.67}$$
$$+3\int_{-\infty}^{z_1} d\sigma\int [d\alpha d\beta d\gamma d\rho]\beta\bar{q}(z_{12}^{\alpha})F^{\mu+}(z_{2\sigma}^{\beta})[(1-\epsilon)b^\mu\gamma^+ + (1+\epsilon)\gamma^+\gamma_T^{\mu\nu}b_\nu]q(z_{21}^{\beta+\gamma})\Bigg\}.$$

Or in the terms of standard operators

$$\mathbf{E}' \;=\; 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F - \frac{C_A}{2}\right)b_\mu\Bigg\{-\int_{-\infty}^{z_2} d\sigma\int d\alpha d\rho\Big[(1-\epsilon)\mathcal{T}^\mu_{\gamma^+}(z_{12}^{\alpha},\sigma,z_{12}^{\alpha+\rho}) - (1+\epsilon)\mathcal{T}^\mu_{\gamma^+\gamma_T^{\nu\mu}}(z_{12}^{\alpha},\sigma,z_{12}^{\alpha+\rho})\Big]$$
$$+3\int_{-\infty}^{z_1} d\sigma\int [d\alpha d\beta d\gamma d\rho]\beta\Big[(1-\epsilon)\mathcal{T}^\mu_{\gamma^+}(z_{12}^{\alpha},z_{2\sigma}^{\beta},z_{21}^{\beta+\gamma}) - (1+\epsilon)\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_{12}^{\alpha},z_{2\sigma}^{\beta},z_{21}^{\beta+\gamma})\Big]\Bigg\}. \tag{12.68}$$

Now, we would like to compare with the calculation made in the momentum space. In the momentum space calculation we made a total shift such, that the first point in $\mathcal{T}$ is z_1 and the integral over σ is till z_2. Also the integral over Feynman parameter was reduced to 3 (i.e. we have to integrate one of them explicitly). Let us make such transformations here.

For **E**-part:

$$\int_{-\infty}^{z_1} d\sigma \int [d\alpha d\beta d\gamma d\rho]\beta\mathcal{T}(z_{12}^{\alpha}, z_{2\sigma}^{\beta}, z_{21}^{\beta+\gamma}) = -\int [d\alpha d\beta d\gamma d\rho]\bar{q}Aq(z_1, z_{12}^{\rho+\gamma}, z_{12}^{\rho}) \tag{12.69}$$
$$= \int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma d\rho](\rho+\gamma)\mathcal{T}(z_1, z_{1\sigma}^{\rho+\gamma}, z_{12}^{\rho}) = \int_{-\infty}^{z_2} d\sigma \int d\beta d\gamma \int_0^{1-\beta-\gamma} d\alpha(\beta+\gamma)\mathcal{T}(z_1, z_{1\sigma}^{\beta+\gamma}, z_{12}^{\beta})$$
$$= \int_{-\infty}^{z_2} d\sigma \int d\beta d\gamma(1-\beta-\gamma)(\beta+\gamma)\mathcal{T}(z_1, z_{1\sigma}^{\beta+\gamma}, z_{12}^{\beta})$$
$$= \int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\alpha(\beta+\gamma)\mathcal{T}(z_1, z_{1\sigma}^{\beta+\gamma}, z_{12}^{\beta})$$

For **E**′-part:

$$\int_{-\infty}^{z_2} d\sigma \int d\alpha d\rho\mathcal{T}(z_{12}^{\alpha}, \sigma, z_{12}^{\alpha+\rho}) = -\int d\alpha d\rho\bar{q}Aq(z_{12}^{\alpha}, z_2, z_{12}^{\alpha+\rho}) \tag{12.70}$$
$$= -\int d\alpha d\rho\bar{q}Aq(z_1, z_{21}^{\alpha}, z_{12}^{\rho}) = \int_{-\infty}^{z_2} d\sigma \int d\alpha d\rho\, \bar{\alpha}\, \bar{q}Aq(z_1, z_{\sigma 1}^{\alpha}, z_{12}^{\rho})$$
$$= \int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\, \bar{\alpha}\, \mathcal{T}(z_1, z_{\sigma 1}^{\alpha}, z_{12}^{\beta}) = \int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma]\, (\beta+\gamma)\, \mathcal{T}(z_1, z_{1\sigma}^{\beta+\gamma}, z_{12}^{\beta})$$

Combining together we obtain

$$\mathbf{E}+\mathbf{E}' = 2ia_s\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\left(C_F - \frac{C_A}{2}\right)b_{\mu}\int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma] \tag{12.71}$$
$$\Big[-(1-\epsilon)(\beta+\gamma)(1-4\alpha)\mathcal{T}_{\gamma^+}^{\mu}(z_1, z_{1\sigma}^{\beta+\gamma}, z_{12}^{\beta}) + (1+\epsilon)(\beta+\gamma)\mathcal{T}_{\gamma^+\gamma_T^{\mu\nu}}^{\nu}(z_1, z_{1\sigma}^{\beta+\gamma}, z_{12}^{\beta})\Big].$$

It coincides with the calculation made in the momentum space!!!

The similar integration by parts procedure for the $\mathbf{E}'^*$ part gives

$$\mathbf{E}_{\bar{\mathbf{q}}}'^* + \mathbf{E}_{\mathbf{A}}'^* + \mathbf{E}_{\mathbf{q}}'^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\left(C_F - \frac{C_A}{2}\right)\Big\{\int [d\alpha d\beta d\gamma]\bar{q}(z_{21}^{\alpha+\beta})A_{\mu}(z_1)((1-\epsilon)\gamma^+b^{\mu} - (1+\epsilon)\gamma^+b_{\nu}\gamma^+\gamma_T^{\mu\nu})q(z_{21}^{\alpha})$$
$$-3\int [d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^{\beta+\gamma})A_{\mu}(z_{12}^{\beta})((1-\epsilon)\gamma^+b^{\mu} - (1+\epsilon)\gamma^+b_{\nu}\gamma^+\gamma_T^{\mu\nu})q(z_{21}^{\alpha})\Big\} \tag{12.72}$$

Passing to $F^{\mu+}$ we get in the terms of standard operators

$$\mathbf{E}' = 2iga_s\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\left(C_F - \frac{C_A}{2}\right)b_{\mu}\Big\{-\int_{-\infty}^{z_1}\int [d\alpha d\beta d\gamma]((1-\epsilon)\mathcal{T}_{\gamma^+}^{\mu}(z_{21}^{\alpha+\beta}, \sigma, z_{21}^{\alpha}) + (1+\epsilon)\mathcal{T}_{\gamma^+\gamma^{\mu\nu}}^{\nu}(z_{21}^{\alpha+\beta}, \sigma, z_{21}^{\alpha}))$$
$$+3\int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma d\rho]\beta((1-\epsilon)\mathcal{T}_{\gamma^+}^{\mu}(z_{12}^{\beta+\gamma}, z_{1\sigma}^{\beta}, z_{21}^{\alpha}) + (1+\epsilon)\mathcal{T}_{\gamma^{\mu\nu}}^{\nu}(z_{12}^{\beta+\gamma}, z_{1\sigma}^{\beta}, z_{21}^{\alpha})\Big\} \tag{12.73}$$

The final result is

$$\mathbf{E}+\mathbf{E}' = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\Big\{\int_{-\infty}^{z_1}\int[d\alpha d\beta d\gamma d\rho]4(1-\epsilon)\beta\mathcal{T}^\mu_{\gamma^+}(z_{12}^\alpha,z_{2\sigma}^\beta,z_{21}^{\beta+\gamma}) \tag{12.74}$$

$$+\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\Big[-(1-\epsilon)\mathcal{T}^\mu_{\gamma^+}(z_{12}^\alpha,\sigma,z_{12}^{\alpha+\beta})+(1+\epsilon)\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_{12}^\alpha,\sigma,z_{12}^{\alpha+\beta})\Big]\Big\},$$

$$\mathbf{E}^*+\mathbf{E}'^* = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\Big\{\int_{-\infty}^{z_2}\int[d\alpha d\beta d\gamma d\rho]4(1-\epsilon)\beta\mathcal{T}^\mu_{\gamma^+}(z_{12}^{\beta+\gamma},z_{1\sigma}^\beta,z_{21}^{\alpha}) \tag{12.75}$$

$$+\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\Big[-(1-\epsilon)\mathcal{T}^\mu_{\gamma^+}(z_{21}^{\alpha+\beta},\sigma,z_{21}^{\alpha})-(1+\epsilon)\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_{21}^{\alpha+\beta},\sigma,z_{21}^{\alpha})\Big]\Big\}.$$

F. Elaboration of diagrams E

The diagrams **E** are naturally split into three terms

$$\mathbf{EE}_1 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\int[d\alpha d\beta d\gamma d\rho]4(1-\epsilon)\beta\Big\{ \tag{12.76}$$

$$\int_{-\infty}^{z_1}d\sigma\mathcal{T}^\mu_{\gamma^+}(z_{12}^\alpha,z_{2\sigma}^\beta,z_{21}^{\beta+\gamma})+\int_{-\infty}^{z_2}d\sigma\mathcal{T}^\mu_{\gamma^+}(z_{12}^{\beta+\gamma},z_{1\sigma}^\beta,z_{21}^{\alpha})\Big\},$$

$$\mathbf{EE}_2 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\int[d\alpha d\beta d\gamma](\epsilon-1)\Big\{ \tag{12.77}$$

$$\int_{-\infty}^{z_2}d\sigma\mathcal{T}^\mu_{\gamma^+}(z_{12}^\alpha,\sigma,z_{12}^{\alpha+\beta})+\int_{-\infty}^{z_1}d\sigma\mathcal{T}^\mu_{\gamma^+}(z_{21}^{\alpha+\beta},\sigma,z_{21}^{\alpha})\Big\},$$

$$\mathbf{EE}_3 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)b_\mu\int[d\alpha d\beta d\gamma](1+\epsilon)\Big\{ \tag{12.78}$$

$$\int_{-\infty}^{z_2}d\sigma\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_{12}^\alpha,\sigma,z_{12}^{\alpha+\beta})-\int_{-\infty}^{z_1}d\sigma\mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_{21}^{\alpha+\beta},\sigma,z_{21}^{\alpha})\Big\}.$$

As usual we consider them one by one.

For the matrix element of the first one we have

$$\langle\mathbf{EE}_1\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]4(1-\epsilon)\beta\Big\{ \tag{12.79}$$

$$\int_{-\infty}^{z_1}d\sigma T(z_{12}^\alpha,z_{2\sigma}^\beta,z_{21}^{\beta+\gamma})+\int_{-\infty}^{z_2}d\sigma T(z_{12}^{\beta+\gamma},z_{1\sigma}^\beta,z_{21}^{\alpha})\Big\}.$$

Reflecting the second term and together with the reflection of $\sigma\to-\sigma$ we sum together the integrals

$$\langle\mathbf{EE}_1\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[d\alpha d\beta d\gamma d\rho]4(1-\epsilon)\beta\int_{-\infty}^{\infty}d\sigma T(z_{12}^\alpha,z_{2\sigma}^\beta,z_{21}^{\beta+\gamma}). \tag{12.80}$$

Making the Fourier, and integrating over σ and z we obtain

$$\{\mathbf{EE}_1\} = [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left(C_F-\frac{C_A}{2}\right)\int[dx]\int[d\alpha d\beta d\gamma d\rho] \tag{12.81}$$

$$4(1-\epsilon)\beta T(x_1,x_2,x_2)2\delta(\beta x_2)\delta\left(2x+x_1(1-2\alpha)-\bar\beta x_2-x_3(1-2\beta-2\gamma)\right).$$

Integrating over $x_{1,2}$

$$\{\mathbf{EE}_1\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int dx_3 \int [d\alpha d\beta d\gamma d\rho] \\ 4(1-\epsilon) T(-x_3, 0, x_3)\delta(x - \rho x_3). \tag{12.82}$$

Note, that β disappears due to $\delta(\beta x_2)$. Making the change of variables, and integrating over the rest of Feynman variables we get

$$\{\mathbf{EE}_1\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int d\xi \int_0^1 dy \delta(x - y\xi) 2(1-\epsilon)\bar{y}^2 T(-\xi, 0, \xi). \tag{12.83}$$

Considering the second one:

$$\langle \mathbf{EE}_2 \rangle = [ib^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma](\epsilon - 1) \Bigg\{ \\ \int_{-\infty}^{z_2} d\sigma T(z_{12}^\alpha, \sigma, z_{12}^{\alpha+\beta}) + \int_{-\infty}^{z_1} d\sigma T(z_{21}^{\alpha+\beta}, \sigma, z_{21}^\alpha) \Bigg\} \\ = [ib^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int [d\alpha d\beta d\gamma](\epsilon - 1) \int_{-\infty}^{\infty} d\sigma T(z_{12}^\alpha, \sigma, z_{12}^{\alpha+\beta}). \tag{12.84}$$

The Fourier is

$$\{\mathbf{EE}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \\ \int [dx] \int [d\alpha d\beta d\gamma](\epsilon - 1) T(x_1, x_2, x_3) 2\delta(x_2)\delta\left(2x + x_1(1-2\alpha) + x_3(1 - 2\alpha - 2\beta)\right). \tag{12.85}$$

Integrating over $x_{1,2}$

$$\{\mathbf{EE}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int dx_3 \int [d\alpha d\beta d\gamma](\epsilon - 1) T(-x_3, 0, x_3)\delta\left(x - \beta x_3\right). \tag{12.86}$$

After integration over rest Feynman variables and convenient change of variables

$$\{\mathbf{EE}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int_0^1 dy \int d\xi \delta\left(x - y\xi\right)(\epsilon - 1)\bar{y} T(-\xi, 0, \xi). \tag{12.87}$$

The third contribution is obviously zero. Indeed, it has σ only as a second argument. Thus, it will generate $\delta(x_2)$, and $\Delta T(-x_3, 0, x_3) = 0$. I.e.

$$\{\mathbf{EE}_3\} = 0. \tag{12.88}$$

G. Final expression for the diagrams E

We have

$$\{\mathbf{E} + \mathbf{E}^*\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] \\ 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \left(C_F - \frac{C_A}{2}\right) \int_0^1 dy \int d\xi \delta\left(x - y\xi\right)(1 - \epsilon)\bar{y}(1 - 2y) T(-\xi, 0, \xi). \tag{12.89}$$

XIII. DIAGRAMS D

A. Diagram D

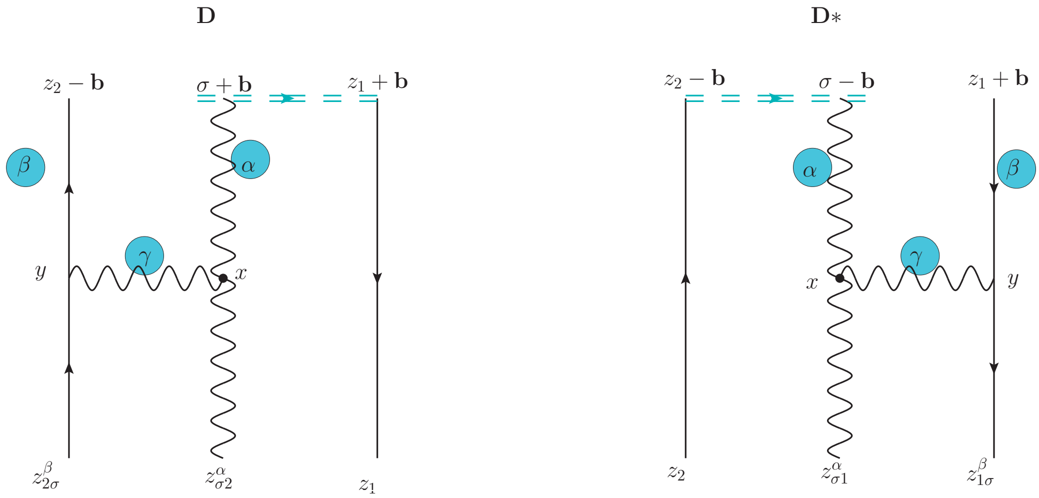
$$\text{3GluonVertex} = gf^{ABC}A^A_\mu(\partial_\alpha B^B_\beta)B^C_\gamma\Big(2g^{\mu\beta}g^{\alpha\gamma} - g^{\mu\alpha}g^{\beta\gamma} - 2g^{\mu\gamma}g^{\alpha\beta}\Big)$$

The diagram is

$$\mathbf{D} = v^{\mu\alpha\beta\gamma}_{A'B'C'}$$
$$\Big\{\bar{q}(z_1 n + \boldsymbol{b})[ig\int_{-\infty}^{z_1} d\sigma n^\nu t^A B^A_\nu(n\sigma + \boldsymbol{b})]\gamma^+\psi(z_2 n - \boldsymbol{b})\Big\}\Big(ig\int d^d y\bar{\psi}(y)\ \not{B}(y)q(y)\Big)\Big(ig\int d^d x A^{A'}_\mu(x)(\partial_\alpha B^{B'}_\beta(x))B^{C'}_\gamma(x)\Big)$$
$$+v^{\mu\alpha\beta\gamma}_{A'B'C'}$$
$$\Big\{\bar{q}(z_1 n + \boldsymbol{b})[ig\int_{-\infty}^{z_1} d\sigma n^\nu t^A B^A_\nu(n\sigma + \boldsymbol{b})]\gamma^+\psi(z_2 n - \boldsymbol{b})\Big\}\Big(ig\int d^d y\bar{\psi}(y)\ \not{B}(y)q(y)\Big)\Big(ig\int d^d x A^{A'}_\mu(x)(\partial_\alpha B^{B'}_\beta(x))B^{C'}_\gamma(x)\Big)$$

where

$$v^{\mu\alpha\beta\gamma}_{ABC} = f_{ABC}\Big(2g^{\mu\beta}g^{\alpha\gamma} - g^{\mu\alpha}g^{\beta\gamma} - 2g^{\mu\gamma}g^{\alpha\beta}\Big).$$



Step I: Substituting propagators

$$\begin{aligned}\mathbf{D} \;=\;& (ig)^3\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}}\int d^dx d^dy\int_{-\infty}^{z_1}d\sigma\Bigg[\\ & \bar{q}(z_1+\boldsymbol{b})\Bigg\{t^C n_\gamma\gamma^+\frac{\gamma^+ z_2 - \not{b} - \not{y}}{[-(z_2 n-\boldsymbol{b}-y)^2]^{2-\epsilon}}\gamma^\beta t^B\frac{x_\alpha - y_\alpha}{[-(y-x)^2]^{2-\epsilon}}\frac{v^{\mu\alpha\beta\gamma}_{ABC}}{[-(\sigma n+\boldsymbol{b}-x)^2]^{1-\epsilon}}\Bigg\}A^A_\mu(x)q(y)\\ & +\bar{q}(z_1+\boldsymbol{b})\Bigg\{t^B n_\beta\gamma^+\frac{\gamma^+ z_2 - \not{b} - \not{y}}{[-(z_2 n-\boldsymbol{b}-y)^2]^{2-\epsilon}}\gamma^\gamma t^C\frac{v^{\mu\alpha\beta\gamma}_{ABC}}{[-(y-x)^2]^{1-\epsilon}}\frac{x_\alpha - n_\alpha\sigma - b_\alpha}{[-(\sigma n+\boldsymbol{b}-x)^2]^{2-\epsilon}}\Bigg\}A^A_\mu(x)q(y)\Bigg]\end{aligned} \quad (13.1)$$

Step II: After minimal simplifications

$$\begin{aligned}\mathbf{D} &= ig^3\frac{C_A}{2}\frac{\Gamma^2(2-\epsilon)\Gamma(1-\epsilon)}{16\pi^{3d/2}}\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\Bigg[\\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(x)\frac{\gamma^+(\not{b}+\not{y})\gamma^\beta(x_\alpha-y_\alpha)n_\gamma\Big(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\Big)}{[-(z_2n-\boldsymbol{b}-y)^2]^{2-\epsilon}[-(\sigma n+\boldsymbol{b}-x)^2]^{1-\epsilon}[-(y-x)^2]^{2-\epsilon}}q(y) \\ &-\bar{q}(z_1+\boldsymbol{b})A_\mu(x)\frac{\gamma^+(\not{b}+\not{y})\gamma^\gamma(x_\alpha-n_\alpha\sigma-b_\alpha)n_\beta\Big(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\Big)}{[-(z_2n-\boldsymbol{b}-y)^2]^{2-\epsilon}[-(\sigma n+\boldsymbol{b}-x)^2]^{2-\epsilon}[-(y-x)^2]^{1-\epsilon}}q(y)\Bigg].\end{aligned} \tag{13.2}$$

Her we used

$$f^{ABC}t^Bt^C=\frac{i}{2}C_At^A,\qquad f^{ABC}t^Ct^B=-\frac{i}{2}C_At^A, \tag{13.3}$$

Step III: Contracting Lorentz indices we get

$$\begin{aligned}\mathbf{D} &= 2ig^3\frac{C_A}{2}\frac{\Gamma^2(2-\epsilon)\Gamma(1-\epsilon)}{16\pi^{3d/2}}\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\Bigg[\\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(x)\frac{\gamma^+(\not{b}+\not{y})\gamma^\mu(x^+-y^+)-y^+\gamma^+(x^\mu-y^\mu)}{[-(z_2n-\boldsymbol{b}-y)^2]^{2-\epsilon}[-(\sigma n+\boldsymbol{b}-x)^2]^{1-\epsilon}[-(y-x)^2]^{2-\epsilon}}q(y) \\ &+\bar{q}(z_1+\boldsymbol{b})A_\mu(x)\frac{y^+\gamma^+(x_\mu-b_\mu)+\gamma^+(\not{b}+\not{y})\gamma^\mu x^+}{[-(z_2n-\boldsymbol{b}-y)^2]^{2-\epsilon}[-(\sigma n+\boldsymbol{b}-x)^2]^{2-\epsilon}[-(y-x)^2]^{1-\epsilon}}q(y)\Bigg].\end{aligned} \tag{13.4}$$

Step IV: This loop integral is topologically similar to the diagram **C**. So, we make the similar change of variables

$$\begin{aligned}(x-\sigma n-b)^2 &= \alpha \\ (z_2n-b-y)^2 &= \beta \\ (x-y)^2 &= 1-\alpha-\beta=\gamma\end{aligned}$$

Next we shift

$$\begin{aligned}x &\to x+\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma n+\frac{\beta\gamma}{\lambda}z_2n+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right), \\ y &\to y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_2n+\frac{\alpha\gamma}{\lambda}\sigma n-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right), \\ \lambda &= \alpha\beta+\beta\gamma+\gamma\alpha.\end{aligned} \tag{13.5}$$

$$\begin{aligned}\mathbf{D} &= 2ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\frac{C_A}{2}\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon} \\ &\frac{\bar{q}(z_1+\boldsymbol{b})A_\mu(x+\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}} \\ &\Big\{\gamma\gamma^+(2\frac{\alpha\gamma}{\lambda}\not{b}+\not{y})\gamma^\mu(x^+-y^+)-\gamma y^+\gamma^+(x^\mu-y^\mu+2\frac{\alpha\beta}{\lambda}b^\mu) \\ &+\alpha y^+\gamma^+(x_\mu-2\frac{\beta\gamma}{\lambda}b_\mu)+\alpha\gamma^+(2\frac{\alpha\gamma}{\lambda}\not{b}+\not{y})\gamma^\mu x^+\Big\}q(y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).\end{aligned} \tag{13.6}$$

Step V: This diagram is to be expanded up to one-derivative terms. The next expansion order gives contribution to $\boldsymbol{b}^2$ (the left derivative act on the gluon field)

$$\begin{aligned}\mathbf{D} &= 2ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\frac{C_A}{2}\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon} \\ &\frac{\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}} \\ &(1+x^\nu\overleftarrow{\partial_\nu}+y^\nu\overrightarrow{\partial_\nu})\Big\{\gamma\gamma^+(2\frac{\alpha\gamma}{\lambda}\not{b}+\not{y})\gamma^\mu(x^+-y^+)-\gamma y^+\gamma^+(x^\mu-y^\mu+2\frac{\alpha\beta}{\lambda}b^\mu) \\ &+\alpha y^+\gamma^+(x_\mu-2\frac{\beta\gamma}{\lambda}b_\mu)+\alpha\gamma^+(2\frac{\alpha\gamma}{\lambda}\not{b}+\not{y})\gamma^\mu x^+\Big\}q(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).\end{aligned} \tag{13.7}$$

Step VI: We split this diagram into term with derivative and without derivative

$$\mathbf{D}=\mathbf{D}_0+\mathbf{D}_1 \tag{13.8}$$

The contribution with no derivative reads

$$\begin{aligned}\mathbf{D}_0 &= 2ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\frac{C_A}{2}\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon} \\ &\frac{\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}} \\ &\Big\{\gamma\gamma^+\not{y}\gamma^\mu(x^+-y^+)-\gamma y^+\gamma^+(x^\mu-y^\mu)+\alpha y^+\gamma^+x_\mu+\alpha\gamma^+\not{y}\gamma^\mu x^+\Big\}q(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).\end{aligned} \tag{13.9}$$

This contribution is zero for the $A_+=0$ and $\gamma^+\gamma^+=0$, which arise since the integral is $\sim g^{\mu\nu}$. Thus

$$\mathbf{D}_0=0,\qquad \mathbf{D}=\mathbf{D}_1.$$

Therefore, we drop this contribution and consider only the contribution with a single derivative. We have

$$\begin{aligned}\mathbf{D} &= 2ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\frac{C_A}{2}\int d^dxd^dy\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon} \\ &\frac{\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{5-3\epsilon}} \\ &(x^\nu\overleftarrow{\partial_\nu}+y^\nu\overrightarrow{\partial_\nu})\Big\{\gamma2\frac{\alpha\gamma}{\lambda}\gamma^+\not{b}\gamma^\mu(x^+-y^+)-\gamma2\frac{\alpha\beta}{\lambda}y^+\gamma^+b^\mu \\ &-\alpha2\frac{\beta\gamma}{\lambda}y^+\gamma^+b_\mu+\alpha2\frac{\alpha\gamma}{\lambda}\gamma^+\not{b}\gamma^\mu x^+\Big\}q(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).\end{aligned} \tag{13.10}$$

Step VII: We integrate with the help of (7.12)

$$\begin{aligned}\mathbf{D} &= 2ig^3\frac{4^\epsilon\Gamma(-\epsilon)}{16\pi^{d/2}}\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\frac{(\alpha\beta\gamma)^\epsilon}{2\lambda^3} \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \\ &\Big\{\gamma2\frac{\alpha\gamma}{\lambda}\gamma^+\not{b}\gamma^\mu(\beta\overleftarrow{\partial_+}-\alpha\overrightarrow{\partial_+})-\gamma2\frac{\alpha\beta}{\lambda}\gamma^+b^\mu(\gamma\overleftarrow{\partial_+}+\bar{\beta}\overrightarrow{\partial_+}) \\ &-\alpha2\frac{\beta\gamma}{\lambda}\gamma^+b_\mu(\gamma\overleftarrow{\partial_+}+\bar{\beta}\overrightarrow{\partial_+})+\alpha2\frac{\alpha\gamma}{\lambda}\gamma^+\not{b}\gamma^\mu(\bar{\alpha}\overleftarrow{\partial_+}+\gamma\overrightarrow{\partial_+})\Big\}q(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).\end{aligned} \tag{13.11}$$

Step VIII: We simplify

$$\begin{aligned}\mathbf{D} &= 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\frac{\alpha\beta\gamma}{\lambda^3} \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right)) \\ &\Big\{\frac{\gamma}{\lambda}\gamma^+\not{b}\gamma^\mu(\beta\overleftarrow{\partial}_+-\alpha\overrightarrow{\partial}_+)-\frac{\beta}{\lambda}\gamma^+b^\mu(\gamma\overleftarrow{\partial}_++\bar{\beta}\overrightarrow{\partial}_+) \\ &-\frac{\beta}{\lambda}\gamma^+b_\mu(\gamma\overleftarrow{\partial}_++\bar{\beta}\overrightarrow{\partial}_+)+\frac{\alpha}{\lambda}\gamma^+\not{b}\gamma^\mu(\bar{\alpha}\overleftarrow{\partial}_++\gamma\overrightarrow{\partial}_+)\Big\}q(\frac{\alpha\beta+\beta\gamma}{\lambda}z_2+\frac{\alpha\gamma}{\lambda}\sigma-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right)).\end{aligned} \tag{13.12}$$

Step IX: We change to dual variables (10.13) and drop the primes

$$\begin{aligned}\mathbf{D} &= 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma] \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(z_{\sigma 2}^\alpha+\boldsymbol{b}(1-2\alpha)) \\ &\Big\{\gamma^+\not{b}\gamma^\mu(\alpha\overleftarrow{\partial}_+-\beta\overrightarrow{\partial}_+)-\gamma^+b^\mu(\alpha\overleftarrow{\partial}_++\bar{\beta}\overrightarrow{\partial}_+)-\gamma^+b_\mu(\alpha\overleftarrow{\partial}_++\bar{\beta}\overrightarrow{\partial}_+)+\gamma^+\not{b}\gamma^\mu(\bar{\alpha}\overleftarrow{\partial}_++\beta\overrightarrow{\partial}_+)\Big\}q(z_{2\sigma}^\beta-\boldsymbol{b}(1-2\beta)).\end{aligned} \tag{13.13}$$

Step VIII: We simplify

$$\begin{aligned}\mathbf{D} &= 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma] \\ &\bar{q}(z_1+\boldsymbol{b})A_\mu(z_{\sigma 2}^\alpha+\boldsymbol{b}(1-2\alpha))\Big\{\gamma^+\not{b}\gamma^\mu\overleftarrow{\partial}_+-2\gamma^+b^\mu(\alpha\overleftarrow{\partial}_++\bar{\beta}\overrightarrow{\partial}_+)\Big\}q(z_{2\sigma}^\beta-\boldsymbol{b}(1-2\beta)).\end{aligned} \tag{13.14}$$

Step IX: The variables $\boldsymbol{b}$ in the arguments of the fields could be dropped

$$\text{DY}\qquad\begin{aligned}\mathbf{D} &= 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma] \\ &\bar{q}(z_1)A_\mu(z_{\sigma 2}^\alpha)\Big\{\gamma^+\not{b}\gamma^\mu\overleftarrow{\partial}_+-2\gamma^+b^\mu(\alpha\overleftarrow{\partial}_++\bar{\beta}\overrightarrow{\partial}_+)\Big\}q(z_{2\sigma}^\beta),\end{aligned} \tag{13.15}$$

$$\text{SIDIS}\qquad\begin{aligned}\mathbf{D} &= 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{+\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma] \\ &\bar{q}(z_1)A_\mu(z_{\sigma 2}^\alpha)\Big\{\gamma^+\not{b}\gamma^\mu\overleftarrow{\partial}_+-2\gamma^+b^\mu(\alpha\overleftarrow{\partial}_++\bar{\beta}\overrightarrow{\partial}_+)\Big\}q(z_{2\sigma}^\beta).\end{aligned} \tag{13.16}$$

B. Diagram D*

The diagram is

$$\begin{aligned}&\mathbf{D}^*=v_{A'B'C'}^{\mu\alpha\beta\gamma} \\ &\left(ig\int d^dxA_\mu^{A'}(x)(\partial_\alpha B_\beta^{B'}(x))B_\gamma^{C'}(x)\right)\left(ig\int d^dy\bar{q}(y)\not{B}(y)\psi(y)\right)\Big\{\bar{\psi}(z_1n+\boldsymbol{b})[-ig\int_{-\infty}^{z_2}d\sigma n^\nu t^A B_\nu^A(n\sigma-\boldsymbol{b})]\gamma^+q(z_2n-\boldsymbol{b})\Big\} \\ &+v_{A'B'C'}^{\mu\alpha\beta\gamma} \\ &\left(ig\int d^dxA_\mu^{A'}(x)(\partial_\alpha B_\beta^{B'}(x))B_\gamma^{C'}(x)\right)\left(ig\int d^dy\bar{q}(y)\not{B}(y)\psi(y)\right)\Big\{\bar{\psi}(z_1n+\boldsymbol{b})[-ig\int_{-\infty}^{z_2}d\sigma n^\nu t^A B_\nu^A(n\sigma-\boldsymbol{b})]\gamma^+q(z_2n-\boldsymbol{b})\Big\}\end{aligned}$$

where

$$v_{ABC}^{\mu\alpha\beta\gamma}=f_{ABC}\Big(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\Big).$$

Step I: Substituting propagators

$$\mathbf{D}^* = -(ig)^3 \frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}} \frac{\Gamma(2-\epsilon)}{2\pi^{d/2}} \frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}} \int d^dx d^dy \int_{-\infty}^{z_2} d\sigma \Bigg[\tag{13.17}$$
$$\bar{q}(y)\left\{ t^B t^C n_\gamma \gamma^\beta \frac{\not{y} - \gamma^+ z_1 - \not{b}}{[-(z_1 n + \boldsymbol{b} - y)^2]^{2-\epsilon}} \gamma^+ \frac{y_\alpha - x_\alpha}{[-(y-x)^2]^{2-\epsilon}} \frac{v^{\mu\alpha\beta\gamma}_{ABC}}{[-(\sigma n - \boldsymbol{b} - x)^2]^{1-\epsilon}} \right\} A^A_\mu(x) q(z_2 n - \boldsymbol{b})$$
$$+\bar{q}(y)\left\{ t^C t^B n_\beta \gamma^\gamma \frac{\not{y} - \gamma^+ z_1 - \not{b}}{[-(z_1 n + \boldsymbol{b} - y)^2]^{2-\epsilon}} \gamma^+ \frac{v^{\mu\alpha\beta\gamma}_{ABC}}{[-(y-x)^2]^{1-\epsilon}} \frac{n_\alpha \sigma - b_\alpha - x_\alpha}{[-(\sigma n - \boldsymbol{b} - x)^2]^{2-\epsilon}} \right\} A^A_\mu(x) q(z_2 n - \boldsymbol{b}) \Bigg]$$

Step II: After minimal simplification we get

$$\mathbf{D}^* = ig^3 \frac{\Gamma^2(2-\epsilon)\Gamma(1-\epsilon)}{16\pi^{3d/2}} \frac{C_A}{2} \int d^dx d^dy \int_{-\infty}^{z_2} d\sigma \Bigg[\tag{13.18}$$
$$\bar{q}(y)\left\{ \frac{n_\gamma \gamma^\beta (\not{y} - \not{b}) \gamma^+ (y_\alpha - x_\alpha) \Big(2g^{\mu\beta} g^{\alpha\gamma} - g^{\mu\alpha} g^{\beta\gamma} - 2g^{\mu\gamma} g^{\alpha\beta} \Big)}{[-(z_1 n + \boldsymbol{b} - y)^2]^{2-\epsilon} [-(y-x)^2]^{2-\epsilon} [-(\sigma n - \boldsymbol{b} - x)^2]^{1-\epsilon}} \right\} A^A_\mu(x) q(z_2 n - \boldsymbol{b})$$
$$-\bar{q}(y)\left\{ \frac{n_\beta \gamma^\gamma (\not{y} - \not{b}) \gamma^+ (n_\alpha \sigma - b_\alpha - x_\alpha) \Big(2g^{\mu\beta} g^{\alpha\gamma} - g^{\mu\alpha} g^{\beta\gamma} - 2g^{\mu\gamma} g^{\alpha\beta} \Big)}{[-(z_1 n + \boldsymbol{b} - y)^2]^{2-\epsilon} [-(y-x)^2]^{1-\epsilon} [-(\sigma n - \boldsymbol{b} - x)^2]^{2-\epsilon}} \right\} A^A_\mu(x) q(z_2 n - \boldsymbol{b}) \Bigg]$$

Step III: Contracting indices

$$\mathbf{D}^* = 2ig^3 \frac{\Gamma^2(2-\epsilon)\Gamma(1-\epsilon)}{16\pi^{3d/2}} \frac{C_A}{2} \int d^dx d^dy \int_{-\infty}^{z_2} d\sigma \Bigg[\tag{13.19}$$
$$\bar{q}(y)\left\{ \frac{\gamma^\mu (\not{y} - \not{b}) \gamma^+ (y_+ - x_+) - \gamma^+ y^+ (y^\mu - x^\mu)}{[-(z_1 n + \boldsymbol{b} - y)^2]^{2-\epsilon} [-(y-x)^2]^{2-\epsilon} [-(\sigma n - \boldsymbol{b} - x)^2]^{1-\epsilon}} \right\} A^A_\mu(x) q(z_2 n - \boldsymbol{b})$$
$$-\bar{q}(y)\left\{ \frac{\gamma^+ y^+ (b_\mu + x_\mu) + \gamma^\mu (\not{y} - \not{b}) \gamma^+ x^+}{[-(z_1 n + \boldsymbol{b} - y)^2]^{2-\epsilon} [-(y-x)^2]^{1-\epsilon} [-(\sigma n - \boldsymbol{b} - x)^2]^{2-\epsilon}} \right\} A^A_\mu(x) q(z_2 n - \boldsymbol{b}) \Bigg]$$

Step IV: This loop integral is topologically similar to the diagram C^*. So, we make the similar change of variables

$$(\sigma n - b - x)^2 = \alpha$$
$$(y - z_1 n - b)^2 = \beta$$
$$(x - y)^2 = 1 - \alpha - \beta = \gamma$$

Next we shift

$$x \rightarrow x + \frac{\alpha\beta + \alpha\gamma}{\lambda}\sigma + \frac{\beta\gamma}{\lambda} z_1 - \boldsymbol{b}\left(1 - 2\frac{\beta\gamma}{\lambda}\right), \tag{13.20}$$
$$y \rightarrow y + \frac{\alpha\beta + \beta\gamma}{\lambda} z_1 + \frac{\alpha\gamma}{\lambda}\sigma + \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma}{\lambda}\right),$$
$$\lambda = \alpha\beta + \beta\gamma + \gamma\alpha.$$

We obtain the expression

$$\mathbf{D}^* = 2ig^3 \frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}} \frac{C_A}{2} \int d^dx d^dy \int_{-\infty}^{z_2} d\sigma \int [d\alpha d\beta d\gamma] \alpha^{-\epsilon} \beta^{1-\epsilon} \gamma^{-\epsilon} \tag{13.21}$$
$$\bar{q}(y + \frac{\alpha\beta + \beta\gamma}{\lambda} z_1 + \frac{\alpha\gamma}{\lambda}\sigma + \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma}{\lambda}\right)) \Bigg[$$
$$\left\{ \frac{\gamma\gamma^\mu (\not{y} - 2\frac{\alpha\gamma}{\lambda} \not{b}) \gamma^+ (y_+ - x_+) - \gamma\gamma^+ y^+ (y^\mu - x^\mu + 2\frac{\alpha\beta}{\lambda} b^\mu) - \alpha\gamma^+ y^+ (2\frac{\beta\gamma}{\lambda} b_\mu + x_\mu) - \alpha\gamma^\mu (\not{y} - 2\frac{\alpha\gamma}{\lambda} \not{b}) \gamma^+ x^+}{[-\bar{\beta} x^2 - \bar{\alpha} y^2 + 2\gamma(xy) + \frac{4\alpha\beta\gamma}{\lambda} \boldsymbol{b}^2 + i0]^{2-\epsilon}} \right\}$$
$$A^A_\mu(x + \frac{\alpha\beta + \alpha\gamma}{\lambda}\sigma + \frac{\beta\gamma}{\lambda} z_1 - \boldsymbol{b}\left(1 - 2\frac{\beta\gamma}{\lambda}\right)) q(z_2 n - \boldsymbol{b}) \Bigg]$$

Step V: We expend the fields. Just like in the diagram D case, we observe that no-derivative term vanishes. We got (right derivative on the gluon field)

$$\begin{aligned}\mathbf{D}^* &= 2ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\frac{C_A}{2}\int d^dx d^dy\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon} \\ &\bar{q}(\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))(y_\mu\overleftarrow{\partial^\mu}+x_\mu\overrightarrow{\partial^\mu})\Bigg[\\ &\left\{\frac{\gamma\gamma^\mu(\not{y}-2\frac{\alpha\gamma}{\lambda}\not{b})\gamma^+(y_+-x_+)-\gamma\gamma^+y^+(y^\mu-x^\mu+2\frac{\alpha\beta}{\lambda}b^\mu)-\alpha\gamma^+y^+(2\frac{\beta\gamma}{\lambda}b_\mu+x_\mu)-\alpha\gamma^\mu(\not{y}-2\frac{\alpha\gamma}{\lambda}\not{b})\gamma^+x^+}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{2-\epsilon}}\right\} \\ &A_\mu^A(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))q(z_2n-\boldsymbol{b})\Bigg]\end{aligned} \tag{13.22}$$

Step VI: We drop odd powers of x, y

$$\begin{aligned}\mathbf{D}^* &= -2ig^3\frac{\Gamma(5-3\epsilon)}{16\pi^{3d/2}}\frac{C_A}{2}\int d^dx d^dy\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon} \\ &\bar{q}(\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))(y_\mu\overleftarrow{\partial^\mu}+x_\mu\overrightarrow{\partial^\mu})\Bigg[\\ &\left\{\frac{\gamma2\frac{\alpha\gamma}{\lambda}\gamma^\mu\not{b}\gamma^+(y_+-x_+)+\gamma2\frac{\alpha\beta}{\lambda}\gamma^+y^+b^\mu+\alpha2\frac{\beta\gamma}{\lambda}\gamma^+y^+b_\mu-\alpha2\frac{\alpha\gamma}{\lambda}\gamma^\mu\not{b}\gamma^+x^+}{[-\bar{\beta}x^2-\bar{\alpha}y^2+2\gamma(xy)+\frac{4\alpha\beta\gamma}{\lambda}\boldsymbol{b}^2+i0]^{2-\epsilon}}\right\} \\ &A_\mu^A(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))q(z_2n-\boldsymbol{b})\Bigg]\end{aligned} \tag{13.23}$$

Step VII: Integrating

$$\begin{aligned}\mathbf{D}^* &= -2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\alpha^{-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\frac{(\alpha\beta\gamma)^\epsilon}{2\lambda^3} \\ &\bar{q}(\frac{\alpha\beta+\beta\gamma}{\lambda}z_1+\frac{\alpha\gamma}{\lambda}\sigma+\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))\Bigg[\\ &\Bigg\{\gamma2\frac{\alpha\gamma}{\lambda}\gamma^\mu\not{b}\gamma^+(\alpha\overleftarrow{\partial_+}-\beta\overrightarrow{\partial_+})+\gamma2\frac{\alpha\beta}{\lambda}\gamma^+b^\mu(\bar{\beta}\overleftarrow{\partial_+}+\gamma\overrightarrow{\partial_+}) \\ &+\alpha2\frac{\beta\gamma}{\lambda}\gamma^+b_\mu(\bar{\beta}\overleftarrow{\partial_+}+\gamma\overrightarrow{\partial_+})-\alpha2\frac{\alpha\gamma}{\lambda}\gamma^\mu\not{b}\gamma^+(\gamma\overleftarrow{\partial_+}+\bar{\alpha}\overrightarrow{\partial_+})\Bigg\} \\ &A_\mu^A(\frac{\alpha\beta+\alpha\gamma}{\lambda}\sigma+\frac{\beta\gamma}{\lambda}z_1-\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))q(z_2n-\boldsymbol{b})\Bigg]\end{aligned} \tag{13.24}$$

Step VIII: Passing to dual variables

$$\begin{aligned}\mathbf{D}^* &= -2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\bar{q}(z_{1\sigma}^\beta+\boldsymbol{b}(1-2\beta)) \\ &\left\{\gamma^\mu\not{b}\gamma^+(\beta\overleftarrow{\partial_+}-\alpha\overrightarrow{\partial_+})+\gamma^+b^\mu(\bar{\beta}\overleftarrow{\partial_+}+\alpha\overrightarrow{\partial_+})+\gamma^+b_\mu(\bar{\beta}\overleftarrow{\partial_+}+\alpha\overrightarrow{\partial_+})-\gamma^\mu\not{b}\gamma^+(\beta\overleftarrow{\partial_+}+\bar{\alpha}\overrightarrow{\partial_+})\right\} \\ &A_\mu^A(z_{\sigma1}^\alpha-\boldsymbol{b}(1-2\alpha))q(z_2n-\boldsymbol{b})\end{aligned} \tag{13.25}$$

Step IX: Thus we get

$$\mathbf{D}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\bar q(z_{1\sigma}^\beta) \tag{13.26}$$
$$\Big\{\gamma^\mu \not b\gamma^+\overrightarrow{\partial_+}-2\gamma^+b^\mu(\bar\beta\overleftarrow{\partial_+}+\alpha\overrightarrow{\partial_+})\Big\}A_\mu^A(z_{\sigma1}^\alpha)q(z_2n)$$

C. Refining diagrams D and D*

The diagrams are

$$\mathbf{D} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma]\bar q(z_1)A_\mu(z_{\sigma2}^\alpha)\Big\{\gamma^+ \not b\gamma^\mu\overleftarrow{\partial_+}-2\gamma^+b^\mu(\alpha\overleftarrow{\partial_+}+\bar\beta\overrightarrow{\partial_+})\Big\}q(z_{2\sigma}^\beta) \tag{13.27}$$
$$\mathbf{D}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma]\bar q(z_{1\sigma}^\beta)\Big\{\gamma^\mu \not b\gamma^+\overrightarrow{\partial_+}-2\gamma^+b^\mu(\bar\beta\overleftarrow{\partial_+}+\alpha\overrightarrow{\partial_+})\Big\}A_\mu^A(z_{\sigma1}^\alpha)q(z_2n) \tag{13.28}$$

We recombine the gamma-algebra by (11.29)

$$\mathbf{D} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma] \tag{13.29}$$
$$\bar q(z_1)A_\mu(z_{\sigma2}^\alpha)\Big\{\gamma^+b^\mu((1-2\alpha)\overleftarrow{\partial_+}-2\bar\beta\overrightarrow{\partial_+})-\gamma^+b_\nu\gamma_T^{\mu\nu}\overleftarrow{\partial_+}\Big\}q(z_{2\sigma}^\beta)$$
$$\mathbf{D}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma] \tag{13.30}$$
$$\bar q(z_{1\sigma}^\beta)\Big\{\gamma^+b^\mu(-2\bar\beta\overleftarrow{\partial_+}+(1-2\alpha)\overrightarrow{\partial_+})+\gamma^+b_\nu\gamma_T^{\mu\nu}\overrightarrow{\partial_+})\Big\}A_\mu^A(z_{\sigma1}^\alpha)q(z_2n)$$

Again we must check the expression on the possible ambiguity, just as it was done for diagrams C. Just in the same way we find that terms with the derivative acting on quark and $\sim\gamma_T^{\mu\nu}$ are regular:

$$\int_{-\infty}^{z_2}\int[d\alpha d\beta d\gamma]A_\mu(z_{\sigma2}^\alpha)\frac{2\bar\beta}{\beta}\partial_\sigma q(z_{2\sigma}^\beta)=\int_{-\infty}^0 d\tau\int_1^\infty d\omega\frac{2\omega-1}{\omega^2}A_\mu(\omega\tau+z_2)\partial_\tau q(\tau+z_2), \tag{13.31}$$
$$\int_{-\infty}^{z_2}\int[d\alpha d\beta d\gamma]\partial_\sigma A_\mu(z_{\sigma2}^\alpha)\frac{1}{\bar\alpha}q(z_{2\sigma}^\beta)=\int_{-\infty}^0 d\tau\int_0^1 d\omega\partial_\tau A_\mu(\omega\tau+z_2)q(\tau+z_2), \tag{13.32}$$

where for the change of variables see diagram C. The situation with the terms $\sim\gamma^+\partial_+A_\mu$ is more interesting. The contribution is zero

$$\int_{-\infty}^{z_2}\int[d\alpha d\beta d\gamma]\partial_\sigma A_\mu(z_{\sigma2}^\alpha)\frac{1-2\alpha}{\bar\alpha}q(z_{2\sigma}^\beta)=\int_{-\infty}^0 d\tau\int_0^1 d\omega\partial_\tau A_\mu(\omega\tau+z_2)q(\tau+z_2)\int_0^1 d\alpha(1-2\alpha)=0. \tag{13.33}$$

Probably, it means something, but for a moment being it just implies that the integral is regular.

Next we rewrite the derivative acting on the quark as

$$\int_{-\infty}^{z_1}d\sigma A_\mu(z_{\sigma2}^\alpha)\bar\beta\overrightarrow{\partial_+}q(z_{2\sigma}^\beta) = A_\mu(z_{12}^\alpha)\frac{\bar\beta}{\beta}q(z_{21}^\beta)-\int_{-\infty}^{z_1}(\partial_+A_\mu(z_{\sigma2}^\alpha))\frac{\bar\alpha\bar\beta}{\beta}q(z_{2\sigma}^\beta) \tag{13.34}$$
$$\int_{-\infty}^{z_2}d\sigma\bar\beta(\overrightarrow{\partial_+}\bar q(z_{1\sigma}^\beta))A_\mu(z_{\sigma1}^\alpha) = \bar q(z_{12}^\beta)\frac{\bar\beta}{\beta}A_\mu(z_{21}^\alpha)-\int_{-\infty}^{z_2}\bar q(z_{1\sigma}^\beta)\frac{\bar\alpha\bar\beta}{\beta}(\partial_+A_\mu(z_{\sigma1}^\alpha)) \tag{13.35}$$

Therefore,

$$\mathbf{D} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\Big[-2\frac{\bar\beta}{\beta}\bar q(z_1)\gamma^+b^\mu A_\mu(z_{12}^\alpha)q(z_{21}^\beta) + \int_{-\infty}^{z_1}d\sigma\bar q(z_1)(\partial_+A_\mu(z_{\sigma 2}^\alpha))\gamma^+\Big\{b^\mu\left(2\frac{\bar\alpha}{\beta}-1\right)-b_\nu\gamma_T^{\mu\nu}\Big\}q(z_{2\sigma}^\beta)\Big] \tag{13.36}$$

$$\mathbf{D}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left[-2\frac{\bar\beta}{\beta}\bar q(z_{12}^\beta)\gamma^+b^\mu A_\mu^A(z_{21}^\alpha)q(z_2n) + \int_{-\infty}^{z_2}d\sigma\bar q(z_{1\sigma}^\beta)(\partial_+A_\mu^A(z_{\sigma 1}^\alpha))\gamma^+\Big\{b^\mu\left(2\frac{\bar\alpha}{\beta}-1\right)+b_\nu\gamma_T^{\mu\nu}\Big\}q(z_2n)\right]. \tag{13.37}$$

Let us mention the analogy with the diagrams **C** and $\mathbf{C}^*$.
Substituting the relations for the field in the light-cone gauge we get

$$\mathbf{D} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\int_{-\infty}^{z_1}d\sigma\Big[2\frac{\bar\beta\bar\alpha}{\beta}\bar q(z_1)\gamma^+b_\mu F^{\mu+}(z_{\sigma 2}^\alpha)q(z_{21}^\beta) + \bar q(z_1)F^{\mu+}(z_{\sigma 2}^\alpha)\gamma^+\Big\{b_\mu\left(1-2\frac{\bar\alpha}{\beta}\right)+b_\nu\gamma_T^{\mu\nu}\Big\}q(z_{2\sigma}^\beta)\Big] \tag{13.38}$$

$$\mathbf{D}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\int_{-\infty}^{z_2}d\sigma\left[2\frac{\bar\beta\bar\alpha}{\beta}\bar q(z_{12}^\beta)\gamma^+b_\mu F^{\mu+}(z_{\sigma 1}^\alpha)q(z_2) + \bar q(z_{1\sigma}^\beta)F^{\mu+}(z_{\sigma 1}^\alpha)\gamma^+\Big\{b^\mu\left(1-2\frac{\bar\alpha}{\beta}\right)-b_\nu\gamma_T^{\mu\nu})\Big\}q(z_2)\right]. \tag{13.39}$$

Finally, we have

$$\mathbf{D} = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b_\mu\int[d\alpha d\beta d\gamma]\int_{-\infty}^{z_1}d\sigma\Big[2\frac{\bar\beta\bar\alpha}{\beta}\mathcal{T}^\mu_{\gamma^+}(z_1,z_{\sigma 2}^\alpha,z_{21}^\beta) + \left(1-2\frac{\bar\alpha}{\beta}\right)\mathcal{T}^\mu_{\gamma^+}(z_1,z_{\sigma 2}^\alpha,z_{2\sigma}^\beta)-\mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z_1,z_{\sigma 2}^\alpha,z_{2\sigma}^\beta)\Big] \tag{13.40}$$

$$\mathbf{D}^* = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b_\mu\int[d\alpha d\beta d\gamma]\int_{-\infty}^{z_2}d\sigma\left[2\frac{\bar\beta\bar\alpha}{\beta}\mathcal{T}^\mu_{\gamma^+}(z_{12}^\beta,z_{\sigma 1}^\alpha,z_2) + \left(1-2\frac{\bar\alpha}{\beta}\right)\mathcal{T}^\mu_{\gamma^+}(z_{1\sigma}^\beta,z_{\sigma 1}^\alpha,z_2)+b_\nu\mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z_{1\sigma}^\beta,z_{\sigma 1}^\alpha,z_2)\right]. \tag{13.41}$$

Compare it to **C** and $\mathbf{C}^*$.

D. Elaboration of diagrams D and D*

The diagrams are

$$\mathbf{D} = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_1}d\sigma\int[d\alpha d\beta d\gamma] \; \bar q(z_1)A_\mu(z_{\sigma 2}^\alpha)\Big\{\gamma^+b^\mu((1-2\alpha)\overleftarrow{\partial_+}-2\bar\beta\overrightarrow{\partial_+})-\gamma^+b_\nu\gamma_T^{\mu\nu}\overleftarrow{\partial_+}\Big\}q(z_{2\sigma}^\beta) \tag{13.42}$$

$$\mathbf{D}^* = 2iga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int_{-\infty}^{z_2}d\sigma\int[d\alpha d\beta d\gamma] \; \bar q(z_{1\sigma}^\beta)\Big\{\gamma^+b^\mu(-2\bar\beta\overleftarrow{\partial_+}+(1-2\alpha)\overrightarrow{\partial_+})+\gamma^+b_\nu\gamma_T^{\mu\nu}\overrightarrow{\partial_+})\Big\}A_\mu^A(z_{\sigma 1}^\alpha)q(z_2n) \tag{13.43}$$

In the previous section we found that the expression does not have any problems with rapidity divergences.

Let me split the parts with various derivative and structures. And consider them separately.

$$\mathbf{DD}_1 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b^\mu\int[d\alpha d\beta d\gamma](1-2\alpha) \tag{13.44}$$
$$\left\{\int_{-\infty}^{z_1}d\sigma\bar q(z_1)\gamma^+\partial_+A_\mu(z^\alpha_{\sigma 2})q(z^\beta_{2\sigma})+\int_{-\infty}^{z_2}d\sigma\bar q(z^\beta_{1\sigma})\gamma^+\partial_+A(z^\alpha_{\sigma 1})q(z_2)\right\}$$
$$\mathbf{DD}_2 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b^\mu\int[d\alpha d\beta d\gamma](-2\bar\beta) \tag{13.45}$$
$$\left\{\int_{-\infty}^{z_1}d\sigma\bar q(z_1)\gamma^+A_\mu(z^\alpha_{\sigma 2})\partial_+q(z^\beta_{2\sigma})+\int_{-\infty}^{z_2}d\sigma\partial_+\bar q(z^\beta_{1\sigma})\gamma^+A(z^\alpha_{\sigma 1})q(z_2)\right\}$$
$$\mathbf{DD}_3 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b^\nu\int[d\alpha d\beta d\gamma] \tag{13.46}$$
$$\left\{-\int_{-\infty}^{z_1}d\sigma\bar q(z_1)\gamma_T^{\mu\nu}\partial_+A_\mu(z^\alpha_{\sigma 2})q(z^\beta_{2\sigma})+\int_{-\infty}^{z_2}d\sigma\bar q(z^\beta_{1\sigma})\gamma_T^{\mu\nu}\partial_+A(z^\alpha_{\sigma 1})q(z_2)\right\}$$

These expressions are very similar to expression for diagrams C, with the main change $A\leftrightarrow q$. However, it could be seen as exchange of $\alpha,\beta\leftrightarrow\bar\beta,\bar\alpha$. The later one make everything very different, since the singularity are $\alpha\to 1$ (of diagrams C have a suppressed phase volume (due to the $\beta\in(0,\bar\alpha)$). Here it transforms into singularity at $\beta\to 0$, which has unsuppressed phase volume. We will see that it result to an extra δ-contributions on the boundary.

First elaborate the $\mathbf{DD}_1$ using that $\partial_+A_\mu=-F^{\mu+}$, we get

$$\mathbf{DD}_1 = -2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b^\mu\int[d\alpha d\beta d\gamma](1-2\alpha)\left\{\int_{-\infty}^{z_1}d\sigma\mathcal{T}^\mu_{\gamma^+}(z_1,z^\alpha_{\sigma 2},z^\beta_{2\sigma})+\int_{-\infty}^{z_2}d\sigma\mathcal{T}^\mu_{\gamma^+}(z^\beta_{1\sigma},z^\alpha_{\sigma 1},z_2)\right\} \tag{13.47}$$

The matrix element of it reads (we also make $b\to b/2$, and set $p_+=1$)

$$\langle\mathbf{DD}_1\rangle = [b^\mu\epsilon_{\mu\nu}s^\nu M] \tag{13.48}$$
$$(-2i)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma](1-2\alpha)\left\{\int_{-\infty}^{z_1}d\sigma\tilde T(z_1,z^\alpha_{\sigma 2},z^\beta_{2\sigma})+\int_{-\infty}^{z_2}d\sigma\tilde T(z^\beta_{1\sigma},z^\alpha_{\sigma 1},z_2)\right\}$$
$$= [b^\mu\epsilon_{\mu\nu}s^\nu M]\,(-2i)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2} \tag{13.49}$$
$$\int[d\alpha d\beta d\gamma](1-2\alpha)\left\{\int_{-\infty}^{z}d\sigma\tilde T(z,\sigma\bar\alpha-z\alpha,-z\bar\beta+\sigma\beta)+\int_{-\infty}^{-z}d\sigma\tilde T(z\bar\beta+\sigma\beta,\sigma\bar\alpha+z\alpha,-z)\right\}.$$

In the first term we change $\tilde T(z_1,z_2,z_3)\to\tilde T(-z_3,-z_2,-z_1)$

$$\langle\mathbf{DD}_1\rangle = [b^\mu\epsilon_{\mu\nu}s^\nu M]\,(-2i)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2} \tag{13.50}$$
$$\int[d\alpha d\beta d\gamma](1-2\alpha)\left\{\int_{-\infty}^{z}d\sigma\tilde T(z\bar\beta-\sigma\beta,-\sigma\bar\alpha+z\alpha,-z)+\int_{-\infty}^{-z}d\sigma\tilde T(z\bar\beta+\sigma\beta,\bar\alpha+z\alpha,-z)\right\}.$$

Then we replace $\sigma\to-\sigma$ in the first term and sum then together

$$\langle\mathbf{DD}_1\rangle = \left[p_+^2b^\mu\epsilon_{\mu\nu}s^\nu M\right](-2i)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma](1-2\alpha)\int_{-\infty}^{\infty}d\sigma\tilde T(z\bar\beta+\sigma\beta,\sigma\bar\alpha+z\alpha,-z). \tag{13.51}$$

Performing Fourier we get

$$\begin{aligned}\{\mathbf{DD}_1\} &= \int \frac{dz}{2\pi} e^{-i2xz}\langle \mathbf{DD}_1\rangle \qquad (13.52)\\ &= [b^\mu \epsilon_{\mu\nu} s^\nu M]\,(-2i) a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}(2\pi)\int [d\alpha d\beta d\gamma](1-2\alpha)\int [dx]\delta(2x+x_1+x_2-x_3)\delta(\beta x_1+\bar\alpha x_2)T(x_1,x_2,x_3)\\ &= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M]\,(-2) a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\int [d\alpha d\beta d\gamma](1-2\alpha)\int [dx]\delta\left(x+\frac{x_1+x_2-x_3}{2}\right)\delta(\beta x_1+\bar\alpha x_2)T(x_1,x_2,x_3)\end{aligned}$$

Next we integrate the δ-functions

$$\begin{aligned}\{\mathbf{DD}_1\} &= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M]\,(-2) a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\int [d\alpha d\beta d\gamma](1-2\alpha)\int dx_{2,3}\delta\,(x-x_3)\,\delta(\gamma x_2-\beta x_3)T(-x_3-x_2,x_2,x_3)\\ &= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M]\,(-2) a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\int dx_2\int [d\alpha d\beta d\gamma]\delta\left(x-\frac{\gamma}{\beta}x_2\right)\frac{1-2\alpha}{\beta}T(-x-x_2,x_2,x).\end{aligned}$$

Finally, I replace $\gamma/\beta=\xi$, $x_2=y$

$$\begin{aligned}\{\mathbf{DD}_1\} &= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M]\,(-2) a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\int dy\int_0^\infty d\xi\delta(x-\xi y)T(-x-y,y,x)\int_0^{1/(1+\xi)} d\beta(2\beta(1+\xi)-1)\\ &= 0. \qquad (13.53)\end{aligned}$$

It is rather expected result! It is in the synergy with the calculation of the previous section, that shows that in the decomposition $\int_{-\infty}^{z_1}=\int_{-\infty}^{z_2}+\int_{z_2}^{z_1}$ the first term vanish. Indeed, we known that the calculation is independent on the middle-point. Thus we can recut the integration from the very beginning.

$$\{\mathbf{DD}_1\}=0. \qquad (13.54)$$

We consider the contribution $\mathbf{DD}_2$. First we integrate by parts the derivative over quark field. We recall that it is a regular integral. The next formulas, are the same as for $\mathbf{CC}_2$ with replacement $\alpha,\beta\to\bar\beta,\bar\alpha$. We have

$$\begin{aligned}&-2\bar\beta\int_{-\infty}^{z_1} A_\mu(z_{\sigma 2}^\alpha)\partial_+ q(z_{2\sigma}^\beta) = \frac{-2\bar\beta}{\beta}\int_{-\infty}^{z_1} A_\mu(z_{\sigma 2}^\alpha)\partial_\sigma q(z_{2\sigma}^\beta) \qquad (13.55)\\ &= \frac{-2\bar\beta}{\beta}A_\mu(z_{12}^\alpha)q(z_{21}^\beta)+\frac{2\bar\beta}{\beta}\int_{-\infty}^{z_1}\partial_\sigma A_\mu(z_{2\sigma}^\alpha)q(z_{2\sigma}^\beta) = \frac{-2\bar\beta}{\beta}A_\mu(z_{12}^\alpha)q(z_{21}^\beta)+\frac{2\bar\alpha\bar\beta}{\beta}\int_{-\infty}^{z_1}\partial_+ A_\mu(z_{\sigma 2}^\alpha)q(z_{2\sigma}^\beta).\end{aligned}$$

We would like to make a $F^{\mu+}$ from the A_μ we do it as

$$A_\mu(z_{12}^\alpha) = -\int_{-\infty}^0 d\lambda F^{\mu+}(\lambda+z_1+z_{21}\alpha) = -\int_{-\infty}^{z_2} d\lambda F^{\mu+}(\lambda+z_{12}\bar\alpha). \qquad (13.56)$$

We also rewrite

$$\bar\beta\int_{-\infty}^{z_1}\partial_+ A_\mu(z_{\sigma 2}^\alpha)q(z_{2\sigma}^\beta) = -\bar\beta\int_{-\infty}^{z_1} F^{\mu+}(z_{\sigma 2}^\alpha)q(z_{2\sigma}^\beta) = -\int_{-\infty}^{z_2} F^{\mu+}(\lambda+z_{12}\bar\alpha)q(\frac{\beta}{\bar\alpha}(\lambda-z_2)+z_{21}^\beta). \qquad (13.57)$$

Such form is much better, since it does not leave a possibility to have ambiguity at $\alpha\to 1$. Thus we have

$$-2\bar\beta\int_{-\infty}^{z_1} A_\mu(z_{\sigma 2}^\alpha)\partial_+ q(z_{2\sigma}^\beta) = \frac{2\bar\beta}{\beta}\int_{-\infty}^{z_2} d\lambda\left(F^{\mu+}(\lambda+z_{12}\bar\alpha)q(z_{21}^\beta)-F^{\mu+}(\lambda+z_{12}\bar\alpha)q(\frac{\beta}{\bar\alpha}(\lambda-z_2)+z_{21}^\beta)\right) \qquad (13.58)$$

Note, that at $\beta\to 0$ the bracket is 0, and thus the sum is well-defined. In order to preserve this property for each term (since we going to manipulate them separately) we introduce a "plus" distribution. The "plus" distributions is understood as usual

$$(f(\beta))_+ = f(\beta)-\delta(\beta)\int_0^1 d\beta' f(\beta'). \qquad (13.59)$$

Similarly, we have for the diagram $\mathbf{D}^*$

$$-2\bar{\beta}\int_{-\infty}^{z_2}\partial_+\bar{q}(z_{1\sigma}^{\beta})A_\mu(z_{\sigma 1}^{\alpha})=\left(\frac{2\bar{\beta}}{\beta}\right)_+\int_{-\infty}^{z_1}d\lambda\left(\bar{q}(z_{12}^{\beta})F^{\mu+}(\lambda+z_{21}\bar{\alpha})-\bar{q}\left((\lambda-z_1)\frac{\beta}{\bar{\alpha}}+z_{12}^{\beta}\right)F^{\mu+}(\lambda+z_{21}\bar{\alpha})\right) \quad (13.60)$$

We consider both terms separately. We introduce

$$\mathbf{DD}_{21} = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b^\mu\int[d\alpha d\beta d\gamma]\left(\frac{2\bar{\beta}}{\beta}\right)_+\Big\{ \quad (13.61)$$
$$\int_{-\infty}^{z_2}d\lambda\mathcal{T}^\mu_{\gamma^+}(z_1,\lambda+z_{12}\bar{\alpha},z_{21}^{\beta})+\int_{-\infty}^{z_1}d\lambda\mathcal{T}^\mu_{\gamma^+}(z_{12}^{\beta},\lambda+z_{21}\bar{\alpha},z_2)\Big\}$$

$$\mathbf{DD}_{22} = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b^\mu\int[d\alpha d\beta d\gamma]\left(\frac{-2\bar{\beta}}{\beta}\right)_+\Big\{ \quad (13.62)$$
$$\int_{-\infty}^{z_2}d\lambda\mathcal{T}^\mu_{\gamma^+}(z_1,\lambda+z_{12}\bar{\alpha},\frac{\beta}{\bar{\alpha}}(\lambda-z_2)+z_{21}^{\beta})+\int_{-\infty}^{z_1}d\lambda\mathcal{T}^\mu_{\gamma^+}((\lambda-z_1)\frac{\beta}{\bar{\alpha}}+z_{12}^{\beta},\lambda+z_{21}\bar{\alpha},z_2)\Big\}$$

The matrix element is

$$\langle\mathbf{DD}_{21}\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{2\bar{\beta}}{\beta}\right)_+\Big\{ \quad (13.63)$$
$$\int_{-\infty}^{z_2}d\lambda T(z_1,\lambda+z_{12}\bar{\alpha},z_{21}^{\beta})+\int_{-\infty}^{z_1}d\lambda T(z_{12}^{\beta},\lambda+z_{21}\bar{\alpha},z_2)\Big\}$$

$$\langle\mathbf{DD}_{22}\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{-2\bar{\beta}}{\beta}\right)_+\Big\{ \quad (13.64)$$
$$\int_{-\infty}^{z_2}d\lambda T(z_1,\lambda+z_{12}\bar{\alpha},\frac{\beta}{\bar{\alpha}}(\lambda-z_2)+z_{21}^{\beta})+\int_{-\infty}^{z_1}d\lambda T((\lambda-z_1)\frac{\beta}{\bar{\alpha}}+z_{12}^{\beta},\lambda+z_{21}\bar{\alpha},z_2)\Big\}$$

Take the second term, revert, change $\sigma\to-\sigma$ and sum together

$$\langle\mathbf{DD}_{21}\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{2\bar{\beta}}{\beta}\right)_+\int_{-\infty}^{\infty}d\lambda T(z,\lambda+2z\bar{\alpha},-(1-2\beta)z) \quad (13.65)$$

$$\langle\mathbf{DD}_{22}\rangle = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{-2\bar{\beta}}{\beta}\right)_+ \quad (13.66)$$
$$\int_{-\infty}^{\infty}d\lambda T(z,\lambda+2z\bar{\alpha},\frac{\beta}{\bar{\alpha}}(\lambda+z)-(1-2\beta))$$

Compare it to $\langle\mathbf{CC}_{21}\rangle$ and $\langle\mathbf{CC}_{22}\rangle$.

We take the Fourier and obtain

$$\{\mathbf{DD}_{21}\} = [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{2\bar{\beta}}{\beta}\right)_+\int[dx] \quad (13.67)$$
$$2\delta(2x+x_1+2x_2\bar{\alpha}-x_3(1-2\beta))\delta(x_2)T(x_1,x_2,x_3),$$
$$\{\mathbf{DD}_{22}\} = [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{-2\bar{\beta}}{\beta}\right)_+\int[dx] \quad (13.68)$$
$$2\delta\left(2x+x_1+2x_2\bar{\alpha}+x_3(2\beta-\frac{\gamma}{\bar{\alpha}})\right)\delta\left(x_2+\frac{\beta}{\bar{\alpha}}x_3\right)T(x_1,x_2,x_3).$$

Simplifying δ-functions

$$\{\mathbf{DD}_{21}\} = [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{2\bar{\beta}}{\beta}\right)_+\int[dx]2\delta(2x-2\bar{\beta}x_3)\delta(x_2)T(x_1,x_2,x_3), \quad (13.69)$$

$$\{\mathbf{DD}_{22}\} = [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma]\left(\frac{-2\bar{\beta}}{\beta}\right)_+\int[dx]2\delta\left(2x+2x_1\right)\delta\left(x_2+\frac{\beta}{\bar{\alpha}}x_3\right)T(x_1,x_2,x_3). \quad (13.70)$$

We integrate out

$$\{\mathbf{DD}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int [d\alpha d\beta d\gamma] \left(\frac{2\bar\beta}{\beta}\right)_+ \int dx_3 \delta(x - \bar\beta x_3) T(-x_3, 0, x_3), \quad (13.71)$$

$$\{\mathbf{DD}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int [d\alpha d\beta d\gamma] \left(\frac{-2\bar\beta}{\beta}\right)_+ \int dx_2 \delta\left(\frac{\beta}{\bar\alpha} x + \frac{\gamma}{\bar\alpha} x_2\right) T(-x, x_2, x - x_2) \quad (13.72)$$

Again comparing to $\{\mathbf{CC}_{21}\}$ and $\{\mathbf{CC}_{22}\}$, we see complete agreement. Here comes the difference. We integrate over α in the first case

$$\{\mathbf{DD}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int dx_3 \int_0^1 d\beta \left(\frac{2\bar\beta}{\beta}\right)_+ \bar\beta \delta(x - \bar\beta x_3) T(-x_3, 0, x_3), \quad (13.73)$$

Note, that **the δ-contribution is not canceled**, in contrast to $\{\mathbf{CC}_{21}\}$-case. Unfolding the $(..)_+$ distribution, we obtain

$$\{\mathbf{DD}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int dx_3 \int_0^1 d\beta \frac{2\bar\beta^2}{\beta} \delta(x - \bar\beta x_3) T(-x_3, 0, x_3), \quad (13.74)$$

The second integral is convenient to rewrite as $(x_2 \to x - x_3)$

$$\{\mathbf{DD}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int dx_3 \int [d\alpha d\beta d\gamma] \left(\frac{-2\bar\beta}{\beta}\right)_+ \delta\left(x - \frac{\gamma}{\bar\alpha} x_3\right) T(-x, x - x_3, x_3). \quad (13.75)$$

We do the change of variables

$$\xi = \frac{\gamma}{\bar\alpha}, \qquad \Rightarrow \qquad \xi \in (0,1), \qquad \beta \in (0, \bar\xi), \qquad d\alpha = \frac{\beta}{(1-\xi)^2}, \quad (13.76)$$

and obtain

$$\{\mathbf{DD}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int dx_3 \int_0^1 d\xi \int_0^{\bar\xi} d\beta \left(\frac{-2\bar\beta}{\beta}\right)_+ \frac{\beta}{(1-\xi)^2} \delta(x - \xi x_3) T(-x, x - x_3, x_3). \quad (13.77)$$

Shortcut Making the change of variables (we ignore $(..)_+$ here since it cancel between terms)

$$\{\mathbf{DD}_{21}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int d\xi \int_0^1 dy \frac{2y^2}{\bar y} \delta(x - y\xi) T(-\xi, 0, \xi), \quad (13.78)$$

$$\{\mathbf{DD}_{22}\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int d\xi \int_0^1 dy \frac{-1-y}{1-y} \delta(x - y\xi) T(-x, x - \xi, \xi). \quad (13.79)$$

I have a problem here. We miss a part of the expression here. In the expression (13.58) we have expected that at $\beta \to 0$ the integral vanish, alike it happens in $\mathbf{CC}_2$ at $\alpha \to 1$. But in fact these case are different, in the diagram CC it was indeed zero, since the integral over β shrinks to 0. Here it is not zero, but is an ambiguity $0 \times \infty$. I have spend 2 weeks (literally) resolving it, but have not been able yet. The result is $\sim T(-x, 0, x)$, i.e. it restore the famous δ-function contribution in the evolution kernel. !See next section! I will return to it later.

So far, it seems to be correct to state that each term is regular by means of the $(..)_+$ distribution

$$\{\mathbf{DD}_{21}\}_+ = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int d\xi \int_0^1 dy \left(\frac{2y^2}{\bar y}\right)_+ \delta(x - y\xi) T(-\xi, 0, \xi), \quad (13.80)$$

$$\{\mathbf{DD}_{22}\}_+ = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int d\xi \int_0^1 dy \left(\frac{-1-y}{1-y}\right)_+ \delta(x - y\xi) T(-x, x - \xi, \xi). \quad (13.81)$$

The difference in comparison to the previous result is

$$\{\mathbf{DD}_{21}\}_+ + \{\mathbf{DD}_{22}\}_+ - (\{\mathbf{DD}_{21}\} + \{\mathbf{DD}_{22}\}) = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \Big[2T(-x, 0, x)\Big]. \quad (13.82)$$

Finally, we consider the third contribution:

$$\mathbf{DD}_3 = -2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} b^\mu \int [d\alpha d\beta d\gamma] \left\{ \int_{-\infty}^{z_1} d\sigma \mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z_1, z^\alpha_{\sigma 2}, z^\beta_{2\sigma}) - \int_{-\infty}^{z_2} d\sigma \mathcal{T}^\nu_{\gamma^+\gamma_T^{\mu\nu}}(z^\beta_{1\sigma}, z^\alpha_{\sigma 1}, z_2) \right\} \tag{13.83}$$

The matrix element is

$$\langle \mathbf{DD}_3 \rangle = [ib^\mu \epsilon_{\mu\nu} s^\nu M](-2)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int [d\alpha d\beta d\gamma] \left\{ \int_{-\infty}^{z_1} d\sigma \Delta T(z_1, z^\alpha_{\sigma 2}, z^\beta_{2\sigma}) - \int_{-\infty}^{z_2} d\sigma \Delta T(z^\beta_{1\sigma}, z^\alpha_{\sigma 1}, z_2) \right\} \tag{13.84}$$

Reflecting the last term we get

$$\langle \mathbf{DD}_3 \rangle = [ib^\mu \epsilon_{\mu\nu} s^\nu M](-2)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int [d\alpha d\beta d\gamma] \int_{-\infty}^{\infty} d\sigma \Delta T(z, \sigma\bar{\alpha} - \alpha z, \sigma\beta - \bar{\beta} z) \tag{13.85}$$

Making Fourier, we get

$$\{\mathbf{DD}_3\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M](-2)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int [d\alpha d\beta d\gamma] \int [dx] \Delta T(x_1, x_2, x_3) 2\delta(\bar{\alpha} x_2 + x_3 \beta)\delta(2x + x_1 - \alpha x_2 - \bar{\beta} x_3) \tag{13.86}$$

Simplifying

$$\{\mathbf{DD}_3\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M](-2)a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int [d\alpha d\beta d\gamma] \int dx_3 \Delta T(-x, x - x_3, x_3) \frac{\delta(x - \frac{\gamma}{\bar{\alpha}} x_3)}{\bar{\alpha}} \tag{13.87}$$

Changing variables we get

$$\{\mathbf{DD}_3\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} \int_0^1 dy \int d\xi \delta(x - y\xi)(-1)\Delta T(-x, x - \xi, \xi). \tag{13.88}$$

E. The consideration of DD_2 (again)

Let me evaluate $\mathbf{DD}_2$ in a different order

$$\mathbf{DD}_2 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} b^\mu \int [d\alpha d\beta d\gamma](-2\bar{\beta}) \left\{ \int_{-\infty}^{z_1} d\sigma \bar{q}(z_1)\gamma^+ A_\mu(z^\alpha_{\sigma 2})\partial_+ q(z^\beta_{2\sigma}) + \int_{-\infty}^{z_2} d\sigma \partial_+ \bar{q}(z^\beta_{1\sigma})\gamma^+ A(z^\alpha_{\sigma 1}) q(z_2) \right\}. \tag{13.89}$$

In particular I am not going to use integration by parts. I rewrite

$$\mathbf{DD}_2 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2} b^\mu \int [d\alpha d\beta d\gamma](2\bar{\beta}) \int_{-\infty}^0 d\lambda \left\{ \int_{-\infty}^{z_1} d\sigma \bar{q}(z_1)\gamma^+ F^{\mu +}(\lambda + z^\alpha_{\sigma 2})\partial_+ q(z^\beta_{2\sigma}) + \int_{-\infty}^{z_2} d\sigma \partial_+ \bar{q}(z^\beta_{1\sigma})\gamma^+ F^{\mu +}(\lambda + z^\alpha_{\sigma 1}) q(z_2) \right\}. \tag{13.90}$$

The derivatives can be written as

$$\mathbf{DD}_2 = 2ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}b^\mu\int[d\alpha d\beta d\gamma](2\bar\beta)\int_{-\infty}^0 d\lambda \tag{13.91}$$
$$\left\{\int_{-\infty}^{z_1}d\sigma\frac{d}{d\omega}\mathcal{T}^\mu_{\gamma^+}(z_1,\lambda+z^\alpha_{\sigma 2},z^\beta_{2\sigma}+\omega)+\int_{-\infty}^{z_2}d\sigma\frac{d}{d\omega}\mathcal{T}^\mu_{\gamma^+}(z^\beta_{1\sigma}+\omega,\lambda+z^\alpha_{\sigma 1},z_2)\right\}_{\omega=0}.$$

The matrix element and Fourier gives

$$\{\mathbf{DD}_2\} = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma](2\bar\beta)\int_{-\infty}^0 d\lambda\int[dx]T(x_1,x_2,x_3)\int\frac{dz}{2\pi}e^{-2ixz} \tag{13.92}$$
$$\left\{\int_{-\infty}^{z_1}d\sigma\frac{d}{d\omega}e^{-i(x_1z_1+x_2(\lambda+z^\alpha_{\sigma 2})+x_3(z^\beta_{2\sigma}+\omega))}+\int_{-\infty}^{z_2}d\sigma\frac{d}{d\omega}e^{-i(x_1(z^\beta_{1\sigma}+\omega)+x_2(\lambda+z^\alpha_{\sigma 1})+x_3z_2)}\right\}_{\omega=0}$$

Differentiating

$$\{\mathbf{DD}_2\} = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma](2\bar\beta)\int_{-\infty}^0 d\lambda\int[dx]T(x_1,x_2,x_3)\int\frac{dz}{2\pi}e^{-2ixz}(-i) \tag{13.93}$$
$$\left\{\int_{-\infty}^{z}d\sigma x_3e^{-iz(x_1-x_2\alpha-\bar\beta x_3)}e^{-i(x_2\lambda+\sigma(x_2\bar\alpha+x_3\beta)}+\int_{-\infty}^{-z}d\sigma x_1e^{-iz(x_3+x_2\alpha+\bar\beta x_1)}e^{-i(x_2\lambda+\sigma(x_2\bar\alpha+x_1\beta)}\right\}.$$

Next we integrate over λ and σ

$$\{\mathbf{DD}_2\} = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2} \tag{13.94}$$
$$\int[d\alpha d\beta d\gamma](2\bar\beta)\int[dx]T(x_1,x_2,x_3)\int\frac{dz}{2\pi}e^{-2ixz}\Bigg[$$
$$\frac{ix_3e^{-iz(x_1+x_2(1-2\alpha)-x_3(1-2\beta)}}{(x_2+i0)(\bar\alpha x_2+x_3\beta+i0)}+\frac{ix_1e^{iz(x_3+x_2(1-2\alpha)-x_1(1-2\beta)}}{(x_2+i0)(\bar\alpha x_2+x_1\beta+i0)}\Bigg].$$

We split the numerators as

$$\frac{1}{(x_2+i0)(\bar\alpha x_2+x_{1,3}\beta+i0)}=\frac{1}{x_{1,3}\beta-x_2\alpha}\left(\frac{1}{x_2+i0}-\frac{1}{x_{1,3}\beta+\bar\alpha x_2+i0}\right) \tag{13.95}$$

In the second term I change $x_{123}\to -x_{321}$, and get

$$\{\mathbf{DD}_2\} = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2} \tag{13.96}$$
$$\int[d\alpha d\beta d\gamma](2\bar\beta)\int[dx]T(x_1,x_2,x_3)\int\frac{dz}{2\pi}e^{-2ixz}\frac{ix_3e^{-iz(x_1+x_2(1-2\alpha)-x_3(1-2\beta))}}{x_3\beta-x_2\alpha}\Bigg\{$$
$$\left(\frac{1}{x_2+i0}-\frac{1}{x_2-i0}\right)-\left(\frac{1}{\bar\alpha x_2+x_3\beta+i0}-\frac{1}{\bar\alpha x_2+x_3\beta-i0}\right)\Bigg\}.$$

The terms in brackets are $(-2\pi i)\delta$-functions

$$\{\mathbf{DD}_2\} = [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma](2\bar\beta)\int[dx] \tag{13.97}$$
$$T(x_1,x_2,x_3)\int\frac{dz}{2\pi}e^{-2ixz}\frac{i(-2\pi i)x_3e^{-iz(x_1+x_2(1-2\alpha)-x_3(1-2\beta))}}{x_3\beta-x_2\alpha}\Big\{\delta(x_2)-\delta(\bar\alpha x_2+x_3\beta)\Big\}.$$

We also integrate over z

$$\{\mathbf{DD}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int [dx] T(x_1, x_2, x_3) \int [d\alpha d\beta d\gamma] 2\delta(2x + x_1 + x_2(1-2\alpha) - x_3(1-2\beta)) \frac{2\bar\beta x_3}{x_3\beta - x_2\alpha} \Big\{ \delta(x_2) - \delta(\bar\alpha x_2 + x_3\beta) \Big\}. \quad (13.98)$$

With the help of $[dx]$ we simplify

$$\{\mathbf{DD}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int [dx] T(x_1, x_2, x_3) \int [d\alpha d\beta d\gamma] \delta(x + x_1 + \bar\alpha x_2 + x_3\beta) \frac{2\bar\beta x_3}{x_3\beta - x_2\alpha} \Big\{ \delta(x_2) - \delta(\bar\alpha x_2 + x_3\beta) \Big\} \quad (13.99)$$

$$= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int [dx] T(x_1, x_2, x_3) \int [d\alpha d\beta d\gamma] \delta(x + x_1 + \bar\alpha x_2 + x_3\beta) \frac{2\bar\beta x_3}{-x - x_1 - x_2} \Big\{ \delta(x_2) - \delta(\bar\alpha x_2 + x_3\beta) \Big\} \quad (13.100)$$

$$= [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int [dx] T(x_1, x_2, x_3) \int [d\alpha d\beta d\gamma] \delta(x + x_1 + \bar\alpha x_2 + x_3\beta) \frac{2\bar\beta x_3}{x_3 - x} \Big\{ \delta(x_2) - \delta(x + x_1) \Big\}.$$

Finally, let me integrate over x_1 using $[dx]$

$$\{\mathbf{DD}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int dx_{23} T(x_1, x_2, x_3) \int [d\alpha d\beta d\gamma] \delta(x - \alpha x_2 - x_3\bar\beta) \frac{2\bar\beta x_3}{x_3 - x} \Big\{ \delta(x_2) - \delta(x - x_2 - x_3) \Big\}. \quad (13.101)$$

If I integrate over everything except Millin convolution, and change variables I get

$$\{\mathbf{DD}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int d\xi \int_0^1 dy \delta(x - y\xi) \Big\{ \frac{2y^2\xi}{\xi - x} T(-\xi, 0, \xi) - \frac{(1+y)\xi}{\xi - x} T(-x, x-\xi, \xi) \Big\} \quad (13.102)$$

Which is

$$\{\mathbf{DD}_2\} = [i\pi b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int d\xi \int_0^1 dy \delta(x - y\xi) \Big\{ \frac{2y^2}{1-y} T(-\xi, 0, \xi) - \frac{(1+y)}{1-y} T(-x, x-\xi, \xi) \Big\} \quad (13.103)$$

It coincides with the previous one.

Let me invent a regulator that would resolve QS function. I do it by introducing $e^{-is_1\lambda - is_2\sigma}$ in the first term of (13.93) and $e^{+is_1\lambda + is_2\sigma}$ in the second. Integrating over λ and σ I get

$$\{\mathbf{DD}_2\} = [i b^\mu \epsilon_{\mu\nu} s^\nu M] 2 a_s \Gamma(-\epsilon) \mathbf{B}^\epsilon \frac{C_A}{2} \int [d\alpha d\beta d\gamma] (2\bar\beta) \int [dx] T(x_1, x_2, x_3) \int \frac{dz}{2\pi} e^{-2ixz} \Bigg[\frac{i x_3 e^{-iz(x_1 + x_2(1-\alpha) - x_3(1-2\beta))}}{(x_2 + s_1 + i0)(\bar\alpha x_2 + x_3\beta + s_2 + i0)} + \frac{i x_1 e^{iz(x_3 + x_2(1-2\alpha) - x_1(1-2\beta))}}{(x_2 - s_1 + i0)(\bar\alpha x_2 + x_1\beta - s_2 + i0)} \Bigg]. \quad (13.104)$$

Splitting the ratios I get

$$\frac{1}{(x_2 \pm s_1 + i0)(\bar{\alpha}x_2 + x_i \pm s_2\beta + i0)} = \frac{1}{x_i\beta - x_2\alpha \pm (s_2 - s_1)}\left(\frac{1}{x_2 \pm s_1 + i0} - \frac{1}{x_i\beta + \bar{\alpha}x_2 \pm s_2 + i0}\right). \quad (13.105)$$

In the second term I change $x_{123} \to -x_{321}$, and get

$$\begin{aligned}\{\mathbf{DD}_2\} &= [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2} \quad (13.106)\\ &\int[d\alpha d\beta d\gamma](2\bar{\beta})\int[dx]T(x_1,x_2,x_3)\int\frac{dz}{2\pi}e^{-2ixz}\frac{ix_3e^{-iz(x_1+x_2(1-2\alpha)-x_3(1-2\beta))}}{x_3\beta - x_2\alpha + s_2 - s_1}\Bigg\{\\ &\left(\frac{1}{x_2+s_1+i0} - \frac{1}{x_2+s_1-i0}\right) - \left(\frac{1}{\bar{\alpha}x_2+x_3\beta+s_3+i0} - \frac{1}{\bar{\alpha}x_2+x_3\beta+s_3-i0}\right)\Bigg\}.\end{aligned}$$

The terms in brackets are $(-2\pi i)\delta$-functions

$$\begin{aligned}\{\mathbf{DD}_2\} &= [ib^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[d\alpha d\beta d\gamma](2\bar{\beta})\int[dx] \quad (13.107)\\ &T(x_1,x_2,x_3)\int\frac{dz}{2\pi}e^{-2ixz}\frac{i(-2\pi i)x_3e^{-iz(x_1+x_2(1-2\alpha)-x_3(1-2\beta))}}{x_3\beta - x_2\alpha + s_2 - s_1}\Bigg\{\delta(x_2+s_1) - \delta(\bar{\alpha}x_2+x_3\beta+s_2)\Bigg\}.\end{aligned}$$

We also integrate over z

$$\begin{aligned}\{\mathbf{DD}_2\} &= [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int[dx]T(x_1,x_2,x_3) \quad (13.108)\\ &\int[d\alpha d\beta d\gamma]2\delta(2x+x_1+x_2(1-2\alpha)-x_3(1-2\beta))\frac{2\bar{\beta}x_3}{x_3\beta - x_2\alpha + s_2 - s_1}\Bigg\{\\ &\delta(x_2+s_1) - \delta(\bar{\alpha}x_2+x_3\beta+s_2)\Bigg\}.\end{aligned}$$

Integrating over x_1 and simplifying a bit

$$\begin{aligned}\{\mathbf{DD}_2\} &= [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int dx_{23}T(-x_2-x_3,x_2,x_3) \quad (13.109)\\ &\int[d\alpha d\beta d\gamma]\delta(x-\alpha x_2-\bar{\beta}x_3)\frac{2\bar{\beta}x_3}{x_3 - x + s_2 - s_1}\Bigg\{\delta(x_2+s_1) - \delta(\bar{\alpha}x_2+x_3\beta+s_2)\Bigg\}.\end{aligned}$$

I integrate over x_2 using δ-functions and change the variables as: In the first term I make a change of variables

$$x_3 = \xi + \frac{\alpha}{\bar{\beta}}s_1, \quad (13.110)$$

in the second

$$x_3 = \xi + \frac{\alpha}{\gamma}s_2 \quad (13.111)$$

$$\begin{aligned}\{\mathbf{DD}_2\} &= [i\pi b^\mu\epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\frac{C_A}{2}\int d\xi \quad (13.112)\\ &\int[d\alpha d\beta d\gamma]\Bigg\{T(-\xi+\frac{\gamma}{\bar{\beta}}s_1,-s_1,\xi+\frac{\alpha}{\bar{\beta}}s_1)\delta(x-\bar{\beta}\xi)\frac{2\bar{\beta}(x+s_1\alpha)}{x\beta - s_1\gamma + s_2\bar{\beta}}\\ &-\delta(\bar{\alpha}x-\gamma\xi)T\left(-s_2-x,x-\xi-\frac{\bar{\beta}}{\gamma}s_2,\xi+\frac{\alpha}{\gamma}s_2\right)\frac{2\bar{\beta}(\bar{\alpha}x+s_2\alpha)}{x\beta+\bar{\beta}s_2-\gamma s_1}\Bigg\}.\end{aligned}$$

One can check that at $s_{1,2}=0$ this expression turn to the previous one. Simultaneously each term is regular at $\beta\to 0$. Let me make the further change of variables $\bar\beta = y$ (in the first term) and $\gamma/\bar\alpha = y$ in the second term. I get

$$\begin{aligned}\{\mathbf{DD}_2\} &= [i\pi b^\mu \epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\int d\xi \int_0^1 dy\delta(x-\xi y) \\ &\left\{\int_0^y d\alpha T\left(-\xi+\left(1-\frac{\alpha}{y}\right)s_1, -s_1, \xi+\frac{\alpha}{y}s_1\right)\frac{2y(x+s_1\alpha)}{x\bar y + s_1(\alpha-y)+s_2 y}\right. \\ &\left.-\int_0^1 d\alpha T\left(-s_2-x, x-\xi-s_2\frac{y+\bar\alpha y}{y\bar\alpha}, \xi+\frac{\alpha}{\bar\alpha y}s_2\right)\frac{2(\bar\alpha x+s_2\alpha)(\bar\alpha y+\alpha)}{x\bar\alpha\bar y+(y+\bar\alpha y)s_2 - y\bar\alpha s_1}\right\}.\end{aligned} \tag{13.113}$$

Some rescaling in α

$$\begin{aligned}\{\mathbf{DD}_2\} &= [i\pi b^\mu \epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\int d\xi \int_0^1 dy\delta(x-\xi y) \\ &\left\{\int_0^1 d\alpha T\left(-\xi+\bar\alpha s_1, -s_1, \xi+\alpha s_1\right)\frac{2y^2(x+s_1 y\alpha)}{x\bar y + y(s_2-\bar\alpha s_1)}\right. \\ &\left.-\int_0^1 d\alpha T\left(-s_2-x, x-\xi-s_2\frac{y+\bar\alpha y}{y\bar\alpha}, \xi+\frac{\alpha}{\bar\alpha y}s_2\right)\frac{2(\bar\alpha x+s_2\alpha)(\bar\alpha y+\alpha)}{x\bar\alpha\bar y+(y+\bar\alpha y)s_2 - y\bar\alpha s_1}\right\}.\end{aligned} \tag{13.114}$$

In the second term $\alpha\to y\alpha/(1+y\alpha)$

$$\begin{aligned}\{\mathbf{DD}_2\} &= [i\pi b^\mu \epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\int d\xi \int_0^1 dy\delta(x-\xi y) \\ &\left\{\int_0^1 d\alpha T\left(-\xi+\bar\alpha s_1, -s_1, \xi+\alpha s_1\right)\frac{2y^2(x+s_1 y\alpha)}{x\bar y + y(s_2-\bar\alpha s_1)}\right. \\ &\left.-\int_0^\infty d\alpha T\left(-s_2-x, x-\xi-s_2(1+\alpha), \xi+\alpha s_2\right)\frac{2y^2(1+\alpha)(x+s_2 y\alpha)}{(1+\alpha y)^3(x\bar y+y(s_2-s_1+s_2\alpha)}\right\}.\end{aligned} \tag{13.115}$$

This sum is regular (at $s\to 1$) everywhere except $y=1$! This is this lost δ-function. Unfortunately, the regulator is too difficult to take the integral explicitly. I have taken it by Mathematica, in the limit $y\to 1$.

F. Final expression for diagrams D

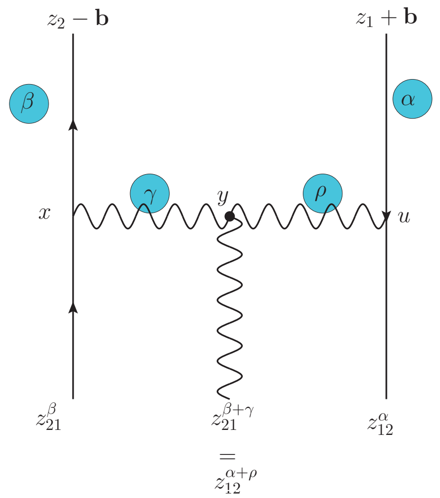
There is a forgotten common minus, which comes from the 3-gluon vertex.Here correct version:

$$\begin{aligned}\{\mathbf{D}+\mathbf{D}^*\} &= [i\pi b^\mu \epsilon_{\mu\nu}s^\nu M]2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \frac{C_A}{2}\left\{-2T(-x,0,x)\right. \\ &\left.+\int d\xi\int_0^1 dy\delta(x-\xi y)\left\{\frac{-2y^2T(-\xi,0,\xi)+(1+y)T(-x,x-\xi,\xi)}{1-y}+\Delta T(-x,x-\xi,\xi)\right\}\right\}\end{aligned} \tag{13.116}$$

XIV. DIAGRAM F

A. Diagram F

F



The diagram has the form

$$\mathbf{F} = v^{\mu\alpha\beta\gamma}_{A'B'C'} \tag{14.1}$$
$$\Big(ig\int d^du\bar{q}(u)\ \not{B}(u)\psi(u)\Big)\Big(\bar{\psi}(z_1n+\boldsymbol{b})\gamma^+\psi(z_2n-\boldsymbol{b})\Big)\Big(ig\int d^dx\bar{\psi}(x)\ \not{B}(x)q(x)\Big)\Big(ig\int d^dyA^{A'}_{\mu}(y)(\partial_\alpha B^{B'}_{\beta}(y))B^{C'}_{\gamma}(y)\Big)$$
$$+v^{\mu\alpha\beta\gamma}_{A'B'C'}$$
$$\Big(ig\int d^du\bar{q}(u)\ \not{B}(u)\psi(u)\Big)\Big(\bar{\psi}(z_1n+\boldsymbol{b})\gamma^+\psi(z_2n-\boldsymbol{b})\Big)\Big(ig\int d^dx\bar{\psi}(x)\ \not{B}(x)q(x)\Big)\Big(ig\int d^dyA^{A'}_{\mu}(y)(\partial_\alpha B^{B'}_{\beta}(y))B^{C'}_{\gamma}(y)\Big)$$

Step I: Substituting propagators

$$\begin{aligned}\mathbf{F} \ =\ & (ig)^3\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(2-\epsilon)}{2\pi^{d/2}}\frac{-\Gamma(1-\epsilon)}{4\pi^{d/2}}\int d^dxd^dyd^du\bar{q}(u)\Bigg[\\ & t^Bt^C\gamma^\beta\frac{\not{u}-z_1\gamma^+-\not{b}}{[-(u-z_1n-b)^2]^{2-\epsilon}}\gamma^+\frac{z_2\gamma^+-\not{b}-\not{x}}{[-(x-z_2n+b)^2]^{2-\epsilon}}\gamma^\gamma\frac{y_\alpha-u_\alpha}{[-(y-u)^2]^{2-\epsilon}}\frac{v^{\mu\alpha\beta\gamma}_{ABC}}{[-(x-y)^2]^{1-\epsilon}} \\ & +t^Ct^B\gamma^\gamma\frac{\not{u}-z_1\gamma^+-\not{b}}{[-(u-z_1n-b)^2]^{2-\epsilon}}\gamma^+\frac{z_2\gamma^+-\not{b}-\not{x}}{[-(x-z_2n+b)^2]^{2-\epsilon}}\gamma^\beta\frac{y_\alpha-x_\alpha}{[-(y-x)^2]^{2-\epsilon}}\frac{v^{\mu\alpha\beta\gamma}_{ABC}}{[-(y-u)^2]^{1-\epsilon}}\Bigg]A^A_\mu(y)q(x).\end{aligned} \tag{14.2}$$

Step II: Simplifying a bit

$$\begin{aligned}\mathbf{F} \ =\ & g^3\frac{C_A}{2}\frac{\Gamma^3(2-\epsilon)\Gamma(1-\epsilon)}{32\pi^{2d}}\int d^dxd^dyd^du\bar{q}(u)\Bigg[\\ & \gamma^\beta\frac{\not{u}-\not{b}}{[-(u-z_1n-b)^2]^{2-\epsilon}}\gamma^+\frac{\not{x}+\not{b}}{[-(x-z_2n+b)^2]^{2-\epsilon}}\gamma^\gamma\frac{y_\alpha-u_\alpha}{[-(y-u)^2]^{2-\epsilon}}\frac{\Big(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\Big)}{[-(x-y)^2]^{1-\epsilon}} \\ & -\gamma^\gamma\frac{\not{u}-\not{b}}{[-(u-z_1n-b)^2]^{2-\epsilon}}\gamma^+\frac{\not{x}+\not{b}}{[-(x-z_2n+b)^2]^{2-\epsilon}}\gamma^\beta\frac{y_\alpha-x_\alpha}{[-(y-x)^2]^{2-\epsilon}}\frac{\Big(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\Big)}{[-(y-u)^2]^{1-\epsilon}}\Bigg]A_\mu(y)q(x).\end{aligned} \tag{14.3}$$

Step III: We joining propagators according to the picture

$$\begin{aligned}(u-z_1-b)^2 &\to \alpha\\ (x-z_2+b)^2 &\to \beta\\ (x-y)^2 &\to \gamma\\ (u-y)^2 &\to \rho\end{aligned}$$

We also make a shift

$$\begin{aligned}x &\to x+\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)\\ y &\to y+\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)\\ u &\to u+\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)\end{aligned}$$

$$\lambda=\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho.$$

We obtain

$$\begin{aligned}\mathbf{F} &= g^3\frac{C_A}{2}\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\int d^dxd^dyd^du\int[d\alpha d\beta d\gamma d\rho] \qquad (14.4)\\ &\bar{q}(u+\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right))\\ &\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\rho^{-\epsilon}\frac{\left(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\right)}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(yu)+2\gamma(xy)+4\frac{\alpha\beta\gamma\rho}{\gamma}\boldsymbol{b}^2+i0]^{7-4\epsilon}}\Bigg[\\ &\rho\gamma^\beta(\not{u}-2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^+(\not{x}+2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^\gamma\left(y_\alpha-u_\alpha-\frac{\alpha\beta\gamma}{\lambda}n_\alpha(z_1-z_2)-b_\alpha 2\frac{\alpha\beta\gamma}{\lambda}\right)\\ &-\gamma\gamma^\gamma(\not{u}-2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^+(\not{x}+2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^\beta\left(y_\alpha-x_\alpha+\frac{\alpha\beta\rho}{\lambda}n_\alpha(z_1-z_2)+b_\alpha 2\frac{\alpha\beta\rho}{\lambda}\right)\\ &\Bigg]A_\mu(y+\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right))\\ &q(x+\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)).\end{aligned}$$

Step IV: According to general analysis we do not need any derivatives. Therefore we just drop the x, y, u fomr the arguments

$$\begin{aligned}\mathbf{F} &= g^3\frac{C_A}{2}\frac{\Gamma(7-4\epsilon)}{32\pi^{2d}}\int d^dxd^dyd^du\int[d\alpha d\beta d\gamma d\rho]\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \qquad (14.5)\\ &\alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\rho^{-\epsilon}\frac{\left(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\right)}{[-(\beta+\gamma)x^2-(\gamma+\rho)y^2-(\alpha+\rho)u^2+2\rho(yu)+2\gamma(xy)+4\frac{\alpha\beta\gamma\rho}{\gamma}\boldsymbol{b}^2+i0]^{7-4\epsilon}}\Bigg[\\ &\rho\gamma^\beta(\not{u}-2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^+(\not{x}+2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^\gamma\left(y_\alpha-u_\alpha-\frac{\alpha\beta\gamma}{\lambda}n_\alpha(z_1-z_2)-b_\alpha 2\frac{\alpha\beta\gamma}{\lambda}\right)\\ &-\gamma\gamma^\gamma(\not{u}-2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^+(\not{x}+2\frac{\alpha\gamma\rho}{\lambda}\not{b})\gamma^\beta\left(y_\alpha-x_\alpha+\frac{\alpha\beta\rho}{\lambda}n_\alpha(z_1-z_2)+b_\alpha 2\frac{\alpha\beta\rho}{\lambda}\right)\\ &\Bigg]A_\mu(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right))\\ &q(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)).\end{aligned}$$

Step V: Leaving only the even power so x, y, u we get (we also a bit simplify)

$$\begin{aligned}
\mathbf{F} \; = \; & g^3 \frac{C_A}{2} \frac{\Gamma(7-4\epsilon)}{32\pi^{2d}} \int d^d x d^d y d^d u \int [d\alpha d\beta d\gamma d\rho] \bar{q}(\frac{\alpha\gamma\rho + \alpha\beta\gamma + \alpha\beta\rho}{\lambda} z_1 + \frac{\beta\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)) \qquad (14.6) \\
& \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\rho^{-\epsilon} \frac{\left(2g^{\mu\beta}g^{\alpha\gamma} - g^{\mu\alpha}g^{\beta\gamma} - 2g^{\mu\gamma}g^{\alpha\beta}\right)}{[-(\beta+\gamma)x^2 - (\gamma+\rho)y^2 - (\alpha+\rho)u^2 + 2\rho(yu) + 2\gamma(xy) + 4\frac{\alpha\beta\gamma\rho}{\gamma}\boldsymbol{b}^2 + i0]^{7-4\epsilon}} \Bigg[\\
& 4\frac{\alpha\beta\gamma\rho}{\lambda}\frac{\alpha\beta\gamma^2\rho^2}{\lambda^2}\left[\gamma^\beta \not{b}\gamma^+ \not{b}\gamma^\gamma + \gamma^\gamma \not{b}\gamma^+ \not{b}\gamma^\beta\right](n_\alpha(z_1 - z_2) + 2b_\alpha) \\
& -\frac{\alpha\beta\gamma\rho}{\lambda}\left[\gamma^\beta \not{u}\gamma^+ \not{x}\gamma^\gamma + \gamma^\gamma \not{u}\gamma^+ \not{x}\gamma^\beta\right](n_\alpha(z_1 - z_2) + 2b_\alpha) \\
& -2\frac{\beta\gamma\rho^2}{\lambda}\gamma^\beta \not{b}\gamma^+ \not{x}\gamma^\gamma (y_\alpha - u_\alpha) + 2\frac{\beta\gamma^2\rho}{\lambda}\gamma^\gamma \not{b}\gamma^+ \not{x}\gamma^\beta (y_\alpha - x_\alpha) \\
& +2\frac{\alpha\gamma\rho^2}{\lambda}\gamma^\beta \not{u}\gamma^+ \not{b}\gamma^\gamma (y_\alpha - u_\alpha) - 2\frac{\alpha\gamma^2\rho}{\lambda}\gamma^\gamma \not{u}\gamma^+ \not{b}\gamma^\beta (y_\alpha - x_\alpha) \\
& \Bigg] A_\mu(\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda}\right)) \\
& q(\frac{\alpha\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \alpha\beta\rho + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right)).
\end{aligned}$$

Step VI: The execution of the integral is straightforward

$$\begin{aligned}
\mathbf{F} \; = \; & -ig^3 \frac{C_A}{2} \frac{\Gamma(-\epsilon)}{32\pi^{d/2}} \mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma d\rho] \bar{q}(\frac{\alpha\gamma\rho + \alpha\beta\gamma + \alpha\beta\rho}{\lambda} z_1 + \frac{\beta\gamma\rho}{\lambda} z_2 + \boldsymbol{b}\left(1 - 2\frac{\beta\gamma\rho}{\lambda}\right)) \qquad (14.7) \\
& \alpha^{1-\epsilon}\beta^{1-\epsilon}\gamma^{-\epsilon}\rho^{-\epsilon} \frac{(4\alpha\beta\gamma\rho)^\epsilon}{\lambda^3} \left(2g^{\mu\beta}g^{\alpha\gamma} - g^{\mu\alpha}g^{\beta\gamma} - 2g^{\mu\gamma}g^{\alpha\beta}\right) \Bigg[\\
& 4\frac{\alpha\beta\gamma\rho}{\lambda}\frac{\alpha\beta\gamma^2\rho^2}{\lambda^2}\left[\gamma^\beta \not{b}\gamma^+ \not{b}\gamma^\gamma + \gamma^\gamma \not{b}\gamma^+ \not{b}\gamma^\beta\right](n_\alpha(z_1 - z_2) + 2b_\alpha) \frac{\epsilon\lambda^2}{4\alpha\beta\gamma\rho\boldsymbol{b}^2} \\
& -\frac{\alpha\beta\gamma\rho}{\lambda}\left[\gamma^\beta\gamma^\nu\gamma^+\gamma_\nu\gamma^\gamma + \gamma^\gamma\gamma^\nu\gamma^+\gamma_\nu\gamma^\beta\right](n_\alpha(z_1 - z_2) + 2b_\alpha) \frac{\gamma\rho}{2} \\
& -2\frac{\beta\gamma\rho^2}{\lambda}\gamma^\beta \not{b}\gamma^+\gamma^\alpha\gamma^\gamma \frac{\alpha\gamma}{2} + 2\frac{\beta\gamma^2\rho}{\lambda}\gamma^\gamma \not{b}\gamma^+\gamma^\alpha\gamma^\beta \frac{-\alpha\rho}{2} \\
& +2\frac{\alpha\gamma\rho^2}{\lambda}\gamma^\beta\gamma^\alpha\gamma^+ \not{b}\gamma^\gamma \frac{-\beta\gamma}{2} - 2\frac{\alpha\gamma^2\rho}{\lambda}\gamma^\gamma\gamma^\alpha\gamma^+ \not{b}\gamma^\beta \frac{\beta\rho}{2} \\
& \Bigg] A_\mu(\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho + \alpha\beta\rho}{\lambda}\right)) \\
& q(\frac{\alpha\gamma\rho}{\lambda} z_1 + \frac{\alpha\beta\gamma + \alpha\beta\rho + \beta\gamma\rho}{\lambda} z_2 - \boldsymbol{b}\left(1 - 2\frac{\alpha\gamma\rho}{\lambda}\right)).
\end{aligned}$$

Step VII: Simplifying a bit

$$\begin{aligned}\mathbf{F} &= -i\frac{ga_s}{2}\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma d\rho]\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\ &\frac{(\alpha\beta\gamma\rho)^2}{\lambda^4}\left(2g^{\mu\beta}g^{\alpha\gamma}-g^{\mu\alpha}g^{\beta\gamma}-2g^{\mu\gamma}g^{\alpha\beta}\right)\Big[\\ &\left[\gamma^\beta\not{b}\gamma^+\not{b}\gamma^\gamma+\gamma^\gamma\not{b}\gamma^+\not{b}\gamma^\beta\right](n_\alpha(z_1-z_2)+2b_\alpha)\frac{\epsilon}{\boldsymbol{b}^2} \\ &-\left[\gamma^\beta\gamma^\nu\gamma^+\gamma_\nu\gamma^\gamma+\gamma^\gamma\gamma^\nu\gamma^+\gamma_\nu\gamma^\beta\right]\left(\frac{n_\alpha}{2}(z_1-z_2)+b_\alpha\right) \\ &-\gamma^\beta\not{b}\gamma^+\gamma^\alpha\gamma^\gamma-\gamma^\gamma\not{b}\gamma^+\gamma^\alpha\gamma^\beta-\gamma^\beta\gamma^\alpha\gamma^+\not{b}\gamma^\gamma-\gamma^\gamma\gamma^\alpha\gamma^+\not{b}\gamma^\beta \\ &\Big]A_\mu(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)) \\ &q(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)).\end{aligned} \tag{14.8}$$

Step VII: We evaluate the γ-algebra by Mathematica and get

$$\begin{aligned}\mathbf{F} &= -i\frac{ga_s}{2}\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma d\rho]\bar{q}(\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)) \\ &\frac{(\alpha\beta\gamma\rho)^2}{\lambda^4}\left[4(1-\epsilon)\underbrace{(2b_\mu\gamma^+-\not{b}\gamma^+\gamma^\mu-\gamma^\mu\gamma^+\not{b})}_{4b_\mu\gamma^+}\right] \\ &A_\mu(\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)) \\ &q(\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)).\end{aligned} \tag{14.9}$$

Step VIII: Passing to the dual variables

$$\begin{aligned}\mathbf{F} &= -i8ga_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\mu\int[d\alpha d\beta d\gamma d\rho] \\ &\bar{q}(z_{12}^\alpha+\boldsymbol{b}(1-2\alpha))\gamma^+A_\mu(z_{12}^{\alpha+\rho}+\boldsymbol{b}(1-2\alpha+2\rho))q(z_{21}^\beta-\boldsymbol{b}(1-2\beta)).\end{aligned} \tag{14.10}$$

Step IX: We can drop the $\boldsymbol{b}$ from the arguments

$$\mathbf{F} = -i8ga_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\mu\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha)\gamma^+A_\mu(z_{12}^{\alpha+\rho})q(z_{21}^\beta). \tag{14.11}$$

B. Refining diagram F

The diagram reads

$$\mathbf{F} = -i8ga_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\mu\int[d\alpha d\beta d\gamma d\rho]\bar{q}(z_{12}^\alpha)\gamma^+A_\mu(z_{12}^{\alpha+\rho})q(z_{21}^\beta). \tag{14.12}$$

For future simplicity we split it into two parts, whose topologies corresponds to $\mathbf{E}$ and $\mathbf{E}^*$.

$$\mathbf{F} = -4ga_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\mu\int[d\alpha d\beta d\gamma d\rho]\left[\bar{q}(z_{12}^\alpha)\gamma^+A_\mu(z_{12}^{\alpha+\rho})q(z_{21}^\beta)+\bar{q}(z_{12}^\beta)\gamma^+A_\mu(z_{21}^{\alpha+\rho})q(z_{21}^\alpha)\right]. \tag{14.13}$$

We apply (6.19) to translate the operators to the standard operators. In fact we use (11.41,11.42) we get

$$\mathbf{F} = 4iga_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\mu \int [d\alpha d\beta d\gamma d\rho](\alpha+\rho) \tag{14.14}$$
$$\Big[\int_{-\infty}^{z_2} \bar{q}(z_{12}^\alpha)\gamma^+ F^{\mu+}(z_{1\sigma}^{\alpha+\rho})q(z_{21}^\beta) + \int_{-\infty}^{z_1} \bar{q}(z_{12}^\beta)\gamma^+ F^{\mu+}(z_{2\sigma}^{\alpha+\rho})q(z_{21}^\alpha)\Big]d\sigma.$$

Thus the final expression is

$$\mathbf{F} = 4ia_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon b_\mu \tag{14.15}$$
$$\int [d\alpha d\beta d\gamma d\rho](\alpha+\rho)\Big[\int_{-\infty}^{z_2} \mathcal{T}^\mu_{\gamma^+}(z_{12}^\alpha, z_{1\sigma}^{\alpha+\rho}, z_{21}^\beta) + \int_{-\infty}^{z_1} \mathcal{T}^\mu_{\gamma^+}(z_{12}^\beta, z_{2\sigma}^{\alpha+\rho}, z_{21}^\alpha)\Big]d\sigma.$$

C. Elaborating diagram F

The matrix element is

$$\langle \mathbf{F} \rangle = [ib^\mu \epsilon_{\mu\nu}s^\nu M]4a_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon \tag{14.16}$$
$$\int [d\alpha d\beta d\gamma d\rho](\alpha+\rho)\Big[\int_{-\infty}^{z_2} T(z_{12}^\alpha, z_{1\sigma}^{\alpha+\rho}, z_{21}^\beta) + \int_{-\infty}^{z_1} T(z_{12}^\beta, z_{2\sigma}^{\alpha+\rho}, z_{21}^\alpha)\Big]d\sigma$$
$$= [ib^\mu \epsilon_{\mu\nu}s^\nu M]4a_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma d\rho](\alpha+\rho)\int_{-\infty}^{\infty} T(z_{12}^\alpha, z_{1\sigma}^{\alpha+\rho}, z_{21}^\beta)d\sigma, \tag{14.17}$$

where in the second line we have reflected the second term and added in to the first one. Taking the Fourier we get

$$\{\mathbf{F}\} = [i\pi b^\mu \epsilon_{\mu\nu}s^\nu M]4a_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [dx]\int [d\alpha d\beta d\gamma d\rho] \tag{14.18}$$
$$T(x_1,x_2,x_3)(\alpha+\rho)2\delta((\alpha+\rho)x_2)\delta(2x + x_1(1-2\alpha) + x_2(1-\alpha-\rho) - x_3(1-2\beta)).$$

Integrating over $x_{1,2}$

$$\{\mathbf{F}\} = [i\pi b^\mu \epsilon_{\mu\nu}s^\nu M]4a_s(1-\epsilon)\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon \int dx_3 \int [d\alpha d\beta d\gamma d\rho]\delta(x-(1-\alpha-\beta)x_3)T(-x_3,0,x_3). \tag{14.19}$$

Changing variables and integrating over the rest of Feynaman variables we get

$$\{\mathbf{F}\} = [i\pi b^\mu \epsilon_{\mu\nu}s^\nu M]2a_s\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon \int d\xi \int_0^1 dy\delta(x-y\xi)2(1-\epsilon)(-1)y\bar{y}T(-\xi,0,\xi). \tag{14.20}$$

There is a forgotten common minus, which comes from the 3-gluon vertex.Here correct version:

XV. EVALUATION OF THE QUARK-GLUON MIXING

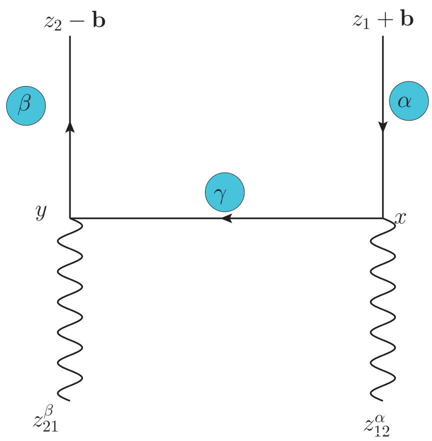
XVI. DIAGRAM M

A. Diagram M

The diagram has the form

$$\mathbf{M} = \left(ig\int d^dx\bar\psi(x)\ A\!\!\!/(x)\psi(x)\right)\left(\bar\psi(z_1n+\boldsymbol{b})\gamma^+\psi(z_2n-\boldsymbol{b})\right)\left(ig\int d^dy\bar\psi(y)\ A\!\!\!/(y)\psi(y)\right) \tag{16.1}$$

M



Step I: Substituting the propagators we obtain

$$\mathbf{M} = (ig)^2\left(\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\right)^3(-1)\int d^dxd^dyA_\mu^A(x)$$
$$\mathrm{Tr}\frac{t^A\gamma^\mu(\not x-\gamma^+z_1-\not b)\gamma^+(\gamma^+z_2-\not b-\not y)\gamma^\nu t^B(\not y-\not x)}{[-(x-z_1n-b)^2+i0]^{2-\epsilon}[-(z_2n-b-y)^2+i0]^{2-\epsilon}[-(x-y)^2+i0]^{2-\epsilon}}A_\nu^B(y), \tag{16.2}$$

where (-1) is due to the fermion permutation.

Step II: Simplifying a bit

$$\mathbf{M} = ig^2\frac{1}{2}\frac{\Gamma^3(2-\epsilon)}{8\pi^{3d/2}}\int d^dxd^dyA_\mu^A(x)A_\nu^A(y)$$
$$\mathrm{Tr}\frac{\gamma^\mu(\not x-\not b)\gamma^+(\not b+\not y)\gamma^\nu(\not y-\not x)}{[-(x-z_1n-b)^2+i0]^{2-\epsilon}[-(z_2n-b-y)^2+i0]^{2-\epsilon}[-(x-y)^2+i0]^{2-\epsilon}}, \tag{16.3}$$

where the factor $1/2$ resulting from the $\mathrm{tr}(t^At^B)=\delta^{AB}/2$.

Step III: The diagram is topologically equivalent to the diagram B. We make the same change of variables, i.e.

$$\begin{aligned}(x-z_1n-b)^2 &\to \alpha\\ (z_2n-b-y)^2 &\to \beta\\ (x-y)^2 &\to 1-\alpha-\beta=\gamma\end{aligned}$$

Next we shift

$$\begin{aligned} x &\to x+\frac{\alpha\beta+\alpha\gamma}{\lambda}z_1n+\frac{\beta\gamma}{\lambda}z_2n+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right),\\ y &\to y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_2n+\frac{\alpha\gamma}{\lambda}z_1n-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right),\\ \lambda &= \alpha\beta+\beta\gamma+\gamma\alpha.\end{aligned} \tag{16.4}$$

We obtain

$$\mathbf{M} = ig^2\frac{1}{2}\frac{\Gamma(6-3\epsilon)}{8\pi^{3d/2}}\int d^dxd^dy \tag{16.5}$$
$$A^A_\mu(x+\frac{\alpha\beta+\alpha\gamma}{\lambda}z_1n+\frac{\beta\gamma}{\lambda}z_2n+\boldsymbol{b}\left(1-2\frac{\beta\gamma}{\lambda}\right))A^A_\nu(y+\frac{\alpha\beta+\beta\gamma}{\lambda}z_2n+\frac{\alpha\gamma}{\lambda}z_1n-\boldsymbol{b}\left(1-2\frac{\alpha\gamma}{\lambda}\right))$$
$$\mathrm{Tr}\frac{\gamma^\mu(\not{x}-\frac{2\beta\gamma}{\lambda}\not{b})\gamma^+(\not{y}+\frac{2\alpha\gamma}{\lambda}\not{b})\gamma^\nu(\not{y}-\not{x}-z_{12}\frac{\alpha\beta}{\lambda}\gamma^+-\frac{2\alpha\beta}{\lambda}\not{b})}{[-(\alpha+\gamma)x^2+2\gamma(xy)-(\beta+\gamma)y^2+\frac{4\alpha\beta\gamma}{\lambda}\mathbf{B}+i0]^{6-3\epsilon}}.$$

Step IV: Now we should understand up to which number of derivatives to expand the fields. The dimension of the measure 8, the dimension of the denominator is 12. So, $8-12=-4$. We would like to get contribution of maximum order +1 (counting in x, y, b), thus we need numerator of dimension +5 at least. Thus, we must expand up to three-derivatives. However, it is clear that the term with three derivative will have too any open indices, which will close to derivative to ∂^2, increasing the twist. However, non-of these three-derivative terms contribute in the final result.

We also make the change to dual variables. And we evaluate it all by Mathematica.

Step V: The expression naturally split into to parts "twist-2" and "twist-3"

$$\mathbf{M_2} = ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]A_\mu(z_{12}^\alpha+(1-2\alpha)\boldsymbol{b})A_\nu(z_{21}^\beta-(1-2\beta)\boldsymbol{b})\Big\{ \tag{16.6}$$
$$g^{\mu\nu}\left(\bar\alpha\partial_1-\bar\beta\partial_2\right)+2\epsilon\frac{b^\mu b^\nu}{\mathbf{B}}\left((1-2\alpha)\partial_1-(1-2\beta)\partial_2\right)-z_{12}g^{\mu\nu}\left(\alpha\partial_1+\bar\beta\partial_2\right)\left(\bar\alpha\partial_1+\beta\partial_2\right)\Big\},$$

where $\partial_{1(2)}$ acts on A_μ (A_ν). The derivatives without explicit indices are ∂_+.

The twist-3 part is

$$\mathbf{M_3} = ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]A_\mu(z_{12}^\alpha+(1-2\alpha)\boldsymbol{b})A_\nu(z_{21}^\beta-(1-2\beta)\boldsymbol{b})\Big\{ \tag{16.7}$$
$$g^{\mu\nu}\Big[-(b\cdot\partial_1)(2\alpha\bar\alpha\partial_1+(1-2\alpha\bar\beta)\partial_2)-(b\cdot\partial_2)((1-2\bar\alpha\beta)\partial_1+2\beta\bar\beta\partial_2)\Big]$$
$$+b^\mu\partial_1^\nu[2\alpha(1-2\alpha)\partial_1+(1-\alpha(1-2\beta))\partial_2]+b^\nu\partial_2^\mu[(1-2\beta(1-2\alpha))\partial_1+2\beta(1-2\beta)\partial_2]$$
$$+b^\mu\partial_2^\nu[(1-2\alpha)(1-2\beta)\partial_1+4\beta\bar\beta\partial_2]+b^\nu\partial_1^\mu[4\alpha\bar\alpha\partial_1+(1-2\alpha)(1-2\beta)\partial_2]\Big\},$$

where $\partial_{1(2)}$ acts on A_μ (A_ν). The derivatives without explicit indices are ∂_+.

B. Elaborating diagram M (twist-2)

The twist-2 and twist-3 parts must be considered separately since they are projected by different projectors. In this section we discuss the twist-2 part.

First we recall that the gluon distribution has only two matrix elements $\sim P_S$ and $\sim P_A$. To project out these components we insert the unity (5.1). We observe that the part projected by P_A is null

$$\mathbf{M_2^A} = 0. \tag{16.8}$$

The traceless component is non-zero, but it does not have a matrix element.

The symmetric-part reads

$$\mathbf{M_2^S} = ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma] A_\mu(z_{12}^\alpha)A_\mu(z_{21}^\beta)\Big\{ \frac{((1-2\epsilon)-(1-3\epsilon)\alpha)\partial_1 - ((1-2\epsilon)-(1-3\epsilon)\beta)\partial_2}{1-\epsilon} - z_{12}(\alpha\partial_1+\bar\beta\partial_2)(\bar\alpha\partial_1+\beta\partial_2)\Big\}. \quad (16.9)$$

Dropping the terms with total derivative, we get much simpler expression

$$\mathbf{M_2^S} = \frac{ia_s}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma]\Big\{1+\gamma-\frac{2\gamma\epsilon}{1-\epsilon}+\gamma^2\partial_\gamma\Big\} A_\mu(\gamma z)\overleftrightarrow{\partial}_+ A_\mu(-\gamma z). \quad (16.10)$$

Here we have also used that $z_1=-z_2=z$.

We substitute

$$A_\mu(z)\overleftrightarrow{\partial}_+ A_\nu(-z) = -F^{\mu+}(z)A_\nu(-z)+A_\mu(z)F^{\nu+}(-z) = \int_{-\infty}^{-z} d\sigma F^{\mu+}(z)F^{\nu+}(\sigma) - \int_{-\infty}^{z} d\sigma F^{\mu+}(\sigma)F^{\nu+}(-z) \quad (16.11)$$

and get

$$\mathbf{M_2^S} = \frac{ia_s}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma]\Big\{1+\gamma-\frac{2\epsilon\gamma}{1-\epsilon}+\gamma^2\partial_\gamma\Big\} \Big[\int_{-\infty}^{-\gamma z} d\sigma F^{\mu+}(\gamma z)F^{\mu+}(\sigma) - \int_{-\infty}^{\gamma z} d\sigma F^{\mu+}(\sigma)F^{\mu+}(-\gamma z)\Big] \quad (16.12)$$

Then we use

$$\langle p|F^{\mu+}(z_1)[z_1,z_2]F^{\mu+}(z_2)|p\rangle = 2p^+\int dx e^{i(z_1-z_2)xp^+}\frac{xp^+}{2}g(x). \quad (16.13)$$

and calculate matrix element

$$\langle\mathbf{M_2^S}\rangle = \frac{ia_s}{2}2p^+\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma]\int dx\frac{xp^+}{2}g(x)\Big\{1+\gamma-\frac{2\epsilon\gamma}{1-\epsilon}+\gamma^2\partial_\gamma\Big\} \Big[\int_{-\infty}^{-\gamma z} d\sigma e^{ix(\gamma z-\sigma)p^+} - \int_{-\infty}^{\gamma z} d\sigma e^{2ix(\sigma+\gamma z)p^+}\Big] \quad (16.14)$$

Integrating

$$\langle\mathbf{M_2^S}\rangle = -\frac{a_s}{2}2p^+\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma]\int dx\frac{xp^+}{2}g(x)\Big\{1+\gamma-\frac{2\epsilon\gamma}{1-\epsilon}+\gamma^2\partial_\gamma\Big\} e^{2ip^+xz\gamma}\Big[\frac{1}{p^+x+i0}+\frac{1}{p^+x-i0}\Big]. \quad (16.15)$$

The factor xp cancels. We integrate over $\{\alpha,\beta\}$

$$\langle\mathbf{M_2^S}\rangle = -a_s2p^+\Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 d\gamma\int dx\frac{g(x)}{2}\bar\gamma\Big\{1+\gamma-\frac{2\epsilon\gamma}{1-\epsilon}+\gamma^2\partial_\gamma\Big\}e^{2ip^+xz\gamma}. \quad (16.16)$$

The term with the derivative over γ is taken by parts

$$\langle\mathbf{M_2^S}\rangle = -a_s2p^+\Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 d\gamma\int dx\frac{g(x)}{2}\Big\{1-2\gamma\bar\gamma-\frac{2\epsilon\gamma}{1-\epsilon}\Big\}e^{2ip^+xz\gamma}. \quad (16.17)$$

Finally we take Fourier (we also rename the variables conveniently)

$$\{\mathbf{M_2^S}\} = -a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 d\gamma\int_0^1 dyd\xi\delta(x-\xi y)g(\xi)\Big\{\frac{1-2y\bar y}{2}-y\bar y\frac{\epsilon}{1-\epsilon}\Big\} \quad (16.18)$$

C. Elaborating diagram M (twist-3 via M2)

The part of $M+2$ contains the twist-3 contribution hidden in the arguments of $\pm\boldsymbol{b}$ of fields. Let me write the expression for M_2 with open indices, but with the removed total-derivative

$$\mathbf{M_2} = ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma]\Big\{\frac{g_T^{\mu\nu}}{2}(1+\gamma+z\gamma^2(\partial_1-\partial_2))+2\gamma\epsilon\frac{b^\mu b^\nu}{\mathbf{B}}\Big\}A_\mu(\gamma z+\gamma\boldsymbol{b})\overleftrightarrow{\partial}_+A_\nu(-\gamma z-\gamma\boldsymbol{b}).\quad(16.19)$$

This can be expanded up to one transverse derivative

$$\mathbf{M_2^\partial} = ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\lambda \int [d\alpha d\beta d\gamma]\gamma\Big\{\frac{g_T^{\mu\nu}}{2}(1+\gamma+z\gamma^2(\partial_1-\partial_2))+2\gamma\epsilon\frac{b^\mu b^\nu}{\mathbf{B}}\Big\}A_\mu(\gamma z)\overleftrightarrow{\partial}_\lambda\overleftrightarrow{\partial}_+A_\nu(-\gamma z).\quad(16.20)$$

Turning to operators

$$\mathbf{M_2^\partial} = ia_s\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\lambda \int [d\alpha d\beta d\gamma]\gamma\Big\{\frac{g_T^{\mu\nu}}{2}(1+\gamma+\gamma^2\partial_\gamma)+2\gamma\epsilon\frac{b^\mu b^\nu}{\mathbf{B}}\Big\}\quad(16.21)$$
$$\Big[\int_{-\infty}^{-\gamma z}d\sigma\mathcal{T}^{\mu(\lambda)\nu}(\gamma z,\sigma)-\int_{-\infty}^{\gamma z}d\sigma\mathcal{T}^{\mu(\lambda)\nu}(\sigma,-\gamma z)\Big].$$

It has been shown in the sec.V F, that only $\mathcal{T}^{\mu\nu\rho}$ part of the operator survives. We get

$$\mathbf{M_2^\partial} = -a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\lambda \int [d\alpha d\beta d\gamma]\gamma\Big\{\frac{g_T^{\mu\nu}}{2}(1+\gamma+\gamma^2\partial_\gamma)+2\gamma\epsilon\frac{b^\mu b^\nu}{\mathbf{B}}\Big\}\quad(16.22)$$
$$\Big[\int_{-\infty}^{-\gamma z}d\sigma\Big(\int_{-\infty}^{\gamma z}+\int_{-\infty}^{\sigma}\Big)\mathcal{T}^{\mu\lambda\nu}(\gamma z,\tau,\sigma)-\int_{-\infty}^{\gamma z}d\sigma\Big(\int_{-\infty}^{\sigma}+\int_{-\infty}^{-\gamma z}\Big)d\tau\mathcal{T}^{\mu\lambda\nu}(\sigma,\tau,-\gamma z)\Big].$$

Let me consider the matrix element of the structure in the square brackets separately. We have

$$\Big\langle\int_{-\infty}^{-\gamma z}d\sigma\Big(\int_{-\infty}^{\gamma z}+\int_{-\infty}^{\sigma}\Big)\mathcal{T}^{\mu\lambda\nu}(\gamma z,\tau,\sigma)-\int_{-\infty}^{\gamma z}d\sigma\Big(\int_{-\infty}^{\sigma}+\int_{-\infty}^{-\gamma z}\Big)d\tau\mathcal{T}^{\mu\lambda\nu}(\sigma,\tau,-\gamma z)\Big\rangle\quad(16.23)$$
$$=\sum t_i^{\mu\rho\nu}\Big[\int_{-\infty}^{-\gamma z}d\sigma\Big(\int_{-\infty}^{\gamma z}+\int_{-\infty}^{\sigma}\Big)F_i(\gamma z,\tau,\sigma)-\int_{-\infty}^{\gamma z}d\sigma\Big(\int_{-\infty}^{\sigma}+\int_{-\infty}^{-\gamma z}\Big)d\tau F_i^{\mu\lambda\nu}(\sigma,\tau,-\gamma z)\Big]$$

Next we substitute $F=\int[dx]\exp(-ipx_iz_i)F(x_1,x_2,x_3)$ and integrate over τ and σ. We con combine the term of this expression together using the relation (5.69) the representation of the δ-function. We get

$$\Big\langle\int_{-\infty}^{-\gamma z}d\sigma\Big(\int_{-\infty}^{\gamma z}+\int_{-\infty}^{\sigma}\Big)\mathcal{T}^{\mu\lambda\nu}(\gamma z,\tau,\sigma)-\int_{-\infty}^{\gamma z}d\sigma\Big(\int_{-\infty}^{\sigma}+\int_{-\infty}^{-\gamma z}\Big)d\tau\mathcal{T}^{\mu\lambda\nu}(\sigma,\tau,-\gamma z)\Big\rangle\quad(16.24)$$
$$=\frac{-2\pi i}{p_+^2}\sum t_i^{\mu\rho\nu}\int[dx]\Big[\frac{e^{-ip_+\gamma z(x_1+x_2-x_3)}(\delta(x_2)-\delta(x_3))}{x_2-x_3}-\frac{e^{-ip_+\gamma z(x_1-x_2-x_3)}(\delta(x_3)-2\delta(x_2+x_3))}{x_2-x_3}\Big]F_i(x_1,x_2,x_3).$$

Taking the Fourier and changing variables we get

$$\Big\{\int_{-\infty}^{-\gamma z}d\sigma\Big(\int_{-\infty}^{\gamma z}+\int_{-\infty}^{\sigma}\Big)\mathcal{T}^{\mu\lambda\nu}(\gamma z,\tau,\sigma)-\int_{-\infty}^{\gamma z}d\sigma\Big(\int_{-\infty}^{\sigma}+\int_{-\infty}^{-\gamma z}\Big)d\tau\mathcal{T}^{\mu\lambda\nu}(\sigma,\tau,-\gamma z)\Big\}\quad(16.25)$$
$$=\frac{\pi i}{p_+^3}\sum t_i^{\mu\rho\nu}\int d\xi\delta(x-\gamma\xi)\Big[2\frac{F_i(-\xi,0,\xi)}{\xi}+\frac{F_i(-\xi,\xi,0)+F_i(0,\xi,-\xi)}{\xi}\Big].$$

Note that in the derivation of this expression we have used only (5.69) that is common for all functions.

Substituting it to the diagram we get

$$\{\mathbf{M_2^\partial}\} = -a_s[i\pi M]\Gamma(-\epsilon)\mathbf{B}^\epsilon b^\lambda \int d\xi\delta(x-\xi\gamma)\int[d\alpha d\beta d\gamma]\gamma\Big\{\frac{g_T^{\mu\nu}}{2}(1+\gamma+\gamma^2\partial_\gamma)+2\gamma\epsilon\frac{b^\mu b^\nu}{\mathbf{B}}\Big\} \quad (16.26)$$
$$\sum t_i^{\mu\lambda\nu}\Big[2\frac{F_i(-\xi,0,\xi)}{\xi}+\frac{F_i(-\xi,\xi,0)+F_i(0,\xi,-\xi)}{\xi}\Big].$$

Applying the parametrization we get

$$\{\mathbf{M_2^\partial}\} = -a_s[i\pi\tilde{s}_\mu b^\mu M]\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_0^1 dy\int d\xi\delta(x-y\xi)\Big\{ \quad (16.27)$$
$$\frac{y(1-3y\bar{y})}{2}\big(G^+(z_1,z_2,z_3)+Y^+(z_1,z_2,z_3)\big)-y^2\bar{y}\frac{3\epsilon}{2-\epsilon}G(z_1,z_2,z_3)\Big\},$$

where (z_1,z_2,z_3) is to be taken as earlier. I.e.

$$G^+(z_1,z_2,z_3)\to\frac{2G^+(-\xi,0,\xi)}{\xi}+\frac{G^+(-\xi,\xi,0)+G^+(0,\xi,-\xi)}{\xi}=\frac{2G^+(-\xi,0,\xi)}{\xi}, \quad (16.28)$$
$$Y^+(z_1,z_2,z_3)\to\frac{2Y^+(-\xi,0,\xi)}{\xi}+\frac{Y^+(-\xi,\xi,0)+Y^+(0,\xi,-\xi)}{\xi}=\frac{2Y^+(-\xi,0,\xi)}{\xi}. \quad (16.29)$$

Thus

$$\{\mathbf{M_2^\partial}\} = -a_s[i\pi\tilde{s}_\mu b^\mu M]\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_0^1 dy\int d\xi\delta(x-y\xi)\Big\{ \quad (16.30)$$
$$y(1-3y\bar{y})\frac{G^+(-\xi,0,\xi)+Y^+(-\xi,0,\xi)}{\xi}-y^2\bar{y}\frac{6\epsilon}{2-\epsilon}\frac{G^+(-\xi,0,\xi)}{\xi}\Big\},$$

D. Elaborating diagram M (twist-3 part)

First we drop the total derivative with respect to $\boldsymbol{b}$ and ∂_+. We get

$$\mathbf{M_3} = \frac{ia_s}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]A_\mu(\gamma z)\Big\{\gamma^2 g^{\mu\nu}(b\cdot\overleftrightarrow{\partial})\overleftrightarrow{\partial}_+ +\gamma(1-2\gamma)\overleftrightarrow{\partial}_+(b^\mu\overleftrightarrow{\partial}_\nu+b^\nu\overleftrightarrow{\partial}_\mu)\Big\}A_\nu(-\gamma z). \quad (16.31)$$

The combination $A_\mu(z)\overleftrightarrow{\partial}_\mu\overleftrightarrow{\partial}_+A_\nu(-z)$ is considered in the previous section. The matrix element of it is given in (16.25). Thus we get

$$\{\mathbf{M_3}\} = \frac{-a_s}{2}[i\pi M]\Gamma(-\epsilon)\mathbf{B}^\epsilon\int d\xi\int\int[d\alpha d\beta d\gamma]\delta(x-\gamma\xi)\Big\{ \quad (16.32)$$
$$\gamma^2 g^{\mu\nu}b_\lambda+\gamma(1-2\gamma)(b^\mu g_{\nu\lambda}+b^\nu g_{\mu\lambda})\Big\}\sum t^{\mu\rho\nu}\left[2\frac{F_i(-\xi,0,\xi)}{\xi}+\frac{F_i(-\xi,\xi,0)+F_i(0,\xi,-\xi)}{\xi}\right].$$

We obtain

$$\{\mathbf{M_3}\} \;=\; -a_s[i\pi\tilde{s}_\mu b^\mu M]\Gamma(-\epsilon)\mathbf{B}^\epsilon \int d\xi \int_0^1 dy\delta(x-\gamma\xi)\Bigg\{ \quad (16.33)$$
$$y\bar{y}(2-3y)\frac{G^+(-\xi,0,\xi)}{\xi} - y\bar{y}(1-3y)\frac{Y^+(-\xi,0,\xi)}{\xi}\Bigg\}.$$

Note, that in the notation T_{3F} it reads

$$\{\mathbf{M_3}\} \;=\; -a_s[i\pi\tilde{s}_\mu b^\mu M]\Gamma(-\epsilon)\mathbf{B}^\epsilon \int d\xi \int_0^1 dy\delta(x-\gamma\xi)\Bigg\{ \quad (16.34)$$
$$y^2\bar{y}\frac{T^+_{3F}(-\xi,0\xi)}{\xi} - 2y\bar{y}(1-2y)\frac{T^+_{3F}(0,-\xi,\xi)}{\xi}\Bigg\}.$$

So, the twist-3 part of the diagram M is

$$\{\mathbf{M_3}\} \;=\; -a_s[i\pi\tilde{s}_\mu b^\mu M]\Gamma(-\epsilon)\mathbf{B}^\epsilon \int d\xi \int_0^1 dy\delta(x-\gamma\xi)\Bigg\{ \quad (16.35)$$
$$y(3-8y+6y^2)\frac{G^+(-\xi,0,\xi)}{\xi} + y^2\frac{Y^+(-\xi,0,\xi)}{\xi} - y^2\bar{y}\frac{6\epsilon}{2-\epsilon}\frac{G^+(-\xi,0\xi)}{\xi}\Bigg\}.$$

E. some old notes

To proceed further we recall that

$$A_\mu(z)\overleftrightarrow{\partial}_+A_\nu(-z) \;=\; -F^{\mu+}(z)A_\nu(-z) + A_\mu(z)F^{\nu+}(-z) = \int_{-\infty}^{-z} d\sigma F^{\mu+}(z)F^{\nu+}(\sigma) - \int_{-\infty}^{z} d\sigma F^{\mu+}(\sigma)F^{\nu+}(-z) \quad (16.36)$$

And that

$$F^{\mu+}(z_1)\overleftrightarrow{\partial}_\rho F^{\nu+}(-z) = \mathcal{T}^{\mu(\rho)\nu}(z_1,z_2) \quad (16.37)$$

It gives

$$\mathbf{M_3} \;=\; -2ia_s\frac{\Gamma(1-\epsilon)}{4\epsilon}\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]\Bigg\{ \quad (16.38)$$
$$\gamma^2 b_\rho g_{\mu\nu} + \gamma(1-2\gamma)\left(b_\mu g_{\rho\nu}+b_\nu g_{\rho\mu}\right)\Bigg\}\Bigg[\int_{-\infty}^{-\gamma z} d\sigma\mathcal{T}^{\mu(\rho)\nu}(\gamma z,\sigma) - \int_{-\infty}^{\gamma z} d\sigma\mathcal{T}^{\mu(\rho)\nu}(\sigma,-\gamma z)\Bigg].$$

Using (5.73) we obtain (here we drop the terms which produces zero, see sec.V F)

$$\mathbf{M_3} \;=\; 2a_s\frac{\Gamma(1-\epsilon)}{4\epsilon}\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]\Bigg\{\gamma^2 b_\rho g_{\mu\nu} + \gamma(1-2\gamma)\left(b_\mu g_{\rho\nu}+b_\nu g_{\rho\mu}\right)\Bigg\}\Bigg[\quad (16.39)$$
$$\int_{-\infty}^{-\gamma z} d\sigma\left(\int_{-\infty}^{\gamma z}+\int_{-\infty}^{\sigma}\right)d\tau\mathcal{T}^{\mu\rho\nu}(\gamma z,\tau,\sigma) - \int_{-\infty}^{\gamma z} d\sigma\left(\int_{-\infty}^{\sigma}+\int_{-\infty}^{-\gamma z}\right)d\tau\mathcal{T}^{\mu\rho\nu}(\sigma,\tau,-\gamma z)\Bigg].$$

Now we apply matrix elements (5.129) and (5.130) we obtain

$$\langle \mathbf{M_3} \rangle = [\tilde{s}_\mu b^\mu M] p_+^3 2 a_s \frac{\Gamma(1-\epsilon)}{4\epsilon} \mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma] \int [dx] \Big\{ -\gamma^2 \Big[\tag{16.40}$$
$$\int_{-\infty}^{-\gamma z} d\sigma \left(\int_{-\infty}^{\gamma z} + \int_{-\infty}^{\sigma} \right) d\tau F(\gamma z, \tau, \sigma) - \int_{-\infty}^{\gamma z} d\sigma \left(\int_{-\infty}^{\sigma} + \int_{-\infty}^{-\gamma z} \right) d\tau F(\sigma, \tau, -\gamma z) \Big]$$
$$+\gamma(1-2\gamma) \Big[\int_{-\infty}^{-\gamma z} d\sigma \left(\int_{-\infty}^{\gamma z} + \int_{-\infty}^{\sigma} \right) d\tau \left(F(\gamma z, \sigma, \tau) + F(\tau, \gamma z, \sigma) \right)$$
$$- \int_{-\infty}^{\gamma z} d\sigma \left(\int_{-\infty}^{\sigma} + \int_{-\infty}^{-\gamma z} \right) d\tau \left(F(\sigma, -\gamma z, \tau) + F(\tau, \sigma, -\gamma z) \right) \Big],$$

where $F(z_1, z_2, z_3)$ is Fourier of $T_{3F}^+ = F$. We apply the representation and take the integral over τ and σ. I obtain

$$\langle \mathbf{M_3} \rangle = [\tilde{s}_\mu b^\mu M] p_+ 2 a_s \frac{\Gamma(1-\epsilon)}{4\epsilon} \mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma] \int [dx] F(x_1,x_2,x_3) \Big\{ -\gamma^2 \Big[\tag{16.41}$$
$$-\frac{e^{ip_+(x_1+x_2-x_3)z\gamma}}{(x_2+i0)(x_3+i0)} - \frac{e^{ip_+(x_1-x_2-x_3)z\gamma}}{(x_2+x_3+i0)(x_3+i0)} + \frac{e^{ip_+(x_1+x_2-x_3)z\gamma}}{(x_2+i0)(x_1+x_2+i0)} + \frac{e^{ip_+(x_1-x_2-x_3)z\gamma}}{(x_2+i0)(x_1+i0)} \Big]$$
$$+\gamma(1-2\gamma) \Big[-\frac{e^{-ip_+(x_1-x_2+x_3)\gamma z}}{(x_2+i0)(x_3+i0)} - \frac{e^{-ip_+(x_1-x_2-x_3)\gamma z}}{(x_2+x_3+i0)(x_3+i0)} - \frac{e^{-ip_+(x_1+x_2-x_3)\gamma z}}{(x_1+i0)(x_3+i0)} - \frac{e^{-ip_+(-x_1+x_2-x_3)\gamma z}}{(x_1+i0)(x_1+x_3+i0)}$$
$$+\frac{e^{-ip_+(x_1-x_2+x_3)\gamma z}}{(x_3+i0)(x_1+x_3+i0)} + \frac{e^{-ip_+(x_1-x_2-x_3)\gamma z}}{(x_1+i0)(x_3+i0)} + \frac{e^{-ip_+(x_1+x_2-x_3)\gamma z}}{(x_1+i0)(x_1+x_2+i0)} + \frac{e^{-ip_+(-x_1+x_2-x_3)\gamma z}}{(x_1+i0)(x_2+i0)} \Big] \Big\}.$$

Next we replace in the terms that integrate over the second Wilson line (e.g. blue terms) $x_{123} \to -x_{321}$ reexpand and apply the representation for δ-function. Here the example of the first terms

$$-\frac{e^{ip_+(x_1+x_2-x_3)z\gamma}}{(x_2+i0)(x_3+i0)} - \frac{e^{ip_+(x_1-x_2-x_3)z\gamma}}{(x_2+x_3+i0)(x_3+i0)} + \frac{e^{ip_+(x_1+x_2-x_3)z\gamma}}{(x_2+i0)(x_1+x_2+i0)} + \frac{e^{ip_+(x_1-x_2-x_3)z\gamma}}{(x_2+i0)(x_1+i0)} \tag{16.42}$$
$$= \frac{e^{-ip_+\gamma z(x_1+x_2-x_3)}}{x_2-x_3} \left(\frac{1}{x_2+i0} - \frac{1}{x_2-i0} - \frac{1}{x_3+i0} + \frac{1}{x_3-i0} \right)$$
$$-\frac{e^{-ip_+\gamma z(x_1-x_2-x_3)}}{x_3} \left(\frac{1}{x_2+i0} - \frac{1}{x_2-i0} - \frac{1}{x_2+x_3+i0} + \frac{1}{x_2+x_3-i0} \right)$$
$$= -2\pi i \left[\frac{e^{-ip_+\gamma z(x_1+x_2-x_3)}}{x_2-x_3} (\delta(x_2) - \delta(x_3)) - \frac{e^{-ip_+\gamma z(x_1-x_2-x_3)}}{x_3} (\delta(x_2) - \delta(x_2+x_3)) \right]$$

We do it

$$\langle \mathbf{M_3} \rangle = [i\pi \tilde{s}_\mu b^\mu M] p_+ (-4) a_s \frac{\Gamma(1-\epsilon)}{4\epsilon} \mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma] \int [dx] F(x_1,x_2,x_3) \Big\{ -\gamma^2 \Big[\tag{16.43}$$
$$\frac{e^{-ip_+\gamma z(x_1+x_2-x_3)}}{x_2-x_3} (\delta(x_2) - \delta(x_3)) - \frac{e^{-ip_+\gamma z(x_1-x_2-x_3)}}{x_3} (\delta(x_2) - \delta(x_2+x_3)) \Big]$$
$$+\gamma(1-2\gamma) \Big[\frac{e^{-ip_+\gamma z(x_1-x_2+x_3)}}{x_2-x_3} (\delta(x_2) - \delta(x_3)) + \frac{e^{-ip_+\gamma z(x_1-x_2-x_3)}}{x_2} (\delta(x_2+x_3) - \delta(x_3))$$
$$+\frac{e^{-ip_+\gamma z(x_1+x_2-x_3)}}{x_1-x_3} (\delta(x_1) - \delta(x_3)) + \frac{e^{-ip_+\gamma z(-x_1+x_2-x_3)}}{x_3} (\delta(x_1+x_3) - \delta(x_1)) \Big] \Big\}.$$

Finally we take Fourier, and obtain

$$\begin{aligned}\{\mathbf{M_3}\} &= [i\pi\tilde{s}_\mu b^\mu M](-2)a_s\frac{\Gamma(1-\epsilon)}{4\epsilon}\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]\int[dx]\Bigg\{-\gamma^2\Bigg[\\ &-2\delta(x-\gamma\xi)\frac{F(-\xi,0,\xi)}{\xi}-\delta(x)\frac{F(-\xi,\xi,0)}{\xi}-\delta(x)\frac{F(0,\xi,-\xi)}{\xi}\Bigg] \\ &+\gamma(1-2\gamma)\Bigg[-\delta(x)\frac{F(-\xi,0,\xi)}{\xi}-2\delta(x-\gamma\xi)\frac{F(-\xi,\xi,0)}{\xi}+\delta(x)\frac{F(0,\xi,-\xi)}{\xi} \\ &-2\delta(x-\gamma\xi)\frac{F(0,-\xi,\xi)}{\xi}+\delta(x)\frac{F(-\xi,\xi,0)}{\xi}+\delta(x)\frac{F(-\xi,0,\xi)}{\xi}\Bigg]\Bigg\}.\end{aligned} \tag{16.44}$$

To proceed we observe that terms $\sim\delta(x)$ could be integrated over γ. Here,

$$\int_0^1[d\alpha d\beta d\gamma]\gamma(1-2\gamma)=0.$$

Also

$$\begin{aligned}\int_{-1}^1 d\xi\left(\frac{F(-\xi,\xi,0)}{\xi}+\frac{F(0,\xi,-\xi)}{\xi}\right)&=\int_{-1}^1 d\xi\left(\frac{F(-\xi,\xi,0)}{\xi}+\frac{F(\xi,-\xi,0)}{\xi}\right)= \\ &\int_{-1}^1 d\xi\left(\frac{F(-\xi,\xi,0)}{\xi}-\frac{F(-\xi,\xi,0)}{\xi}\right)=0.\end{aligned} \tag{16.45}$$

Therefore, only integrals with normal convolution remain. We get

$$\begin{aligned}\{\mathbf{M_3}\} &= [i\pi\tilde{s}_\mu b^\mu M](-2)a_s\frac{\Gamma(1-\epsilon)}{4\epsilon}\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]\int d\xi\delta(x-\gamma\xi)\Bigg\{2\gamma^2\frac{F(-\xi,0,\xi)}{\xi} \\ &-2\gamma(1-2\gamma)\Bigg[\frac{F(0,-\xi,\xi)}{\xi}+\frac{F(-\xi,\xi,0)}{\xi}\Bigg]\Bigg\}.\end{aligned} \tag{16.46}$$

The last two terms can be also simplified. Also we integrate over Feynman variables and get

$$\{\mathbf{M_3}\} = [i\pi\tilde{s}_\mu b^\mu M]a_s\frac{\Gamma(1-\epsilon)}{\epsilon}\mathbf{B}^\epsilon\int_0^1 dy\int d\xi\delta(x-y\xi)\Bigg\{-\bar{y}y^2\frac{F(-\xi,0,\xi)}{\xi}+2y\bar{y}(1-2y)\frac{F(0,-\xi,\xi)}{\xi}\Bigg\} \tag{16.47}$$

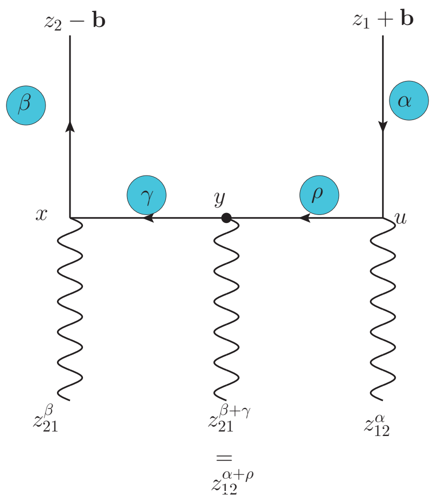
XVII. DIAGRAM N

A. Diagram N

The diagram has the form

$$\mathbf{N} = \Big(ig\int d^dy\bar{\psi}(y)\ \not{A}(y)\psi(y)\Big)\Big(ig\int d^du\bar{\psi}(u)\ \not{A}(u)\psi(u)\Big)\Big(\bar{\psi}(z_1n+\boldsymbol{b})\gamma^+\psi(z_2n-\boldsymbol{b})\Big)\Big(ig\int d^dx\bar{\psi}(x)\ \not{A}(x)\psi(x)\Big)$$

N



Step I: Substitute propagators

$$\mathbf{N} = (-1)(ig)^3\left(\frac{i\Gamma(2-\epsilon)}{2\pi^{d/2}}\right)^4\int d^dxd^dyd^duA_\mu^A(x)A_\rho^B(y)A_\nu^C(u)\mathrm{Tr}(t^At^Bt^C) \tag{17.1}$$
$$\mathrm{Tr}\left[\gamma^\mu\frac{\not{x}-\not{y}}{[-(x-y)^2+i0]^{2-\epsilon}}\gamma^\rho\frac{\not{y}-\not{u}}{[-(y-u)^2+i0]^{2-\epsilon}}\gamma^\nu\frac{\not{u}-z_1\gamma^+-\not{b}}{[-(u-z_1n-b)^2+i0]^{2-\epsilon}}\gamma^+\frac{z_2\gamma^+-\not{b}-\not{x}}{[-(z_2n-b-x)^2+i0]^{2-\epsilon}}\right],$$

where the common minus comes from the fermion statistics.

Step II: Minimal simplifications

$$\mathbf{N} = -ig^3\frac{\Gamma^4(2-\epsilon)}{16\pi^{2d}}\int d^dxd^dyd^duA_\mu^A(x)A_\rho^B(y)A_\nu^C(u)\frac{d^{ABC}+if^{ABC}}{4} \tag{17.2}$$
$$\mathrm{Tr}\left[\gamma^\mu\frac{\not{x}-\not{y}}{[-(x-y)^2+i0]^{2-\epsilon}}\gamma^\rho\frac{\not{y}-\not{u}}{[-(y-u)^2+i0]^{2-\epsilon}}\gamma^\nu\frac{\not{u}-\not{b}}{[-(u-z_1n-b)^2+i0]^{2-\epsilon}}\gamma^+\frac{\not{b}+\not{x}}{[-(z_2n-b-x)^2+i0]^{2-\epsilon}}\right]$$

Step III: It has the same topology as diagram E. In order to evaluate it we shift

$$\begin{aligned}(u-z_1-b)^2 &\to \alpha\\ (x-z_2+b)^2 &\to \beta\\ (x-y)^2 &\to \gamma\\ (u-y)^2 &\to \rho\end{aligned}$$

We also make a shift

$$\begin{aligned}x &\to x+\frac{\alpha\gamma\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\alpha\beta\rho+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho}{\lambda}\right)\\ y &\to y+\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}z_1+\frac{\alpha\beta\gamma+\beta\gamma\rho}{\lambda}z_2-\boldsymbol{b}\left(1-2\frac{\alpha\gamma\rho+\alpha\beta\rho}{\lambda}\right)\\ u &\to u+\frac{\alpha\gamma\rho+\alpha\beta\gamma+\alpha\beta\rho}{\lambda}z_1+\frac{\beta\gamma\rho}{\lambda}z_2+\boldsymbol{b}\left(1-2\frac{\beta\gamma\rho}{\lambda}\right)\end{aligned}$$

$$\lambda = \alpha\gamma\rho + \alpha\beta\gamma + \alpha\beta\rho + \beta\gamma\rho.$$

We get

$$\begin{aligned}\mathbf{N} \;=\; & -ig^3 \frac{\Gamma(8-4\epsilon)}{16\pi^{2d}} \int d^dx d^dy d^du \int [d\alpha d\beta d\gamma d\rho](\alpha\beta\gamma\rho)^{1-\epsilon} A^A_\mu(x) A^B_\rho(y) A^C_\nu(u) \frac{d^{ABC}+if^{ABC}}{4} \\ & \mathrm{Tr}\frac{\gamma^\mu(\not{x}-\not{y}+z_{21}\frac{\alpha\beta\rho}{\lambda}\gamma^+ - 2\frac{\alpha\beta\gamma}{\lambda}\not{b})\gamma^\rho(\not{y}-\not{u}+z_{21}\frac{\alpha\beta\gamma}{\lambda}\gamma^+ - 2\frac{\alpha\beta\gamma}{\lambda}\not{b})\gamma^\nu(\not{u}-2\frac{\beta\gamma\rho}{\lambda}\not{b})\gamma^+(\not{x}+2\frac{\alpha\gamma\rho}{\lambda}\not{b})}{[-(\alpha+\rho)u^2+2\rho(uy)-(\beta+\gamma)x^2+2\gamma(xy)-(\gamma+\rho)y^2+\frac{4\alpha\beta\gamma\rho}{\lambda}\mathbf{B}+i0]^{8-4\epsilon}}\Bigg]\end{aligned} \tag{17.3}$$

Step V: In order to understand – till which order we must expand, we count the total dimension. The denominator + measure=-4. We must get the total dimension +1. The least dimension term in the numerator is the one which contains z_{21} (there could be only one entry of z_{21} due to $A_+=0$), it has the dimension +3 (the powers b also count as a dimension). Thus such term has total dimension -1, and we have to expand up to two derivatives.

The calculation is made by Mathematica.

Step IV: In order to make the calculation simple we also change to dual variables

The result reads

$$\begin{aligned}\mathbf{N} \;=\; & -ga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma d\rho] A^A_\mu(z^\beta_{21}) A^B_\sigma(z^{\beta+\gamma}_{21}) A^C_\nu(z^\alpha_{12})(d^{ABC}+if^{ABC})\Bigg\{ \\ & g^{\mu\nu}b^\sigma\left((1+4\beta)\partial_1 - 2(1-2(\beta+\gamma))\partial_2 - (1+4\alpha)\partial_3\right) \\ & +g^{\mu\sigma}b^\nu\left((1-4\beta)\partial_1 - 4(\beta+\gamma)\partial_2 - (1-4\alpha)\partial_3\right) \\ & +g^{\sigma\nu}b^\mu\left((1-4\beta)\partial_1 + 4(1-\beta-\gamma)\partial_2 - (1-4\alpha)\partial_3\right) \\ & +z_{12}g^{\mu\nu}b^\sigma\left(\beta\partial_1^2+\alpha\partial_3^2+(1+\gamma+\rho)\partial_1\partial_3+(1-\beta-\gamma)\partial_2\partial_3+(\beta+\gamma)\partial_1\partial_2\right) \\ & +z_{12}(g^{\mu\sigma}b^\nu+g^{\sigma\nu}b^\mu)\Big[\beta(1-2\beta)\partial_1^2+\alpha(1-2\alpha)\partial_3^2+2(\alpha+\rho)(\beta+\gamma)\partial_2^2 \\ & +(2\alpha(\beta+\gamma)+(1-2\alpha)(\alpha+\rho))\partial_2\partial_3+(2\beta(\alpha+\rho)+(1-2\beta)(\beta+\gamma))\partial_1\partial_3-(\alpha+\beta-4\alpha\beta)\partial_1\partial_3\Big] \\ & +4\epsilon\frac{b^\mu b^\sigma b^\nu}{\mathbf{B}}\left((1-2\beta)\partial_1+(1-2(\beta+\gamma))\partial_2-(1-2\alpha)\partial_3\right)\Bigg\}\end{aligned} \tag{17.4}$$

where $\partial_1(\partial_2,\partial_3)$ is ∂_+ that acts on $A^A_\mu(z^\beta_{21})(A^B_\sigma(z^{\beta+\gamma}_{21}), A^B_\nu(z^\alpha_{12}))$.

B. Elaborating diagram N

In order to simplify this horrible expression we make a total shift by $-(z^\beta_{21}+z^\alpha_{12})/2$. We get

$$A^A_\mu(z^\beta_{21})A^B_\sigma(z^{\beta+\gamma}_{21})A^C_\nu(z^\alpha_{12}) \to A^A_\mu\left(-\frac{z_{12}(\gamma+\rho)}{2}\right)A^B_\sigma\left(\frac{z_{12}(\gamma-\rho)}{2}\right)A^C_\nu\left(\frac{z_{12}(\gamma+\rho)}{2}\right) \tag{17.5}$$

We also drop the total derivative. Since we have three fields we can drop a single derivative with respect to the another two. For symmetrical reasons we drop ∂_2 Practically, it means the replacement $\partial_2 \to -\partial_1-\partial_3$. The expression tremendously simplifies

$$\begin{aligned}\mathbf{N} \;=\; & -ga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma d\rho] A^A_\mu(z^\beta_{21}) A^B_\sigma(z^{\beta+\gamma}_{21}) A^C_\nu(z^\alpha_{12})(d^{ABC}+if^{ABC}) \\ & \Big\{g^{\mu\nu}b^\sigma\left((3-4\gamma)\partial_1-(3-4\rho)\partial_3\right)+g^{\mu\sigma}b^\nu\left((1+4\gamma)\partial_1+(3-4\rho)\partial_3\right)+g^{\sigma\nu}b^\mu\left(-(3-4\gamma)\partial_1-(1+4\rho)\partial_3\right) \\ & z_{12}(g^{\mu\sigma}b^\nu+g^{\sigma\nu}b^\mu)\Big[\gamma(1-2\gamma)\partial_1^2-(\gamma+\rho-4\gamma\rho)\partial_1\partial_3+\rho(1-2\rho)\partial_3^2\Big] \\ & +z_{12}g^{\mu\nu}b^\sigma\left(-\gamma\partial_1^2+(\gamma+\rho)\partial_1\partial_3-\rho\partial_3^2\right)+8\epsilon\frac{b^\mu b^\sigma b^\nu}{\mathbf{B}}\left(\gamma\partial_1-\rho\partial_3\right)\Big\}\end{aligned} \tag{17.6}$$

Simplifying

$$\mathbf{N} = -ga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int [d\alpha d\beta d\gamma d\rho] A^A_\mu(z^\beta_{21}) A^B_\sigma(z^{\beta+\gamma}_{21}) A^C_\nu(z^\alpha_{12})(d^{ABC} + if^{ABC}) \tag{17.7}$$
$$\Big\{ g^{\mu\nu}b^\sigma\left((3-4\gamma)\partial_1 - (3-4\rho)\partial_3\right) + g^{\mu\sigma}b^\nu\left((1+4\gamma)\partial_1 + (3-4\rho)\partial_3\right) + g^{\sigma\nu}b^\mu\left(-(3-4\gamma)\partial_1 - (1+4\rho)\partial_3\right)$$
$$z_{12}(g^{\mu\sigma}b^\nu + g^{\sigma\nu}b^\mu)((1-2\gamma)\partial_1 - (1-2\rho)\partial_3)\left(\gamma\partial_1 - \rho\partial_3\right)$$
$$-z_{12}g^{\mu\nu}b^\sigma(\partial_1 - \partial_3)\left(\gamma\partial_1 - \rho\partial_3\right) + 8\epsilon\frac{b^\mu b^\sigma b^\nu}{\mathbf{B}}\left(\gamma\partial_1 - \rho\partial_3\right)\Big\}.$$

We have found that it is convenient to make the following change of variables

$$\gamma + \beta = \phi, \qquad \alpha = \bar{\alpha}. \tag{17.8}$$

The domain of these variables is

$$\phi \in (0,1), \qquad 0 < \beta < \phi < \alpha < 1, \qquad J = 1. \tag{17.9}$$

We have

$$\mathbf{N} = -ga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 d\phi \int_0^\phi d\beta \int_\phi^1 d\alpha A^A_\mu(z^\beta_{21}) A^B_\sigma(z^\phi_{21}) A^C_\nu(z^\alpha_{21})(d^{ABC} + if^{ABC}) \tag{17.10}$$
$$\Big\{ g^{\mu\nu}b^\sigma\left((3-4\beta+4\phi)\partial_1 - (3-4\alpha-4\phi)\partial_3\right)$$
$$+g^{\mu\sigma}b^\nu\left((1-4\beta+4\phi)\partial_1 + (3-4\alpha+4\phi)\partial_3\right) + g^{\sigma\nu}b^\mu\left(-(3+4\beta-4\phi)\partial_1 - (1+4\alpha-4\phi)\partial_3\right)$$
$$z_{12}(g^{\mu\sigma}b^\nu + g^{\sigma\nu}b^\mu)((1-2\beta+2\phi)\partial_1 - (1-2\alpha+2\phi)\partial_3)\left((\beta-\phi)\partial_1 + (\alpha-\phi)\partial_3\right)$$
$$-z_{12}g^{\mu\nu}b^\sigma(\partial_1 - \partial_3)\left((\beta-\phi)\partial_1 + (\alpha-\phi)\partial_3\right) + 8\epsilon\frac{b^\mu b^\sigma b^\nu}{\mathbf{B}}\left((\beta-\phi)\partial_1 + (\alpha-\phi)\partial_3\right)\Big\}.$$

This expression can be significantly simplified by rewriting ϕd_i via $\phi\partial_2$. We get

$$\mathbf{N} = -ga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 d\phi \int_0^\phi d\beta \int_\phi^1 d\alpha A^A_\mu(z^\beta_{21}) A^B_\sigma(z^\phi_{21}) A^C_\nu(z^\alpha_{21})(d^{ABC} + if^{ABC}) \tag{17.11}$$
$$\Big\{ g^{\mu\nu}b^\sigma(3\partial_1 - 3\partial_3) + g^{\mu\sigma}b^\nu(\partial_1 + 3\partial_3) + g^{\sigma\nu}b^\mu(-3\partial_1 - \partial_3)$$
$$+4\left(g^{\mu\nu}b^\sigma - g^{\mu\sigma}b^\nu - g^{\sigma\nu}b^\mu\right)\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right)$$
$$z_{12}(g^{\mu\sigma}b^\nu + g^{\sigma\nu}b^\mu)((1-2\alpha)\partial_3 - 2\phi\partial_2 - (1+2\beta)\partial_1)\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right)$$
$$+z_{12}g^{\mu\nu}b^\sigma(\partial_1 - \partial_3)\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right) + 8\epsilon\frac{b^\mu b^\sigma b^\nu}{\mathbf{B}}\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right)\Big\}.$$

Finally, we change $z_{12}\partial_i$ as a derivative over corresponding parameters

$$\mathbf{N} = -ga_s\Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 d\phi \int_0^\phi d\beta \int_\phi^1 d\alpha A^A_\mu(z^\beta_{21}) A^B_\sigma(z^\phi_{21}) A^C_\nu(z^\alpha_{21})(d^{ABC} + if^{ABC}) \tag{17.12}$$
$$\Big\{ g^{\mu\nu}b^\sigma(3\partial_1 - 3\partial_3) + g^{\mu\sigma}b^\nu(\partial_1 + 3\partial_3) + g^{\sigma\nu}b^\mu(-3\partial_1 - \partial_3)$$
$$+4\left(g^{\mu\nu}b^\sigma - g^{\mu\sigma}b^\nu - g^{\sigma\nu}b^\mu\right)\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right)$$
$$(g^{\mu\sigma}b^\nu + g^{\sigma\nu}b^\mu)((1-2\alpha)\partial_\alpha - 2\phi\partial_\phi - (1+2\beta)\partial_\beta)\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right)$$
$$+g^{\mu\nu}b^\sigma(\partial_\beta - \partial_\alpha)\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right) + 8\epsilon\frac{b^\mu b^\sigma b^\nu}{\mathbf{B}}\left(\beta\partial_1 + \phi\partial_2 + \alpha\partial_3\right)\Big\}.$$

To proceed further I observe that the matrix element

$$\langle \partial_i A^A_\mu(z^\beta_{21}) A^B_\sigma(z^\phi_{21}) A^C_\nu(z^\alpha_{21})(+if^{ABC})\rangle \sim -Mp_+ \int [dx] e^{-2izp_+(\alpha x_3+\beta x_1+\phi x_2)} \frac{F}{(x_j+i0)(x_k+i0)}, \tag{17.13}$$

where $j \neq k \neq i$. The arguments of F are set in accordance to convolution. The terms $1/(x_i+i0)$ are to be combined into the δ-functions. Here we should use the symmetry $F(x_{1,2,3}) = F(-x_{3,2,1})$, **which is hold for all functions.** Thus we split the diagrams into two identical parts, and within one part we change $x_{1,2,3} \to -x_{3,2,1}$. In order to have the identical transformation of the exponent I also transform α, β, ϕ

$$\{\phi, \alpha, \beta\} \to \{\bar\phi, \bar\beta, \bar\alpha\}. \tag{17.14}$$

It turns $e^{-2izp_+(\alpha x_3+\beta x_1+\phi x_2)}$ to itself (accounting $x_1+x_2+x_3=0$). We found that it is convinient to first integrate by part over $\partial_{\alpha,\beta}$, next combine δ-function. Here, term $\sim g^{\mu\nu}$ is proportional δ-function, while the terms $\sim g^{\mu\sigma}$ and $g^{\nu\sigma}$ combines to δ-function. As for terms that are boundary in the integration by pats, $\alpha \to 1(\phi)$ combines to δ-function with $\beta \to 0(\phi)$. We got expression that is the sum of terms

$$2[i\pi \tilde s_\mu b^\mu M]p_+ \int_0^1 d\phi \int_0^\phi d\beta \int_\phi^1 d\alpha \int [dx]\delta(x_i) e^{-2izp_+(\alpha x_3+\beta x_1+\phi x_2)} F(x) f(\alpha,\beta,\gamma;x), \tag{17.15}$$

where x is some composition of x_{123}. Next we take Fourier, and get the terms transformed into

$$[i\pi \tilde s_\mu b^\mu M] \int_0^1 d\phi \int_0^\phi d\beta \int_\phi^1 d\alpha \int [dx]\delta(x_i)\delta(x+(\alpha x_3+\beta x_1+\phi x_2)) F(x) f(\alpha,\beta,\gamma;x). \tag{17.16}$$

We obtain

$$\begin{aligned}\mathbf{N} &= -2[i\pi b_\mu \tilde s^\mu] a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 dy \int d\xi \delta(x-y\xi)\Big\{ \qquad (17.17)\\ &(1-2y)(1-6y\bar y)\frac{G^+(-\xi,0\xi)}{\xi} + (1-2y)\frac{Y^+(-\xi,0\xi)}{\xi} - 6y\bar y(1-2y)\frac{\epsilon}{2-\epsilon}\frac{G^+(-\xi,0\xi)}{\xi}\\ &(1-2y\bar y)\frac{G^-(-\xi,0\xi)+Y^-(-\xi,0,\xi)}{\xi} - 6y\bar y\frac{\epsilon}{2-\epsilon}\frac{G^-(-\xi,0\xi)}{\xi}\Big\}\end{aligned}$$

Finally we obtain The expression which has rather many terms $F(\pm\xi,0,\mp\xi)$, $F(0,\pm\xi,\mp\xi)$ and $F(\pm\xi,\mp\xi,0)$. Using the symmetries of F we simplify it to

$$\begin{aligned}\mathbf{N} &= [i\pi \tilde s^\mu b_\mu M] a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \int d\xi \int_0^1 dy \delta(x-y\xi)\Big\{ \qquad (17.18)\\ &(1-2y)(1-2y\bar y)\frac{F(-\xi,0,\xi)}{\xi} - 4(1-2y)y\bar y\frac{F(0,\xi,-\xi)}{\xi} + \epsilon y\bar y(1-2y)\frac{2F(0,\xi,-\xi)+F(-\xi,0\xi)}{\xi}\Big\}\end{aligned}$$

$b \to b/2$, it gives an extra factor 1/2. It is also not finished. .

XVIII. FROM OPERATORS TO DISTRIBUTIONS

A. Twist-2 part

The twist-2 part appears only in the diagrams $\mathbf{A}$, $\mathbf{A}^*$ and $\mathbf{B}$. The passage from operator expression to the TMD is made by (2.4). Moreover in this expression we can drop the total shift of the variables z. Taking it into account we can generalize the formula (2.4) for the operators with two points as

$$\Phi^{[\Gamma]}_{q\leftarrow h}(x,\boldsymbol{b}) = \int \frac{dz}{2\pi} e^{-2ixzp^+} \langle P,S|\mathcal{U}^\Gamma\left(z,-z;\frac{\boldsymbol{b}}{2}\right)|P,S\rangle. \tag{18.1}$$

There is only a single twist-2 operator $\mathcal{O}_{\gamma^+}(z_1,z_2)$. Its matrix element is parameterized as (3.1)

$$\langle P,S|O^{\gamma^+}(z_1,z_2)|P,S\rangle \;=\; 2p^+\int dx e^{ix(z_1-z_2)p^+} f_1(x). \tag{18.2}$$

Next we each diagrams in the terms of PDFs.

The expression for diagrams in the terms of PDFs are

$$\begin{aligned}
\mathbf{A}_{\text{tw-2}} &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\mathcal{O}_{\gamma^+}(z_1,z_{21}^\alpha)-\mathcal{O}_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}_{\gamma^+}(z_1,z_2), && (18.3)\\
\mathbf{A}^*{}_{\text{tw-2}} &= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\mathcal{O}_{\gamma^+}(z_{12}^\alpha,z_2)-\mathcal{O}_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}_{\gamma^+}(z_1,z_2), && (18.4)\\
\mathbf{B}_{\text{tw-2}} &= 2(1-\epsilon)a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta]\mathcal{O}_{\gamma^+}(z_{12}^\alpha,z_{21}^\beta). && (18.5)
\end{aligned}$$

Evaluating the matrix element, and Fourier transform we get

$$\begin{aligned}
\mathcal{O}_{\gamma^+}(z_1,z_2) &\to 2p^+\int dx e^{2ixz}f_1(x)\to f_1(x)=\int dy\delta(x-y)f_1(y), && (18.6)\\
\mathcal{O}_{\gamma^+}(z_1,z_{21}^\alpha) &\to 2p^+\int dx e^{2ixz\bar\alpha}f_1(x)\to \int dy\delta(x-\bar\alpha y)f_1(y), && (18.7)\\
\mathcal{O}_{\gamma^+}(z_{12}^\alpha,z_2) &\to 2p^+\int dx e^{2ixz\bar\alpha}f_1(x)\to \int dy\delta(x-\bar\alpha y)f_1(y), && (18.8)\\
\mathcal{O}_{\gamma^+}(z_{12}^\alpha,z_{21}^\beta) &\to 2p^+\int dx e^{2ixz(1-\alpha-\beta)}f_1(x)\to \int dy\delta(x-(1-\alpha-\beta)y)f_1(y). && (18.9)
\end{aligned}$$

We substitute to the expression for diagrams and simplify

$$\begin{aligned}
\mathbf{A} &\to 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int dy\int_0^1 d\alpha\Big\{\frac{\bar\alpha}{\alpha}\Big[\delta(x-\bar\alpha y)-\delta(x-y)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\delta(\bar\alpha)\delta(x-y)\Big\}f_1(y) && (18.10)\\
&= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int dy\int_0^1 d\alpha\delta(x-\alpha y)f_1(y)\Big\{\left(\frac{\alpha}{1-\alpha}\right)_+-\delta(\bar\alpha)\,(1+\boldsymbol{\lambda}_\delta)\Big\},\\
\mathbf{A}^* &\to 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int dy\int_0^1 d\alpha\delta(x-\alpha y)f_1(y)\Big\{\left(\frac{\alpha}{1-\alpha}\right)_+-\delta(\bar\alpha)\,(1+\boldsymbol{\lambda}_\delta)\Big\}, && (18.11)\\
\mathbf{B} &= 2(1-\epsilon)a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta d\gamma]\int dy\delta(x-\gamma y)f_1(y) && (18.12)\\
&= 2(1-\epsilon)a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int_0^1 d\alpha\int dy\delta(x-\alpha y)\bar\alpha f_1(y).
\end{aligned}$$

where

$$(f(\alpha))_+ = f(\alpha)-\delta(\bar\alpha)\int_0^1 d\beta f(\beta). \tag{18.13}$$

Altogether it reads

$$\mathbf{A}+\mathbf{A}^*+\mathbf{B} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int dy\int_0^1 d\alpha\delta(x-\alpha y)f_1(y)\Big\{\left(\frac{2\alpha}{1-\alpha}\right)_+ +(1-\epsilon)\bar\alpha-2\delta(\bar\alpha)(1+\boldsymbol{\lambda}_\delta)\Big\}. \tag{18.14}$$

This expression literally coincides with the NLO calculation in the free-quark case (see e.g. [33]) (5.8)). Thus we do not repeat the last step (renormalization, etc.).

We redistribute

$$\left(\frac{2\alpha}{1-\alpha}\right)_+ + \bar\alpha = \left(\frac{1+\alpha^2}{1-\alpha}\right)_+ + \frac{\delta(\bar\alpha)}{2}, \tag{18.15}$$

and obtain

$$\mathbf{A} + \mathbf{A}^* + \mathbf{B} = 2a_s C_F \Gamma(-\epsilon)\mathbf{B}^\epsilon \int dy \int_0^1 d\alpha \delta(x-\alpha y) f_1(y) \Big\{ p(\alpha) - \epsilon\bar\alpha - \delta(\bar\alpha)(\frac{3}{2} + 2\boldsymbol{\lambda}_\delta) \Big\}, \tag{18.16}$$

where

$$p(x) = \left(\frac{1+x^2}{1-x}\right)_+ . \tag{18.17}$$

B. Twist-3 part

In the diagrams that contain twist-3 terms we found the following operators

$$b_\mu \mathcal{O}^\mu_{\gamma^+}(z_1, z_2), \qquad b_\mu \mathcal{T}^\mu_{\gamma^+}(z_1, z_2, z_3), \qquad b_\nu \mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z_1, z_2, z_3).$$

Their matrix elements has been evaluated in [34], which is also copy-pasted in the beginning of this file. Some of these definition are suitable for our applications without modification, but some are should be examined carefully. In particular, the definitions are given for the simetric points $O(z, -z)$ while in our case we have asymmetric combinations. We have found (4.4,4.5) and (3.26)

$$\text{DY} \qquad \frac{b_\mu}{2}\langle P,S|\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)|P,S\rangle \;=\; +i(p^+)^2 M \tilde{s}^\mu b_\mu \Big(\int_{-\infty}^{\frac{z_{12}}{2}} + \int_{-\infty}^{\frac{z_{21}}{2}} \Big) d\tau \tilde{T}\left(\frac{z_{12}}{2}, \tau, \frac{z_{21}}{2}\right), \tag{18.18}$$

$$\text{SIDIS} \qquad \frac{b_\mu}{2}\langle P,S|\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)|P,S\rangle \;=\; -i(p^+)^2 M \tilde{s}^\mu b_\mu \Big(\int_{\frac{z_{12}}{2}}^{\infty} + \int_{\frac{z_{21}}{2}}^{\infty} \Big) d\tau \tilde{T}\left(\frac{z_{12}}{2}, \tau, \frac{z_{21}}{2}\right), \tag{18.19}$$

$$\text{ANY} \qquad \frac{b_\mu}{2}\langle P,S|\mathcal{O}^\mu_{\gamma^\mu_T}(z_1,z_2)|P,S\rangle \;=\; 0, \tag{18.20}$$

$$\text{ANY} \qquad \frac{b_\mu}{2}\langle P,S|\mathcal{T}^\mu_{\gamma^+}(z_1,z_2,z_3)|P,S\rangle \;=\; (p^+)^2 M \tilde{s}^\mu b_\mu \tilde{T}(z_1,z_2,z_3), \tag{18.21}$$

where

$$z_{ij} = z_i - z_j.$$

To derive the expression for the last operator we use

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4}\epsilon_T^{\alpha\beta}(\gamma^-\gamma^+ - \gamma^+\gamma^-)\gamma_\alpha\gamma_\beta, \tag{18.22}$$

from which follows

$$\gamma^+\gamma^5 = \frac{i}{2}\epsilon_T^{\alpha\beta}\gamma^+\gamma_\alpha\gamma_\beta. \tag{18.23}$$

Consequently we have

$$\gamma_T^{\mu\nu} = \frac{1}{2}(\gamma_T^\mu\gamma_T^\nu - \gamma_T^\nu\gamma_T^\mu) = \frac{\epsilon_T^{\mu\nu}}{2}\epsilon_{T,\alpha\beta}\gamma^\alpha\gamma^\beta, \tag{18.24}$$

$$\gamma^+\gamma_T^{\mu\nu} = -i\epsilon_T^{\mu\nu}\gamma^+\gamma^5. \tag{18.25}$$

Note: these calculations are done in $d = 4$, where our background fields live. Thus we have

$$\text{ANY} \qquad \frac{b_\mu}{2}\langle P,S|\mathcal{T}^\nu_{\gamma^+\gamma^{\mu\nu}}(z_1,z_2,z_3)|P,S\rangle \;=\; (p^+)^2 M \tilde{s}^\mu b_\mu \Delta\tilde{T}(z_1,z_2,z_3). \tag{18.26}$$

We continue by different color structures

Note on the half-infinite integrals: The Qui-Sterman function appear in the calculation via the combination

$$(p^+)^2\int \frac{dz}{2\pi}e^{-2ixzp^+}\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\sigma\tilde T(z,\sigma,-z)=(p^+)^2\int \frac{dz}{2\pi}e^{-2ixzp^+}\int_{-\infty}^{\infty}d\sigma\tilde T(z,\sigma,-z) \tag{18.27}$$
$$=(p^+)^2\int \frac{dz}{2\pi}e^{-2ixzp^+}\int_{-\infty}^{\infty}d\sigma\int[dx]e^{-ip^+(z(x_1-x_3)+x_2\sigma)}T(x_1,x_2,x_3)$$
$$=(2\pi)\int[dx]\delta(x_2)\delta(2x+x_1-x_3)T(x_1,x_2,x_3)=2\pi\int dx_{1,2,3}\delta(x_1+x_2+x_3)\delta(x_2)\delta(2x+x_1-x_3)T(x_1,x_2,x_3)$$
$$=2\pi\int dx_{1,3}\delta(x_1+x_3)\delta(2x+x_1-x_3)T(x_1,0,x_3)=2\pi\int dx_3\delta(2x-2x_3)T(-x_3,0,x_3)=\pi T(-x,0,x).$$

On another side we can take both integral literally

$$(p^+)^2\int \frac{dz}{2\pi}e^{-2ixzp^+}\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\sigma\tilde T(z,\sigma,-z)= \tag{18.28}$$
$$=(p^+)^2\int \frac{dz}{2\pi}e^{-2ixzp^+}\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\sigma\int[dx]e^{-ip^+(z(x_1-x_3)+x_2\sigma)}T(x_1,x_2,x_3)$$
$$=\frac{i}{2}\int[dx]T(x_1,x_2,x_3)\left(\frac{\delta(x-x_3)}{x_2+i0}+\frac{\delta(x+x_1)}{x_2+i0}\right)$$
$$=\frac{i}{2}\int[dx]\left(\frac{\delta(x-x_3)T(x_1,x_2,x_3)}{x_2+i0}+\frac{\delta(x+x_1)T(-x_3,-x_2,-x_1)}{x_2+i0}\right)$$
$$=\frac{i}{2}\int[dx]T(x_1,x_2,x_3)\left(\frac{\delta(x-x_3)}{x_2+i0}+\frac{\delta(x-x_3)}{-x_2+i0}\right)$$
$$=-2\pi i\frac{i}{2}\int[dx]T(x_1,x_2,x_3)\delta(x-x_3)=\pi T(-x,0,x),$$

where we have explictelly keep the term that makes integral converge and

$$\frac{1}{x_2+i0}-\frac{1}{x_2-i0}=-2\pi i\delta(x_2).$$

Note that according to our definition (3.34)

$$\frac{i}{2}\int[dx]T(x_1,x_2,x_3)\left(\frac{\delta(x-x_3)}{x_2+i0}+\frac{\delta(x+x_1)}{x_2+i0}\right)=\frac{i}{2}T^{(1)}(x), \tag{18.29}$$

and thus

$$T^{(1)}(x)=-2\pi iT(-x,0,x). \tag{18.30}$$

For simplicity of comparison we also use the notation

$$T(-y,y-x,x)=T_F(x,y)=T_F(y,x),\qquad \Delta T(-y,y-x,x)=\Delta T_F(x,y)=-\Delta T_F(y,x). \tag{18.31}$$

C. Twist-3 part of diagrams A and B

The diagrams A and B

$$\mathbf{A}_{\text{tw-3}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon b_\mu\Big\{\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\bar\alpha\mathcal{O}^\mu_{\gamma^+}(z_1,z_{21}^\alpha)-\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big\}, \tag{18.32}$$
$$\mathbf{A^*}_{\text{tw-3}} = 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon b_\mu\Big\{\int_0^1 d\alpha\frac{\bar\alpha}{\alpha}\Big[\bar\alpha\mathcal{O}^\mu_{\gamma^+}(z_{12}^\alpha,z_2)-\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big]-(1+\boldsymbol{\lambda}_\delta)\,\mathcal{O}^\mu_{\gamma^+}(z_1,z_2)\Big\}, \tag{18.33}$$
$$\mathbf{B} = 2(1-\epsilon)a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\int[d\alpha d\beta]\gamma b_\mu\Big[2\mathcal{O}^\mu_{\gamma^+}(z_{12}^\alpha,z_{21}^\beta)+\frac{2-\bar\alpha\delta(\beta)-\bar\beta\delta(\alpha)}{z_2-z_1}\underbrace{\mathcal{O}_{\gamma^\mu}(z_{12}^\alpha,z_{21}^\beta)}_{\to 0}\Big]. \tag{18.34}$$

We adopt the following chain of transformation

$$\text{Operator}^\mu \to \frac{b_\mu}{2}\langle P,S|\text{Operator}^\mu|P,S\rangle\Big|_{z_1=z,z_2=-z} \to \frac{b_\mu}{2}\int\frac{dz}{2\pi}e^{-2izxp^+}\langle P,S|\text{Operator}^\mu|P,S\rangle\Big|_{z_1=z,z_2=-z}$$

$$\begin{aligned}
\mathcal{O}^\mu_{\gamma^+}(z_1,z_2) &\to i(p^+)M\tilde{s}^\mu b_\mu\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\tau\tilde{T}\left(z,\tau,-z\right) && (18.35)\\
&\to i(p^+)M\tilde{s}^\mu b_\mu\int\frac{dz}{2\pi}\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\tau\int[dx]e^{-ip^+(2xz+(x_1-x_3)z+x_2\tau)}T(x_1,x_2,x_3)\\
&= iM\tilde{s}^\mu b_\mu\frac{i}{2}T^{(1)}(x)=i\pi M\tilde{s}^\mu b_\mu T(-x,0,x)\\
\mathcal{O}^\mu_{\gamma^+}(z_1,z_{21}^\alpha) &\to i(p^+)M\tilde{s}^\mu b_\mu\left(\int_{-\infty}^{\bar{\alpha}z}+\int_{-\infty}^{-\bar{\alpha}z}\right)d\tau\tilde{T}\left(\bar{\alpha}z,\tau,-\bar{\alpha}z\right) && (18.36)\\
&\to i(p^+)M\tilde{s}^\mu b_\mu\int\frac{dz}{2\pi}\left(\int_{-\infty}^{\bar{\alpha}z}+\int_{-\infty}^{-\bar{\alpha}z}\right)d\tau\int[dx]e^{-ip^+(2xz+\bar{\alpha}(x_1-x_3)z+x_2\tau)}T(x_1,x_2,x_3)\\
&= i\pi M\tilde{s}^\mu b_\mu\int dy\delta(x-\bar{\alpha}y)T(-y,0,y)\\
\mathcal{O}^\mu_{\gamma^+}(z_{12}^\alpha,z_2) &\to i(p^+)M\tilde{s}^\mu b_\mu\int_{-\infty}^{\infty}d\tau\tilde{T}\left(\bar{\alpha}z,\tau,-\bar{\alpha}z\right) && (18.37)\\
&\to i(p^+)M\tilde{s}^\mu b_\mu\int\frac{dz}{2\pi}\int_{-\infty}^{\infty}d\tau\int[dx]e^{-ip^+(2xz+\bar{\alpha}(x_1-x_3)z+x_2\tau)}T(x_1,x_2,x_3)\\
&= i\pi M\tilde{s}^\mu b_\mu\int dy\delta(x-\bar{\alpha}y)T(-y,0,y)\\
\mathcal{O}^\mu_{\gamma^+}(z_{12}^\alpha,z_{21}^\beta) &\to i(p^+)M\tilde{s}^\mu b_\mu\int_{-\infty}^{\infty}d\tau\tilde{T}\left(\gamma z,\tau,-\gamma z\right) && (18.38)\\
&\to i(p^+)M\tilde{s}^\mu b_\mu\int\frac{dz}{2\pi}\int_{-\infty}^{\infty}d\tau\int[dx]e^{-ip^+(2xz+\gamma(x_1-x_3)z+x_2\tau)}T(x_1,x_2,x_3)\\
&= i\pi M\tilde{s}^\mu b_\mu\int dy\delta(x-\gamma y)T(-y,0,y)
\end{aligned}$$

So we obtain

$$\begin{aligned}
\mathbf{A}+\mathbf{A}^* &\to 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big[i\pi M\tilde{s}^\mu b_\mu\Big]\int dy\Big\{\int_0^1 d\alpha\frac{2\alpha}{\bar{\alpha}}\Big[\alpha\delta(x-\alpha y)-\delta(x-y)\Big]T(-y,0,y) && (18.39)\\
&\quad -2\delta(\bar{\alpha})\delta(x-y)\left(1+\boldsymbol{\lambda}_\delta\right)T(-y,0,y)\Big\}\\
&= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big[i\pi M\tilde{s}^\mu b_\mu\Big]\int dy\int_0^1 d\alpha\Big\{\left(\frac{2\alpha}{1-\alpha}\right)_+-2\alpha-2\delta(\bar{\alpha})\left(1+\boldsymbol{\lambda}_\delta\right)\Big\}\delta(x-\alpha y)T(-y,0,y).
\end{aligned}$$

$$\begin{aligned}
\mathbf{B} &\to 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big[i\pi M\tilde{s}^\mu b_\mu\Big]\int dy\Big\{(1-\epsilon)\int[d\alpha d\beta d\gamma]2\gamma\delta(x-\gamma y)T(-y,0,y)\Big\} && (18.40)\\
&= 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big[i\pi M\tilde{s}^\mu b_\mu\Big]\int dy\int_0^1 d\alpha\Big\{2(1-\epsilon)\alpha\bar{\alpha}\Big\}\delta(x-\alpha y)T(-y,0,y).
\end{aligned}$$

Altogether it is

$$\begin{aligned}
\mathbf{A}+\mathbf{A}^*+\mathbf{B} &\to 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big[i\pi M\tilde{s}^\mu b_\mu\Big]\int dy\int_0^1 d\alpha\delta(x-y\alpha) && (18.41)\\
&\quad\Big\{\left(\frac{2\alpha}{1-\alpha}\right)_+-2\alpha+2(1-\epsilon)\alpha\bar{\alpha}-2\delta(\bar{\alpha})\left(1+\boldsymbol{\lambda}_\delta\right)\Big\}T(-y,0,y). && (18.42)
\end{aligned}$$

We simplify the plus-distribution

$$\left(\frac{2\alpha}{1-\alpha}\right)_+ - 2\alpha + 2\alpha\bar\alpha = \left(\frac{1+\alpha^2}{1-\alpha}\right)_+ + \frac{\delta(\bar\alpha)}{2} - \bar\alpha - 2\alpha^2 \tag{18.43}$$

$$\mathbf{A}+\mathbf{A}^*+\mathbf{B} \;\to\; 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big[i\pi M\tilde s^\mu b_\mu\Big]\int dy\int_0^1 d\alpha \tag{18.44}$$
$$\Big\{p(\alpha)-\bar\alpha-2\alpha^2-2\epsilon\alpha\bar\alpha-\delta(\bar\alpha)\left(\frac{3}{2}+2\boldsymbol{\lambda}_\delta\right)\Big\}T(-y,0,y).$$

XIX. COMBINING THE RESULT TOGETHER

The Sivers function has the matching at small-b as

$$f_{1T}^{\perp}(x,\boldsymbol{b}) \;=\; \pi T(-x,0,x)+\pi\sum_{f=q,g}\int_0^1 dyd\xi\delta(x-y\xi)C_f(y,\mathbf{L}_\mu)T[x,\xi]. \tag{19.1}$$

The coefficient function is derived in the next section

A. Quark-to-quark part

The diagrams contributing to C_q has three color structures. Let us sum them separately. We get

$$\tilde C_F\otimes T \;=\; 2a_sC_F\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{p(y)-\bar y-2y^2-2\epsilon y\bar y-\delta(\bar y)\left(\frac{3}{2}+2\boldsymbol{\lambda}_\delta\right)\Big\}T(-\xi,0,\xi), \tag{19.2}$$

$$\tilde C_{FA}\otimes T \;=\; 2a_s\left(C_F-\frac{C_A}{2}\right)\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{ \tag{19.3}$$
$$\Big[\bar y+2y^2-\epsilon\bar y(1-2y)\Big]T(-\xi,0,\xi)+(2y-1)T(-x,\xi,x-\xi)-\Delta T(-x,\xi,x-\xi)\Big\},$$

$$\tilde C_A\otimes T \;=\; 2a_s\frac{C_A}{2}\Gamma(-\epsilon)\mathbf{B}^\epsilon\Big\{ \tag{19.4}$$
$$-2T(-x,0,x)+\frac{(1+y)T(-x,x-\xi,\xi)+[-2y(\bar y+y^2)+2\epsilon y\bar y]T(-\xi,0,\xi)}{1-y}+\Delta T(-x,x-\xi,\xi)\Big\}.$$

Even more instructive to sum diagrams by topologies. We have

$$\tilde{C}_{WL}\otimes T = 2a_s\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\Big\{ \tag{19.5}$$

$$\Big[C_F\underbrace{\Big(\Big(\frac{2y}{1-y}\Big)_+ - 2y - 2\delta(y)(1+\boldsymbol{\lambda}_\delta)\Big)}_{\mathbf{A}} + \Big(C_F-\frac{C_A}{2}\Big)\underbrace{2y}_{\mathbf{C}} + \frac{C_A}{2}\underbrace{\Big(\frac{-2y^2}{1-y}-2\delta(\bar{y})\Big)}_{\mathbf{D}}\Big]T(-\xi,0,\xi)$$

$$+\Big(C_F-\frac{C_A}{2}\Big)\underbrace{(2y-1)}_{\mathbf{C}}T(-x,\xi,x-\xi)+\frac{C_A}{2}\underbrace{\frac{1+y}{1-y}}_{\mathbf{D}}T(-x,x-\xi,\xi)$$

$$+\Big(C_F-\frac{C_A}{2}\Big)\underbrace{(-1)}_{\mathbf{C}}\Delta T(-x,\xi,x-\xi)+\frac{C_A}{2}\underbrace{(+1)}_{\mathbf{D}}\Delta T(-x,x-\xi,\xi)\Big\}$$

$$= 2a_s\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\Big\{ \tag{19.6}$$

$$\Big[C_F\Big(\Big(\frac{2y}{1-y}\Big)_+ - 2\delta(y)(1+\boldsymbol{\lambda}_\delta)\Big) + \frac{C_A}{2}\Big(\frac{-2y}{1-y}+2\delta(\bar{y})\Big)\Big]T(-\xi,0,\xi)$$

$$+\Big(C_F-\frac{C_A}{2}\Big)(2y-1)T(-x,\xi,x-\xi)+\frac{C_A}{2}\frac{1+y}{1-y}T(-x,x-\xi,\xi)$$

$$-\Big(C_F-\frac{C_A}{2}\Big)\Delta T(-x,\xi,x-\xi)+\frac{C_A}{2}\Delta T(-x,x-\xi,\xi)\Big\}.$$

We note the incredible simplicity of the first line. The sum ladder-like diagrams is

$$\tilde{C}_{Ladder}\otimes T = 2a_s(1-\epsilon)\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\Big\{\Big[C_F\underbrace{2y\bar{y}}_{\mathbf{B}}+\Big(C_F-\frac{C_A}{2}\Big)\underbrace{\bar{y}(1-2y)}_{\mathbf{E}}+\frac{C_A}{2}\underbrace{(-2)y\bar{y}}_{\mathbf{F}}\Big]T(-\xi,0,\xi)\Big\} \tag{19.7}$$

$$= 2a_s(1-\epsilon)\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\Big(C_F-\frac{C_A}{2}\Big)\bar{y}T(-\xi,0,\xi). \tag{19.8}$$

Summing together these expressions we got

$$\tilde{C}_q\otimes T = 2a_s\Gamma(-\epsilon)\mathbf{B}^{\epsilon}\Big\{ \tag{19.9}$$

$$\Big[C_F\Big(\Big(\frac{1+y^2}{1-y}\Big)_+ - \delta(y)(\frac{3}{2}+2\boldsymbol{\lambda}_\delta)\Big) - \frac{C_A}{2}\Big(\frac{1+y^2}{1-y}-2\delta(\bar{y})\Big)-\epsilon\bar{y}\Big(C_F-\frac{C_A}{2}\Big)\Big]T(-\xi,0,\xi)$$

$$+\Big(C_F-\frac{C_A}{2}\Big)(2y-1)T(-x,\xi,x-\xi)+\frac{C_A}{2}\frac{1+y}{1-y}T(-x,x-\xi,\xi)$$

$$-\Big(C_F-\frac{C_A}{2}\Big)\Delta T(-x,\xi,x-\xi)+\frac{C_A}{2}\Delta T(-x,x-\xi,\xi)\Big\}.$$

Let us extract the evolution kernel for the QS function:

$$\mu^2\frac{d}{d\mu^2}T(x,x) = 2a_sP_q\otimes T \quad (19.10)$$

$$= 2a_s\int_0^1 dy\int d\xi\delta(x-y\xi)\Bigg\{ \left(C_F-\frac{C_A}{2}\right)\Big[\left(\frac{1+y^2}{1-y}\right)_+T(-\xi,0,\xi)+(2y-1)_+T(-x,\xi,x-\xi)-\Delta T(-x,\xi,x-\xi)\Big]$$
$$+\frac{C_A}{2}\Big[\left(\frac{1+y}{1-y}\right)_+T(-x,x-\xi,\xi)+\Delta T(-x,x-\xi,\xi)\Big]\Bigg\},$$

where

$$(f(y))_+=f(y)-\delta(\bar y)\int_0^1 dy'f(y'). \quad (19.11)$$

We have a simple expression

$$\tilde C_q\otimes T = 2a_s\Gamma(-\epsilon)\mathbf{B}^\epsilon\left\{P_q\otimes T-C_F\left(\frac{3}{2}+2\boldsymbol{\lambda}_\delta\right)T(-x,0,x)-\epsilon\bar y\left(C_F-\frac{C_A}{2}\right)T(-\xi,0,\xi)\right\}. \quad (19.12)$$

This expression is to be multiplied by TMD renormalization factor, quark filed renormalization and Rapidity renormalization. We have [33]

$$Z_q = 1-C_Fa_s\left(\frac{2}{\epsilon^2}+\frac{4+2\mathbf{l}_\zeta}{\epsilon}\right)+.. \quad (19.13)$$

$$Z_2 = 1+a_s\frac{C_F}{\epsilon}+... \quad (19.14)$$

$$S = -4C_F\mathbf{B}^\epsilon\Gamma(-\epsilon)\left(\mathbf{L}_\mu-\mathbf{l}_\zeta+2\boldsymbol{\lambda}_\delta-\psi(-\epsilon)-\gamma_E\right)+...\,. \quad (19.15)$$

Combining together

$$\mathbb{Z}=Z_qZ_2^{-1}S^{-1/2} = 1+a_sC_F\left[-\frac{1}{\epsilon}\left(\frac{3}{2}+2\boldsymbol{\lambda}_\delta\right)+\left(-\mathbf{L}_\mu^2+2\mathbf{L}_\mu\mathbf{l}_\zeta-4\mathbf{L}_\mu\boldsymbol{\lambda}_\delta-\zeta_2\right)+...\right]+O(a_s^2). \quad (19.16)$$

Thus we get after the renormalization

$$C_q\otimes T = a_s\Big[-2\mathbf{L}_\mu P_q\otimes T \quad (19.17)$$
$$+C_F\left(-\mathbf{L}_\mu^2+2\mathbf{l}_\zeta\mathbf{L}_\mu+3\mathbf{L}_\mu-\zeta_2\right)T(-x,0,x)+\int_0^1 dy\int d\xi\left(C_F-\frac{C_A}{2}\right)2\bar yT(-\xi,0,\xi)\Big]$$

It has correct logarithmic behavior. In the ζ-prescription

$$C_q\otimes T = a_s\Big[-2\mathbf{L}_\mu P_q\otimes T-C_F\zeta_2T(-x,0,x)+\int_0^1 dy\int d\xi\left(C_F-\frac{C_A}{2}\right)2\bar yT(-\xi,0,\xi)\Big]. \quad (19.18)$$

B. Quark-to-gluon part

Summing together diagrams M and N we get

$$\{\mathbf{N}+\mathbf{M}\} = -[i\pi b_\mu \tilde{s}^\mu] a_s \Gamma(-\epsilon)\mathbf{B}^\epsilon \int_0^1 dy \int d\xi \delta(x-y\xi)\Big\{ \frac{1-2y\bar{y}}{2}\frac{G^+(-\xi,0,\xi)+Y^+(-\xi,0,\xi)+G^-(-\xi,0,\xi)+Y^-(-\xi,0,\xi)}{\xi} -3y\bar{y}\frac{\epsilon}{(2-\epsilon)}\frac{G^+(-\xi,0,\xi))+G^-(-\xi,0,\xi)}{\xi}\Big\} \quad (19.19)$$

There is a strange relative factor 4 between these diagrams. For a moment I multiply diagram M by 2 and divide diagram N by 2. I also recal that $b \to b/2$. We can use that $G^\pm + T^\pm = -T^\pm_{3F}$. Tking into account that our definition of diartribution is different by sign we agree in the logarithmic part with [Qui,Kang]. We get

$$C_g \otimes T = a_s\Big[-2\mathbf{L}_\mu P_g \otimes T - \int_0^1 dy \int d\xi \delta(x-y\xi)\frac{3}{2}y\bar{y}\frac{G^+(-\xi,0,\xi)+G^-(-\xi,0,\xi)}{\xi}\Big], \quad (19.20)$$

where

$$P_g \otimes T = \int_0^1 dy \int d\xi \delta(x-y\xi)\Big(-\frac{1-2y\bar{y}}{4}\Big)\frac{G^+(-\xi,0,\xi)+Y^+(-\xi,0,\xi)+G^-(-\xi,0,\xi)+Y^-(-\xi,0,\xi)}{\xi} \quad (19.21)$$

In the article I have changed the sign of G and Y, in order to have the same definition is [BMP] and [Kang].

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