# TMD factorization and artemide

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# Introduction

**ARTEMIDE**
The only publicly-available code working within **TMD factorization**.

News: a lot of done both in theory and in phenomenology

- Critical revision and cross-checks of TMD factorization theory
- Complete theory justification of $\zeta$-prescription used in `artemide`
- New perturbative calculations: NNLO (tw-2 gluon, polarized, etc.) LO+NLO (tw-3)(!)
- Phenomenology: DY, $\pi$DY, SIDIS, DIS+jet.

`artemide` checklist

- ✓ ...
- ✓ Higgs production and gluon TMDs
- ✓ target-mass corrections for SIDIS
- ✓ joined fit of SIDIS+DY (← we are here)
- power corrections for DY
- SSA and Sivers functions
- ...

## TMD factorization

**... should not be mistaken with resummation approach**

- Resummation: $\Lambda_{QCD} \ll q_T \ll Q$ TMD factorization: $q_T \ll Q$
- Common structure at $\Lambda_{QCD} \ll q_T \ll Q$ for some processes (e.g. unpolarized DY and SIDIS, SSA).

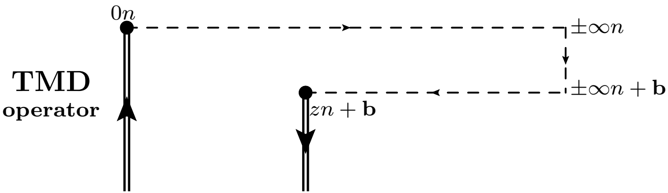
**Pros:**

- Can be applied down to low energies $Q \sim 2$GeV and down to $q_T \sim 0$.
- Well established theory foundation (operators, renormalization, all that).
- Universal evolution
- Gives access to interesting non-perturbative functions.

**Cons:**

- Plenty of new unknown functions.
- Only double-inclusive processes: DY,SIDIS,$ee \to hh + X$, DIS+jet.
- $q_T \ll Q$ and, in addition, $q_T$ is not $p_T$ (next slides)

TMD distributions have very peculiar operator structure



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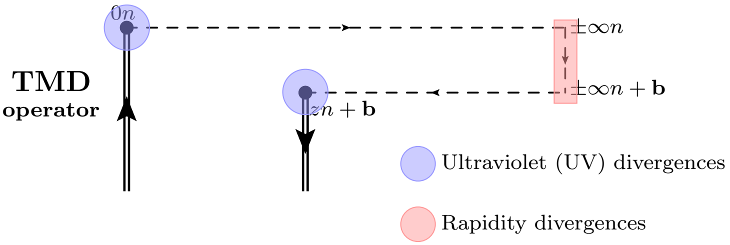


TMD distribution depends on 2 scales

its evolution is given by 2 equations

$$\mu^2\frac{dF(x,b;\mu,\zeta)}{d\mu^2}=\gamma_F(\mu,\zeta)F(x,b;\mu,\zeta),\qquad \zeta\frac{dF(x,b;\mu,\zeta)}{d\zeta}=-\mathcal{D}(\mu,b)F(x,b;\mu,\zeta)$$

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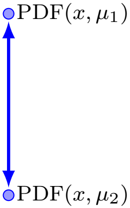
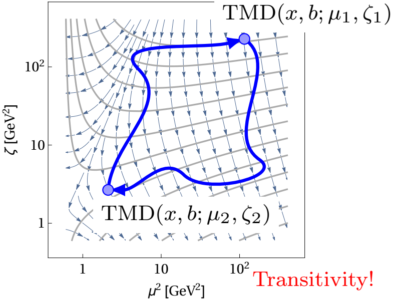
## Double-scale evolution rises a different set questions

### 1D evolution



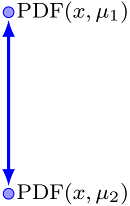

### 2D evolution



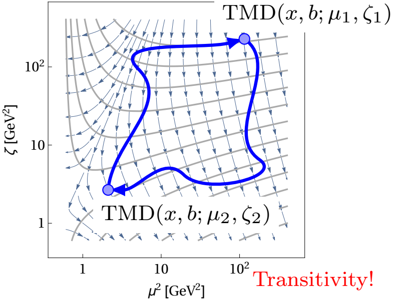

Transitivity!

## Double-scale evolution rises a different set questions

**1D evolution**




**2D evolution**




DIS

$$C(x,Q/\mu)\otimes q(x,\mu)$$

out: $\mu\sim Q$

in: $\mu\sim 2\mathrm{GeV}$

TMD

$$\int dbe^{iq_Tb}C(Q/\mu)F_1(x,b,\mu,\zeta_1)F_1(x,b,\mu,\zeta_2)$$

out: $\mu\sim Q,\quad \zeta_1=\zeta_2=Q^2$

in: $\mu,\zeta_1,\zeta_2\sim$???

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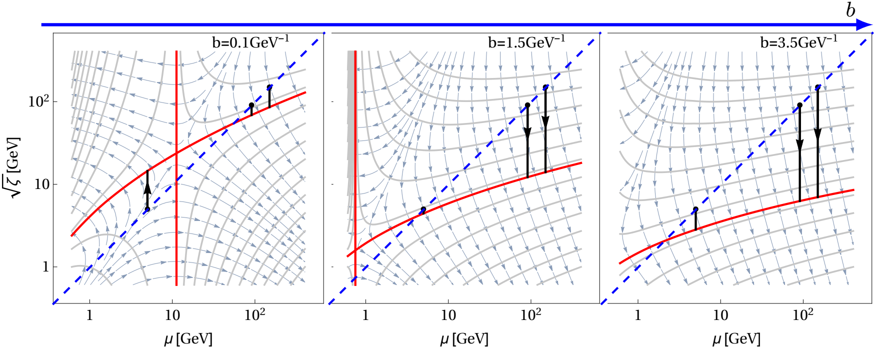
## ζ-prescription




Expression for TMD cross-section is exceptionally simple

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T} = \sigma_0 \int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'}\left(\frac{Q}{\mu}\right)\left(\frac{Q^2}{\zeta_Q}\right)^{-2\mathcal{D}(Q,b)} F_{f\leftarrow h}(x_1,b)F_{f'\leftarrow h}(x_2,b)$$

## Each cross-section requires convolution of 3 NP functions

| N \ q | U | L | T |
|---|---|---|---|
| U | $f_1$ | | $h_1^{\perp}$ |
| L | | $g_1$ | $h_{1L}^{\perp}$ |
| T | $f_{1T}^{\perp}$ | $g_{1T}$ | $h_1$ $h_{1T}^{\perp}$ |

Rapidity anomalous dimension (universal)

$$\frac{d\sigma}{dy dQ^2 d^2\mathbf{q}_T} = \sigma_0 \int d^2b \, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'} \left(\frac{Q}{\mu}\right) \left(\frac{Q^2}{\zeta_Q}\right)^{-2\mathcal{D}(Q,b)} F_{f\leftarrow h}(x_1, b) F_{f'\leftarrow h}(x_2, b)$$

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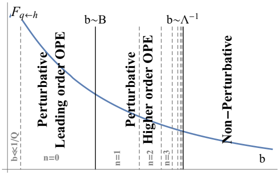
$N^3LO$ (universal)

small-b $N^3LO$

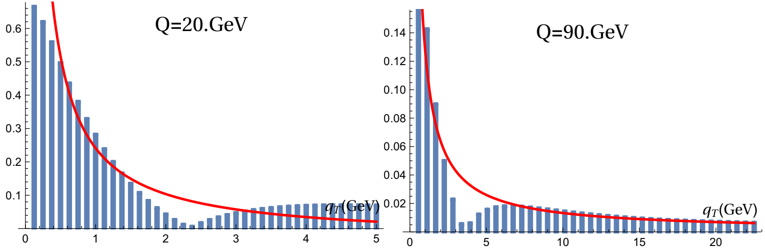
small-b NNLO, NLO, LO depending on TMD




Small-b ↔ large $q_T$
At small-b everything is perturbative turns to resummation

How large is non-perturbative component?

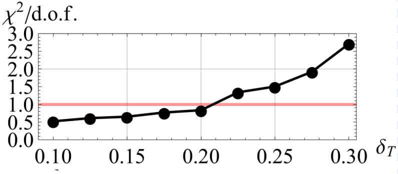
DY(resummation)/DY(TMD)




At $q_T \lesssim 0.1Q$ non-perturbative corrections are essential

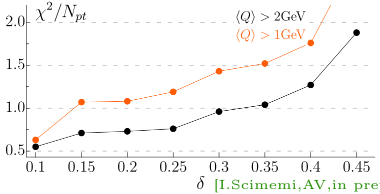
Test by inclusion of the data with
$$q_T < \delta \cdot Q$$

unpolarized DY




[I.Scimemi,AV,1706.01473]

unpolarized SIDIS




[I.Scimemi,AV,in prep.]

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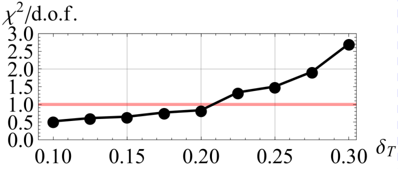


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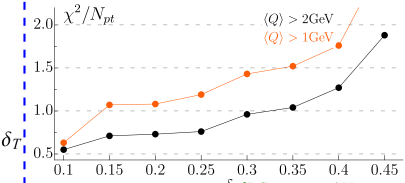
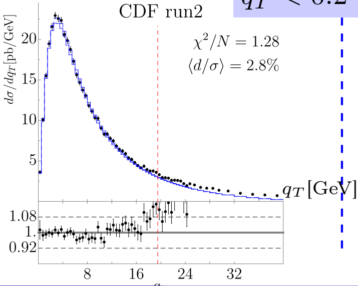
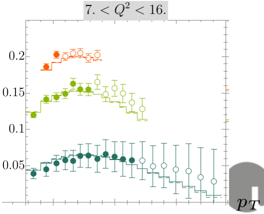
unpolarized SIDIS



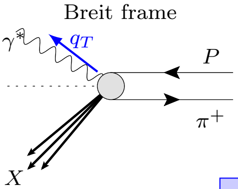

[I.Scimemi,AV,in prep.]

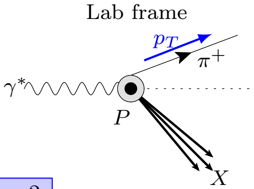
$q_T < 0.2 - 0.25Q$



## Proof of factorization is done in the Breit frame




$$q_T^2 = \frac{p_T^2}{z^2}\frac{1+\gamma^2}{1-\varsigma^2}$$

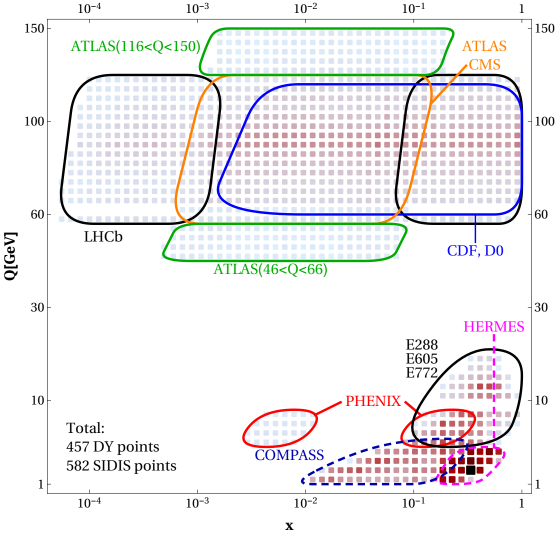
$$\gamma = \frac{2Mx}{Q}$$

$$\varsigma = \gamma\frac{m}{zQ}$$

## In practice: $q_T < 0.25Q$

- Most part of data is not TMD factorisable.
- Low $z$'s are not accessible
- H1, ZEUS data have no TMD points, too low z.

## Global fit of unpolarized DY+SIDIS [I.Scimemi,AV,in prep.]

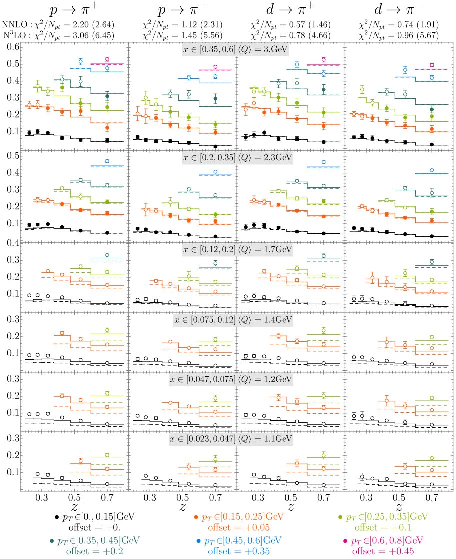


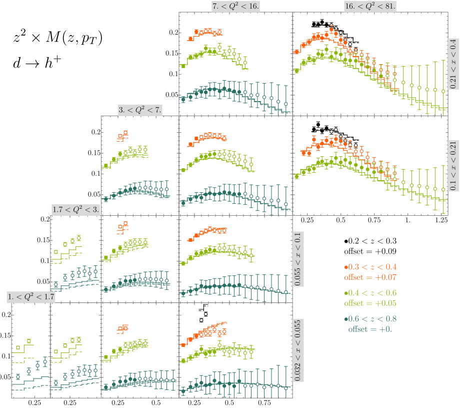

**High energy DY:** 194 points
**Low energy DY:** 263 points
**(Low energy) SIDIS:** 582 points

NNLO: $\chi^2_{global}/N_{pt} = 0.95$
$N^3$LO: $\chi^2_{global}/N_{pt} = 1.05$

`artemide` v.2.02

# Example of SIDIS data

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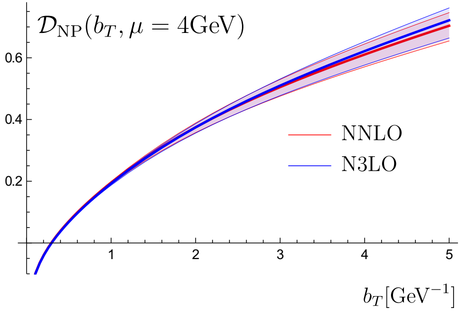
TMD evolution kernel measured in one process suites for all

Using DY extract:
TMDPDF
TMD evolution

→

Using SIDIS extract:
TMDFF

$\chi^2/N_{pt}$=1.00

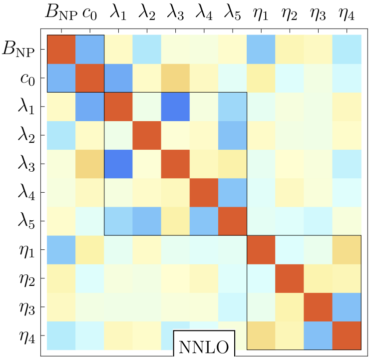



Using DY+SIDIS extract:
TMDPDF
TMDFF
TMD evolution

$\chi^2/N_{pt}$=0.95

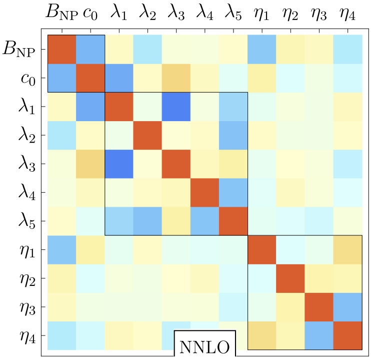
Using $\pi$DY extract:
$\pi$TMDPDF

$\chi^2/N_{pt}$=1.3




Evolution : 2 parameters
TMDPDF : 5 parameters + PDF
TMDFF : 4 parameters + NNFF

Different NP functions are almost decorrelated




Evolution : 2 parameters
TMDPDF : 5 parameters + PDF
TMDFF : 4 parameters + NNFF

Different NP functions are almost decorrelated

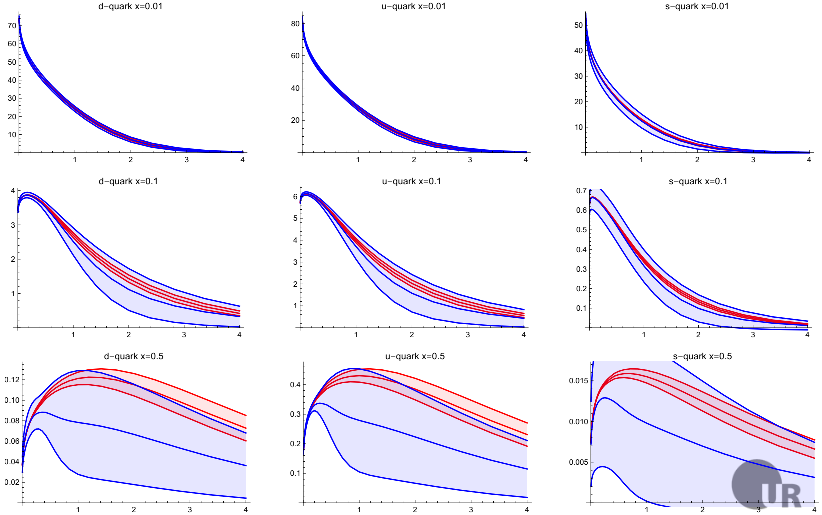
Fit quality essentially depends on the collinear input.

Vary NNPDF within the $1\sigma$ band

$$\chi^2/N_{pt} \in [0.8, 35.]$$

We cannot estimate accurately we the PDF uncertainty.

## The first attempt to estimate a "true" uncertainty band for TMDPDF

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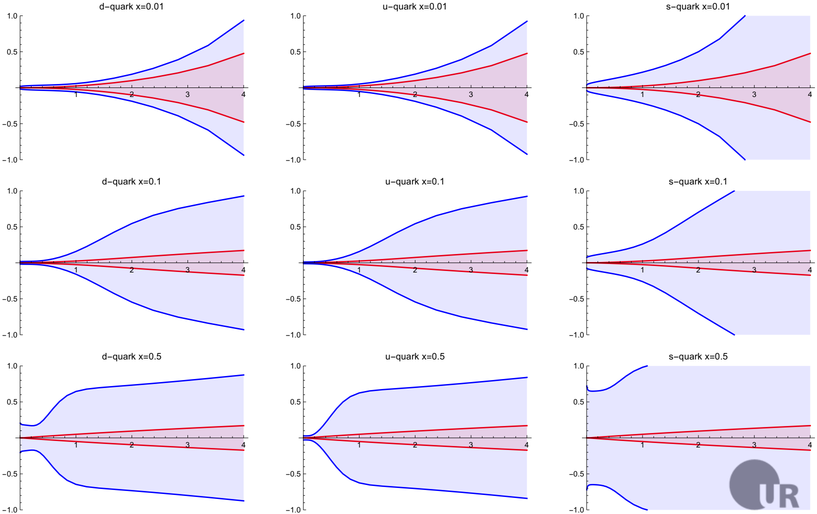
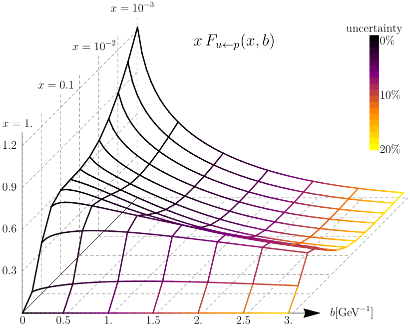
# The first attempt to estimate a “true” uncertainty band for TMDPDF

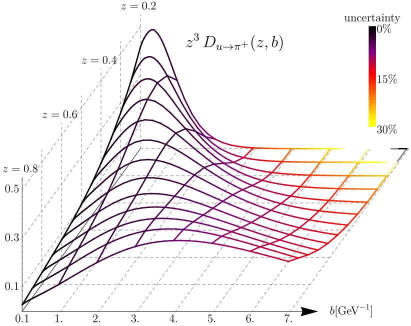


## TMD-distributions do not follow perturbative models

unpolarized TMDPDF




unpolarized TMDFF




... systematic error 10-20%

... evolution does not have this systematics

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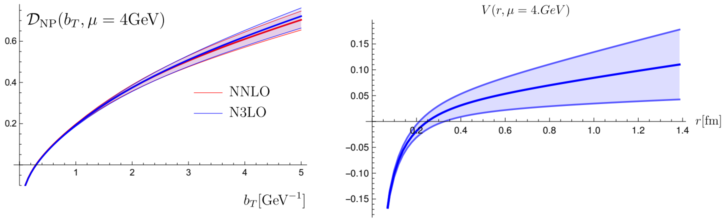
# Example

In stochastic vacuum model




$$V(r)=r\frac{\pi}{4}\mathcal{D}''(0)+\frac{\mathcal{D}'(0)}{2}+\frac{r^2}{2}\int_r^\infty\frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}}$$

QCD string tension: $\sigma=0.014\pm0.01{\rm GeV}^2$ vs. $\sim 0.1-0.2{\rm GeV}^2$

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# Conclusion

TMD distributions are known, and predictions can be generated with it

But there are plenty of unknowns...

- Flavor separation...
- Collinear parts ...
- Power corrections ...
- ...

**artemide**

- Fortran 95
- Python interface (**harpy**)
- Modular structure (from NP-input to cross-section) + LHAPDF interface
- Variety of theory tools and implementations
- Growing functionality

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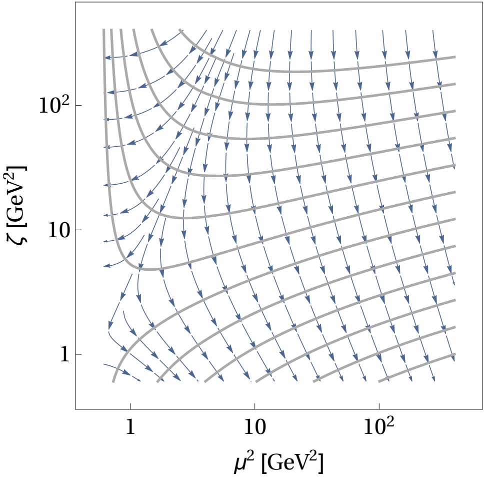
TMD distribution is not defined by a scale $(\mu, \zeta)$
It is defined by an equipotential line.




The scaling is defined by
~~a difference between scales~~
a difference between potentials

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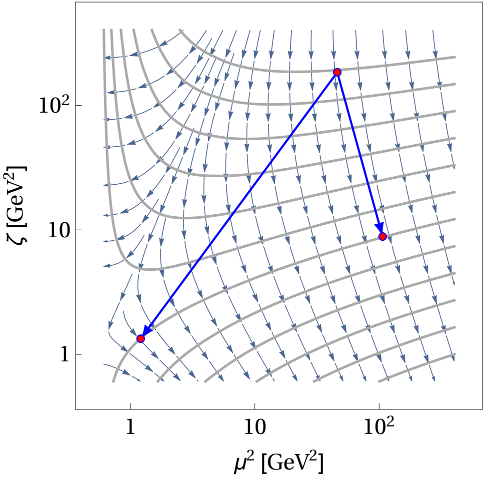
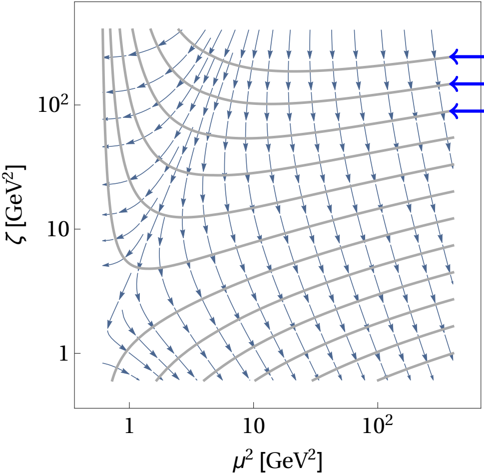



The scaling is defined by
~~a difference between scales~~
a difference between potentials

Evolution factor to both points
is the same
although the scales are
different by $10^2 \text{GeV}^2$

## TMD distributions on the same equipotential line are equivalent.




$\leftarrow$ TMD$(x, b, 1)$

$\leftarrow$ TMD$(x, b, 2)$

$\leftarrow$ TMD$(x, b, 3)$

We can enumerate them by a lines not by $(\mu, \zeta)$

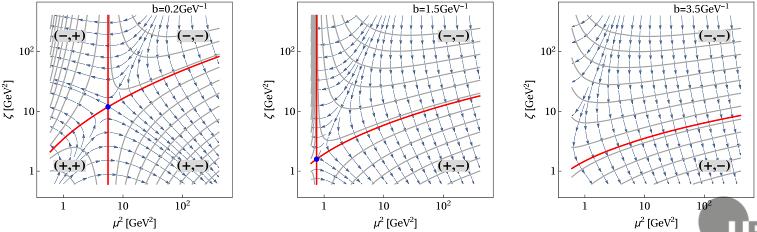
$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

There is a unique line which passes though all $\mu$'s

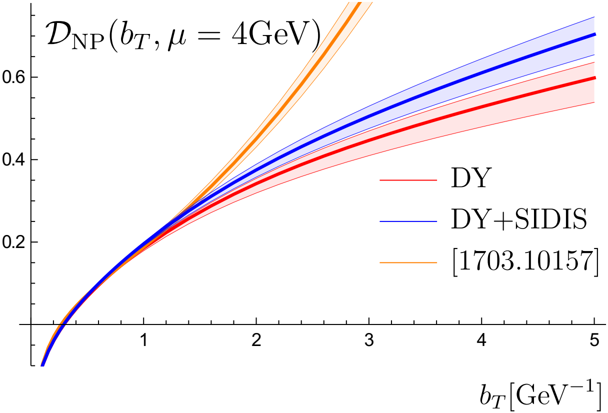
**The optimal TMD distribution**

$$F(x,b) = F(x,b;\mu,\zeta_\mu)$$

where $\zeta_\mu$ is the special line.



## Comparison with other evolutions

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