Collins-Soper kernel: non-perturbative studies

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Challenges of QCD EFTs (QCD.eft 2020)

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This talk is about recent studies of Collins-Soper kernel by non-perturbative methods

CS-kernel = rapidity anomalous dimension (RAD)

$$-\frac{K(\mu,b)}{2} = \mathcal{D}(\mu,b) = -\frac{1}{2}\gamma_\nu(\mu,b) = \frac{1}{2}F_{q\bar{q}}(\mu,b) = ...$$

RAD is an essential part of factorization theorems with TMD

- TMD factorization (DY, SIDIS, $ee \to hhX$)
- $h + \gamma \to \text{jet} + X$ (at small p_T)
- Small-q_T-resummation
- etc... (SCET II)

TMD evolution

$$\zeta\frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(b,\mu)F(x,b;\mu,\zeta) \qquad \Rightarrow \qquad F(x,b;\mu,\zeta_1) = \left(\frac{\zeta_1}{\zeta_2}\right)^{-\mathcal{D}(b,\mu)} F(x,b;\mu,\zeta_2)$$

CSS Resummation formula (1981)

$$\mathcal{S}(Q,b,C_1,C_2) = \int_{C_1^2/b^2}^{C_2^2Q^2}\frac{d\mu^2}{\mu^2}\left[\mathcal{A}(\alpha(\mu),C_1)\ln\left(\frac{C_2^2Q^2}{\mu^2}\right) + \mathcal{B}(\alpha(\mu,C_1,C_2)\right]$$

TMD evolution

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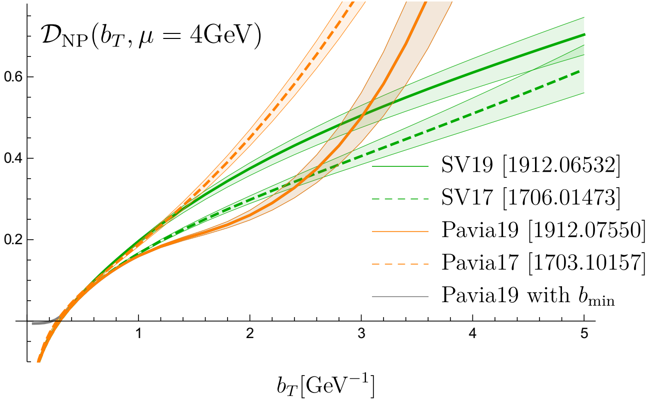
CS-kernel $\mathcal{D}(b,\mu)$ is generally non-perturbative.

BLNY ansatz (BLNY,[0212159])

$$R \to R \times \exp\left[-(\underbrace{g_1 + g_1 g_3 \ln(100 x_1 x_2)}_{\text{TMD distr.}} + \underbrace{g_2 \ln(Q/2Q_0)}_{\text{NP part of CSS}}))b^2\right]$$

Recent extractions

Name	Order	Data set	Model	Comm.
Pavia17	LO	DY+SIDIS	$\mathcal{D}_{\text{pert}}(\mu, b) + g_2 \frac{b^2}{2}$	$b_{\text{min}}^{\text{max}}$-prescription
SV17	NNLO	DY	$\mathcal{D}_{\text{pert}}(\mu, b) + c_0 b^2$	ζ-prescription
SV19	NNLO/N^{3}LO	DY+SIDIS	$\mathcal{D}_{\text{resum}}(\mu, b^*) + c_0 b b^*$	ζ-prescription
Pavia19	NNNLO	DY	$\mathcal{D}_{\text{pert}}(\mu, b) + g_2 \frac{b^2}{4} + g_{2b} \frac{b^4}{4}$	$b_{\text{min}}^{\text{max}}$-prescription



The non-perturbative nature of RAD is undoubted, but it is almost ignored in the theoretical literature.

What is known ...not too much

- Perturbative expression ($b \to 0$) up NNLO (a_s^3) [Li,Zhu,1604.01404],[AV,1610.05791]
- Exact correspondence between RAD and SAD at critical point ($\gamma_s(\mu, v(b)) = 2\mathcal{D}(b, \mu|\epsilon^*)$)[AV,1707.07606]
- Structure of renormalon singularities [Korchemsky,Sterman,9411211],[Tafat,0102237],[Scimemi,AV,1609.06047]

What is interesting to know

- Asymptotic behavior at $b \to \infty$ (b^2, b, const?)
- Asymptotic behavior at $b \to 0$ ($\mathcal{D} \to 0$)
- General size of NP correction
- Any model prediction...
- What is RAD about?
- ...

RAD is a fascinating object to study

- It is measurable with experiment
- It is simple (in comparison to other NP objects, say, PDF)
- It directly tests the properties of QCD vacuum (and only QCD vacuum)

Plan of the talk

- Lattice
- OPE
- NP-modeling

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CS kernel from QCD lattice

There was a set of works on measurement of CS kernel on lattice

- Ji's group [1801.05930][1910.00800][1911.03840] ..most enthusiastic
- MIT group [1811.00026][1901.03685][1910.08569]
- Regensburg^{++} group [1111.4249][1506.07826][2001.in prep] ..most conservative

Lattice restrictions

- Equal-time correlators only
- Very small energies ($P^+ \sim 3$GeV, is an absolute maximum nowadays)
- Not too small distances (lattice artifacts)
- Not too large distances (lattice sizes)

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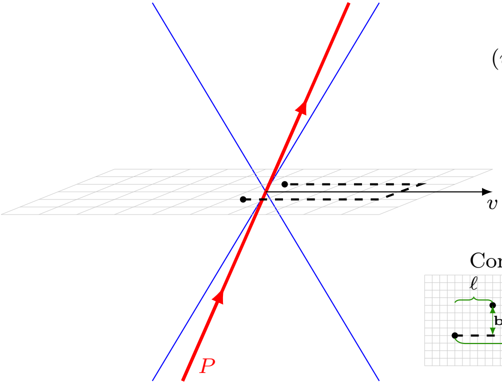
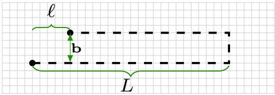
$$\Sigma^{[\Gamma]} = \langle P, S | \bar{q}[\text{staple link}]\Gamma q | P, S \rangle$$



$$(vb) = (bP) = 0$$

$$v^2 < 0$$

Contour parameters



Similarity to a TMD cross-section

At $L \to \infty$ the lattice observable turns into "TMD cross-section"

$$\Sigma_{ij}(\mathbf{b}, \ell v, L, P, S) = \sum_X \langle P, S|J_i(\ell v + b)|X\rangle\langle X|J_j(0)|P, S\rangle$$

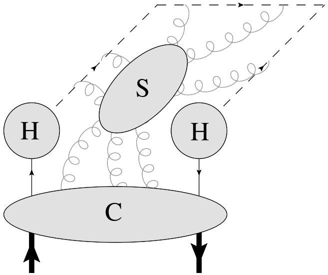
with

$$J_i(x) = H^\dagger(x) q_i(x) \qquad \text{“heavy”-to-light current}$$

$$\mathcal{L}_{HH} = H^\dagger(iv \cdot D)H + \mathcal{O}(L^{-1})$$

- Field H is alike heavy-quark field with $v^2 < 0$ (instant field).

TMD factorization



Using same counting rules as in SCET II we get

$$\Sigma_{ij}(..) = |C_H|^2 \times \Phi_{ij} \times \mathcal{S} + \mathcal{O}\left(\frac{P^-}{P^+}, \frac{b}{L}, \frac{\Lambda}{(vP)}\right)$$

Factorization limit

- Fast-moving hadron (for collinear mode separation)

$$P^\mu = \bar{n}^\mu P^+ + n^\mu \frac{M^2}{2P^+}$$

- "Long" contour (for approximation by "heavy" field)

$$b, \ell \ll L$$

- "Hard" scale (for hard-mode separation)

$$(vP) \gg \Lambda_{\mathrm{QCD}}$$

$$\Sigma_{ij}(..) = |C_H|^2 \times \Phi_{ij} \times \mathcal{S} + \mathcal{O}\left(\frac{P^-}{P^+}, \frac{b}{L}, \frac{\Lambda}{(vP)}\right)$$

Current matching coefficient

$$J_i = C_H(v\mathcal{P})\ (HS)(W\xi_i)$$

At NLO [1811.00026]

$$C_H((v\hat{p}), \mu) = 1 + a_s(\mu)C_F\left(-\frac{1}{2}\ln^2\left(\frac{(v\hat{p})^2}{\mu^2}\right) + \ln\left(\frac{(v\hat{p})^2}{\mu^2}\right) - 2 + \frac{\pi^2}{12}\right) + ...$$

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$$\Sigma_{ij}(..) = |C_H|^2 \times \Phi_{ij} \times \mathcal{S} + \mathcal{O}\left(\frac{P^-}{P^+}, \frac{b}{L}, \frac{\Lambda}{(vP)}\right)$$

TMD distribution

$$\Phi_{ij}(\ell v^-, \mathbf{b}; \mu, \zeta) = \langle P, S|\bar{q}_i W^\dagger(\ell v^- \bar{n} + \mathbf{b}) W^\dagger q_i(0)|P, S\rangle \underbrace{\times \frac{\sqrt{S_{\text{TMD}}}}{\text{Z.b.}_{\text{TMD}}}(\mathbf{b})}_{S^{-1/2}(\mathbf{b},\delta^+)} \times R_{\text{ph.sch.}}(\mathbf{b})$$

Ordinary TMD distribution defined in physical scheme (like in DY or SIDIS)

"vacuum" TMD distribution by instant sources

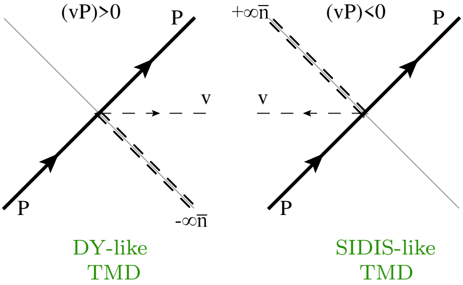
$$\mathcal{S}(\ell v^+, \mathbf{b}; \mu, \zeta) = \langle 0|\frac{\text{Tr}}{N_c} S_n S_v^\dagger(\ell v^+ n + \mathbf{b}) S_v S_n(0)|0\rangle \times \underbrace{\frac{1}{\text{Z.b}}(\mathbf{b}) \times \frac{\text{Z.b.}_{\text{TMD}}}{\sqrt{S_{\text{TMD}}}}(\mathbf{b})}_{\sim S^{-1/2}(\mathbf{b},\delta^+)} \times R^{-1}_{\text{ph.sch.}}(\mathbf{b})$$

TMD-like distribution of vacuum together with remnants of physical scheme definition

- **Note,** the dependence on lv^+ due to SCET II counting.
- It is not an equal-time correlator.
- The factor $R_{\text{ph.sch.}}$ is 1 in perturbation theory, but is generally non-trivial.

Note on the direction of Wilson contours

- Direction of W does depend on sign of (vP)
- C_H depends only on $(vP)^2$



What can be measured on the lattice?

- Enthusiastic picture: Everything! ... Ji's group
- Realistic picture: Ratios

Let's prepare the ratios such that unknown lattice-related content vanishes

$$\frac{\Sigma^{[\Gamma_1]}(\mathbf{b}, \ell, L, v, P_1, S_1, hadron, flavor, a)}{\Sigma^{[\Gamma_2]}(\mathbf{b}, \ell, L, v, P_2, S_2, hadron, flavor, a)} =$$

$$\frac{|C_H(vP_1)|^2}{|C_H(vP_2)|^2} \frac{\Phi^{[\Gamma_1]}(\ell v^-, \mathbf{b}, P_1^+)}{\Phi^{[\Gamma_2]}(\ell v^-, \mathbf{b}, P_2^+)} \frac{\cancel{\mathcal{S}(\ell v^+, \mathbf{b})}}{\cancel{\mathcal{S}(\ell v^+, \mathbf{b})}} \frac{\cancel{Z_{\text{lat.}}(a, L, \ell/L)}}{\cancel{Z_{\text{lat.}}(a, L, \ell/L)}} \frac{1 + ...}{1 + ...}$$

What can be measured on the lattice?

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$$\frac{\Sigma^{[\Gamma_1]}(\mathbf{b},\ell,L,v,P_1,S_1,hadron,flavor,a)}{\Sigma^{[\Gamma_2]}(\mathbf{b},\ell,L,v,P_2,S_2,hadron,flavor,a)}=$$

$$\frac{|C_H(vP_1)|^2}{|C_H(vP_2)|^2}\frac{\Phi^{[\Gamma_1]}(\ell v^-,\mathbf{b},P_1^+)}{\Phi^{[\Gamma_2]}(\ell v^-,\mathbf{b},P_2^+)}\frac{\cancel{\mathcal{S}(\ell v^+,\mathbf{b})}}{\cancel{\mathcal{S}(\ell v^+,\mathbf{b})}}\frac{\cancel{Z_{\text{lat.}}(a,L,\ell/L)}}{\cancel{Z_{\text{lat.}}(a,L,\ell/L)}}\frac{1+...}{1+...}$$

Ideally, one sets $\ell=0$ then Φ is the first Mellin moment of ordinary TMD distribution

$$\Phi^{[\Gamma]}(\ell=0,b;\mu,\zeta)=\int_{-1}^{1}dx\Phi^{[\Gamma]}(x,b;\mu,\zeta)$$

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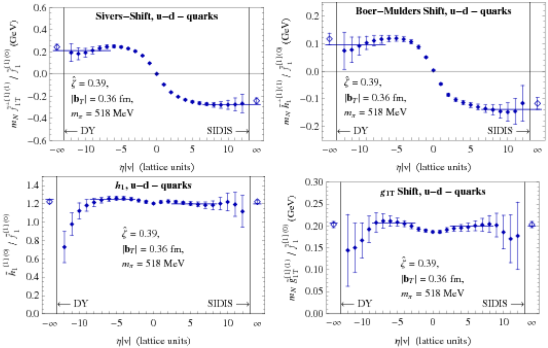
Some checks 1:

Dependence on the sign of (vP) is trivial

- Sivers and Boer-Mulders are sign-change
- Rest are not sign-change
- Magnitude is the same (due to $C_H((vP)^2)$)

$$\frac{C_H(vP)\Phi^{[\Gamma_1]}_{S_1}(b,P^+)}{C_H(vP)\Phi^{[\gamma^+]}_{S_2=0}(b,P^+)}$$

[Musch,et al,1111.4249]



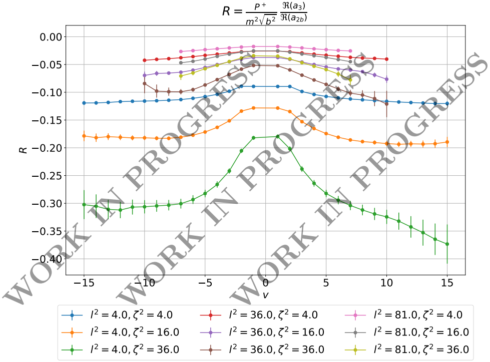
Some checks 2:

Test of power suppression

- $\Sigma^{[\Gamma]} = C_H \times \Phi^{[\Gamma']} \times \mathcal{S}$
- $\Gamma' = \frac{1}{4}\gamma^+\gamma^-\Gamma\gamma^-\gamma^+$
- Non-twist-2 Γ's should be power suppressed

$$\frac{\Phi^{[\Gamma_1]}(b, P^+)}{\Phi^{[\gamma^+]}(b, P^+)}$$

[Regensburg work in progress]



Measurement of Rapidity anomalous dimension

Scales in the factorization formula

$$\Sigma^{[\Gamma]}(...) = |C_H((vp),\mu)|^2 \Phi^{[\Gamma']}(b, P^+; \mu, \zeta_1) \mathcal{S}(b, v; \mu, \zeta_2) \times ...$$

$$\zeta_1 \zeta_2 = \left(\frac{P^+}{v^+}\right)^2 \frac{v^2}{b^2}, \qquad \mu^2 \sim (vp)^2$$

Measurement of Rapidity anomalous dimension

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$$\frac{\Sigma^{[\Gamma]}(b, P_1, ...)}{\Sigma^{[\Gamma]}(b, P_2, ...)} = \frac{|C_H((vP_1), \mu)|^2}{|C_H((vP_2), \mu)|^2} \left(\frac{P_1^+}{P_2^+}\right)^{-2\mathcal{D}(b,\mu)}$$

$$\frac{\Sigma^{\text{Sivers}}(b, P_1, ...)}{\Sigma^{\text{Sivers}}(b, P_2, ...)} = \frac{|C_H((vP_1), \mu)|^2}{|C_H((vP_2), \mu)|^2} \left(\frac{P_1^+}{P_2^+}\right)^{-2\mathcal{D}(b,\mu)}$$

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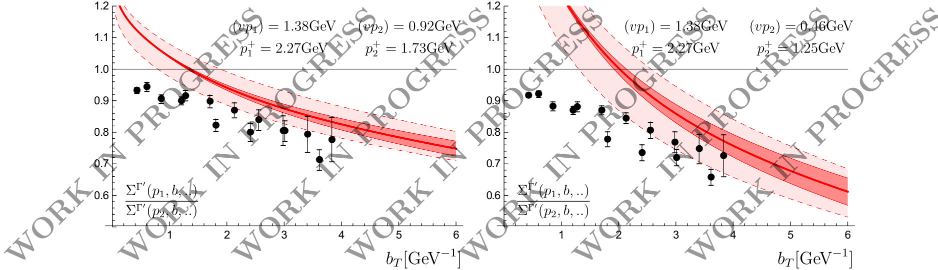

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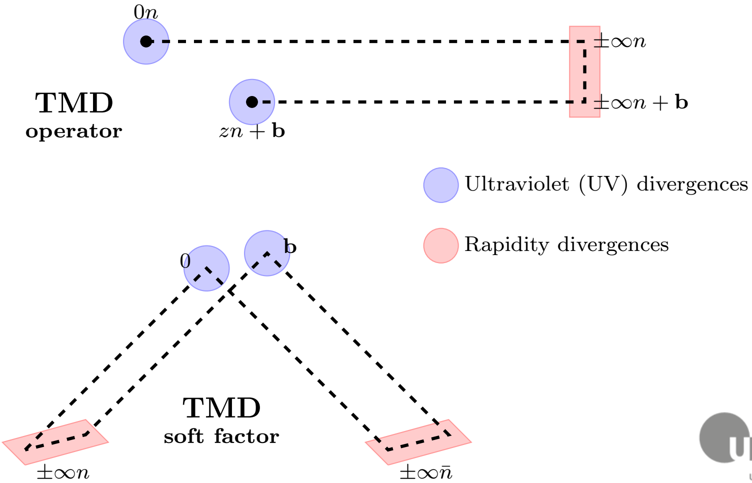
Operator Product Expansion

Rapidity divergences are properties of the operator
Rapidity divergences = UV divergences at the boundary
[AV,1707.07606]



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Remainder: TMD soft factor

$$S(b^2) = \frac{1}{N_c}\langle 0|\mathrm{Tr} P \exp\left(ig\int_C ds^\mu A_\mu(s)\right)|0\rangle,$$

where the contour C is build of 4 segments

$$C: \qquad s^\mu = \begin{cases} C_1 = & n^\mu \sigma, & \sigma \in [0,-\infty], \\ C_2 = & n^\mu \sigma + b^\mu, & \sigma \in [-\infty, 0], \\ C_3 = & \bar{n}^\mu \sigma + b^\mu, & \sigma \in [0,-\infty], \\ C_4 = & \bar{n}^\mu \sigma, & \sigma \in [-\infty, 0]. \end{cases}$$

In any rapidity divergence regularization the RAD is defined as

$$S(\mathbf{b}^2) = \exp\left(A \ln(\text{rap.div}) + B\right),$$

$$\mathcal{D}(\mathbf{b}^2) \sim A = \frac{d \ln S(\mathbf{b})}{d \ln(\text{rap.div.})}.$$

$$\mathcal{D}(\mathbf{b}^2) = \frac{1}{2}\frac{d\ln S}{d\ln \tau^2},$$

$$\tau^2 = \begin{cases} \tau_\delta^2 = 2\delta^+\delta^-, & \delta - \text{regularization}, \\ \tau_b^2 = 2/(b^+b^-), & \text{exponential regularization}, \\ \tau_\Lambda^2 = \frac{2}{\Lambda_+\Lambda_-}, & \text{regularization by cut}, \\ \ldots \quad \ldots \end{cases}$$

All regulators have some bad property

- δ-regularization ⇒ violates gauge symmetry
- exponential regularization ⇒ introduces IR divergences
- analytical regularization ⇒ not-applicable on operator level + IR divergences
- regularization by cut ⇒ difficult integrals

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- δ-regularization ⇒ violates gauge symmetry
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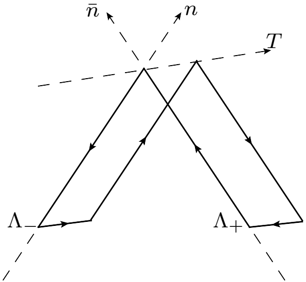
- analytical regularization ⇒ not-applicable on operator level + IR divergences
- regularization by cut ⇒ difficult integrals

Regularization by cut is the best for non-perturbative studies!

Regularization by cut

$$S(b^2, 2\Lambda^+\Lambda^-) = \frac{1}{N_c}\langle 0|\mathrm{Tr}P\exp\left(ig\int_C ds^\mu A_\mu(s)\right)|0\rangle,$$



$$C^\mu = \begin{cases} C_{1\Lambda} = & n^\mu\sigma, & \sigma\in[0,-\Lambda_+], \\ C_{T\Lambda} = & -n^\mu\Lambda_+ + b^\mu\sigma, & \sigma\in[0,1], \\ C_{2\Lambda} = & n^\mu\sigma + b^\mu, & \sigma\in[-\Lambda_+,0], \\ C_{3\Lambda} = & \bar{n}^\mu\sigma + b^\mu, & \sigma\in[0,-\Lambda_-], \\ C_{\bar{T}\Lambda} = & -\bar{n}^\mu\Lambda_- + b^\mu\sigma, & \sigma\in[1,0], \\ C_{4\Lambda} = & \bar{n}^\mu\sigma, & \sigma\in[-\Lambda_-,0]. \end{cases}$$

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Small-b OPE

- Background computation
- Fock-Schwinger gauge

$$[...]_C \quad = \quad 1 + \mathcal{O}(b)$$

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Small-b OPE

- Background computation
- Fock-Schwinger gauge

$$[...]_C \quad = \quad 1 - ig b^\mu \Big\{ \int_{-\Lambda_+}^{0} d\sigma F_{\mu+}(n\sigma) + \int_0^{-\Lambda_-} d\sigma F_{\mu-}(\bar{n}\sigma) \Big\} + \mathcal{O}(b^2)$$

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Small-b OPE

- Background computation
- Fock-Schwinger gauge

$$
\begin{aligned}
[\ldots]_C \quad = \quad & 1 - igb^\mu \Big\{ \int_{-\Lambda_+}^{0} d\sigma F_{\mu+}(n\sigma) + \int_0^{-\Lambda_-} d\sigma F_{\mu-}(\bar{n}\sigma) \Big\} \\
& - ig\frac{b^\mu b^\nu}{2} \Big\{ \int_{-\Lambda_+}^{0} d\sigma \mathcal{D}_\nu^{\text{adj}} F_{\mu+}(n\sigma) + \int_0^{-\Lambda_-} d\sigma \mathcal{D}_\nu^{\text{adj}} F_{\mu-}(\bar{n}\sigma) \Big\} \\
& - g^2 b^\mu b^\nu \Big\{ \int_{-\Lambda_+}^{0} d\sigma \int_\sigma^0 d\tau F_{\mu+}(n\sigma) F_{\nu+}(n\tau) \\
& + \int_{-\Lambda_+} d\sigma \int_0^{-\Lambda_-} d\tau F_{\mu+}(n\sigma) F_{\nu-}(\bar{n}\tau) + \int_0^{-\Lambda_-} d\sigma \int_\sigma^{-\Lambda_-} d\tau F_{\mu-}(\bar{n}\sigma) F_{\nu-}(\bar{n}\tau) \Big\} \\
& + \mathcal{O}(b^3)
\end{aligned}
$$

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Small-b OPE

- Background computation
- Fock-Schwinger gauge

$$\langle 0|[\ldots]_C|0\rangle = \langle 0|1 - igb^\mu\Big\{\int_{-\Lambda_+}^{0} d\sigma F_{\mu+}(n\sigma) + \int_0^{-\Lambda_-} d\sigma F_{\mu-}(\bar{n}\sigma)\Big\}$$

$$-ig\frac{b^\mu b^\nu}{2}\Big\{\int_{-\Lambda_+}^{0} d\sigma \mathcal{D}_\nu^{\text{adj}} F_{\mu+}(n\sigma) + \int_0^{-\Lambda_-} d\sigma \mathcal{D}_\nu^{\text{adj}} F_{\mu-}(\bar{n}\sigma)\Big\}$$

$$-g^2 b^\mu b^\nu\Big\{\int_{-\Lambda_+}^{0} d\sigma \int_\sigma^0 d\tau F_{\mu+}(n\sigma)F_{\nu+}(n\tau)$$

$$+\int_{-\Lambda_+} d\sigma \int_0^{-\Lambda_-} d\tau F_{\mu+}(n\sigma)F_{\nu-}(\bar{n}\tau) + \int_0^{-\Lambda_-} d\sigma \int_\sigma^{-\Lambda_-} d\tau F_{\mu-}(\bar{n}\sigma)F_{\nu-}(\bar{n}\tau)\Big\}$$

$$+\mathcal{O}(b^3)|0\rangle$$

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Small-b OPE

- Background computation
- Fock-Schwinger gauge

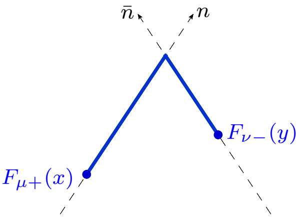
$$
\begin{aligned}
\langle 0|[...]_C|0\rangle \quad = \quad & \langle 0|1 - igb^\mu\Big\{\int_{-\Lambda_+}^{0} d\sigma F_{\mu+}(n\sigma) + \int_0^{-\Lambda_-} d\sigma F_{\mu-}(\bar{n}\sigma)\Big\} \\
& -ig\frac{b^\mu b^\nu}{2}\Big\{\int_{-\Lambda_+}^{0} d\sigma \mathcal{D}_\nu^{\text{adj}} F_{\mu+}(n\sigma) + \int_0^{-\Lambda_-} d\sigma \mathcal{D}_\nu^{\text{adj}} F_{\mu-}(\bar{n}\sigma)\Big\} \\
& -g^2 b^\mu b^\nu \Big\{\int_{-\Lambda_+}^{0} d\sigma \int_\sigma^0 d\tau F_{\mu+}(n\sigma)F_{\nu+}(n\tau) \\
& + \int_{-\Lambda_+} d\sigma \int_0^{-\Lambda_-} d\tau F_{\mu+}(n\sigma)F_{\nu-}(\bar{n}\tau) + \int_0^{-\Lambda_-} d\sigma \int_\sigma^{-\Lambda_-} d\tau F_{\mu-}(\bar{n}\sigma)F_{\nu-}(\bar{n}\tau)\Big\} \\
& +\mathcal{O}(b^3)|0\rangle
\end{aligned}
$$

$$
S \quad = \quad 1 - g^2 \int_{-\Lambda_+}^{0} d\sigma \int_0^{-\Lambda_-} d\tau \langle 0|\frac{\text{Tr}}{N_c} F_{b+}(\sigma n)F_{b-}(\tau\bar{n})|0\rangle + \mathcal{O}(b^4).
$$

Power correction to $\mathcal{D}$ are expressed by multi-gluon-correlates connected by "minimal distance links"

$$\langle 0|F_{x_1\mu_1}(x_1)F_{x_2\mu_2}(x_2)...F_{x_n\mu_n}(x_n)|0\rangle$$

At LO $x_1^2 = x_2^2 = ... = x_n^2 = 0$
At higher-than-LO $x_1^2 \neq x_2^2 \neq ... \neq x_n^2 \neq 0$



b^2-operator

$$\Phi^{\mu\nu}(x,y) = g^2\frac{\text{Tr}}{N_c}\langle 0|F_{x\mu}(x)[x,0][0,y]F_{y\nu}(y)|0\rangle$$

This correlator is parametrized by 2 structures

$$\Phi^{\mu\nu}(x,y) = \left(g^{\mu\nu} - \frac{y^\mu x^\nu}{(xy)}\right)\varphi_1(x,y) + \frac{((xy)x^\mu - x^2y^\mu)((xy)y^\nu - y^2x^\nu)}{(xy)((xy)^2 - x^2y^2)}\varphi_2(x,y).$$

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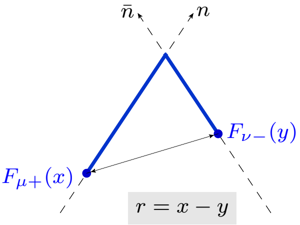
LO simplifications



$$x^2 = y^2 = 0$$

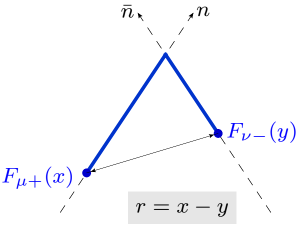
$$\varphi_2(x,y) = 0$$

$$\varphi_1(x,y) = \varphi_1(-r^2)$$

LO b^2 expression

$$S = 1 + b^2 \int_{-2\Lambda_+\Lambda_-}^{0} dr^2 \frac{\varphi(-r^2)}{r^2} \ln\left(\frac{-r^2}{2\Lambda_+ 2\Lambda_-}\right)$$

LO simplifications



$x^2 = y^2 = 0$

$$\varphi_2(x,y) = 0$$

$$\varphi_1(x,y) = \varphi_1(-r^2)$$

LO b^2 expression

$$S = 1 + b^2 \int_{-\infty}^{0} dr^2 \frac{\varphi(-r^2)}{r^2} \ln\left(\frac{-r^2}{2\Lambda_+ 2\Lambda_-}\right)$$

Since $\varphi(r^2) \to 0$ at $r^2 \to \infty$ the limit $\Lambda \to \infty$ is smooth

$$\mathcal{D}(\mathbf{b}) = \frac{\mathbf{b}^2}{2} \int_0^\infty \frac{d\mathbf{r}^2}{\mathbf{r}^2} \varphi_1(\mathbf{r}^2) + \mathcal{O}(a_s, \mathbf{b}^4), \qquad \text{here } \mathbf{r}^2 = -r^2 > 0.$$

Power correction to $\mathcal{D}$

$$\mathcal{D}(\mathbf{b}) = \underbrace{\mathcal{D}_{\text{pert}}(\ln(\mu^2\mathbf{b}^2))}_{\text{known at N}^3\text{LO}} + \mathbf{b}^2 \underbrace{\mathcal{D}_1(\ln(\mu^2\mathbf{b}^2))}_{\text{LO}=\frac{1}{2}\int\frac{\varphi(r^2)}{r^2}dr^2} + \mathbf{b}^4\mathcal{D}_2(\ln(\mu^2\mathbf{b}^2)) + ...$$

Estimation

- At $\mathbf{r}^2 \to 0$, $\varphi_1 \sim \mathbf{r}^2\frac{\pi^2}{36}G_2$
- At $\mathbf{r}^2 \to \infty$, $\varphi_1 \sim \frac{1}{\mathbf{r}^2}$ (at least), $\varphi_1 \sim e^{-2m_\pi r}$(more realistic)

So, order-of-magnitude estimation

$$\mathcal{D}_1 \lesssim \frac{\pi^2}{72}\frac{G_2}{\Lambda^2_{\text{QCD}}} \sim \frac{\pi^2}{72}\frac{G_2}{m_\pi^2} \sim (1. - 6.) \times 10^{-2}\text{GeV}^2$$

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Extracted values ($\times 10^{-2}\text{GeV}^2$)

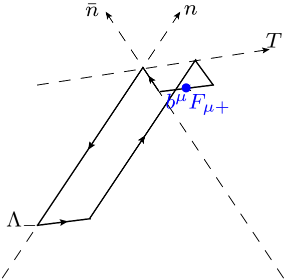
SV17	SV19	Pavia17	Pavia19	BLNY (2001)
0.6-0.7$\pm$ 0.24	2.4-3.2$\pm$0.7	6.5$\pm$0.5	0.9$\pm$0.2	54.-68.

The power correction is relatively small, that is confirmed by recent extractions.

Non-perturbative models

Rapidity anomalous dimension can be defined as a primary object.

- The contour-variation can be made under the sign of functional integration (Mandelstam formula)
- Route to non-perturbative calculation and modeling.



$$\mathcal{D}(b) = \frac{1}{2} \lim_{\Lambda_\pm \to \infty} \frac{S'(b;\Lambda_+\Lambda_-)}{S(b;\Lambda_+\Lambda_-)} + \frac{Z_{\text{cusp}}}{2}$$

$$S'(b) = ig \int_0^1 d\beta \frac{\text{Tr}}{N_c} \langle 0| F_{b+}(-\Lambda_+ b + \beta b) P \exp\left(-ig \int_{C'} dx^\mu A_\mu(x)\right) |0\rangle$$

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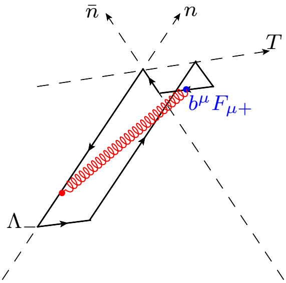
Perturbation theory



Elementary calculation gives

$$\mathcal{D}(b,\mu) = -2a_s(\mu)C_F\Big[\Big(\frac{\mathbf{b}^2}{4}\Big)^{\epsilon}\Gamma(-\epsilon)+\frac{1}{\epsilon}\Big]$$

Coincides with "traditional" LO calculation

- Also checked N_f and $C_F(=0)$ parts of NLO.

Leading term of OPE

Simple calculation

$$\begin{aligned} igF_{b+}W_C &= ig\,b^\mu F_{\mu+}(-\Lambda_+ n) + igb^\mu b^\nu\,\mathcal{D}_\nu^{\text{adj}}F_{\mu+}(-\Lambda_+ n) \\ &+g^2b^\mu b^\nu F_{\mu+}(-\Lambda_+ n)\Big\{\int_{-\Lambda_+}^0 d\sigma F_{\nu+}(n\sigma)+\int_0^{-\Lambda_-}d\sigma F_{\nu-}(\bar{n}\sigma)\Big\}+\mathcal{O}(b^3)\end{aligned}$$

yields the same result

$$\mathcal{D}(\mathbf{b}^2) = \frac{\mathbf{b}^2}{2}\int_0^\infty d\mathbf{r}^2\,\frac{\varphi_1(\mathbf{r}^2)}{\mathbf{r}^2}$$

The operator definition can be used to compute RAD in models of QCD vacuum

Example: **stochastic vacuum model**

- The statement of the model 1: correlators of $F_{\mu\nu}$ are independent on gauge links
- The statement of the model 2: correlators of many $F_{\mu\nu}$ are dominated by pair-correlators
- It trivialize the space of functions: Only two 2-point functions

$$\langle 0|F_{\mu\nu}(0)F_{\rho\lambda}(z)|0\rangle = \Big\{(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})\left(\Delta(z^2) + \Delta_1(z^2)\right)$$
$$+ \left[g_{\mu\lambda}z_\nu z_\rho - g_{\nu\lambda}z_\mu z_\rho - g_{\mu\rho}z_\nu z_\lambda + g_{\nu\rho}z_\mu z_\lambda\right] \frac{\partial\Delta_1(z^2)}{\partial z^2}\Big\}$$

- Allows to relate different observables
- Trivially support confinement. (via non-Abelian Stockes theorem $\rightarrow$ are law)

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Example: **stochastic vacuum model**: large-b behavior

Substituting the expression for RAD we get

$$\mathcal{D} \;=\; \int_0^{\mathbf{b}^2} d\mathbf{y}^2 \left(\frac{\sqrt{\mathbf{b}^2}}{\sqrt{\mathbf{y}^2}} - 1\right) \left\{ -\mathbf{y}^2\Delta_1(\mathbf{y}^2) + \int_0^\infty d\mathbf{r}^2 \left(\Delta(\mathbf{r}^2+\mathbf{y}^2) + \frac{\Delta_1(\mathbf{r}^2+\mathbf{y}^2)}{2}\right) \right\}.$$

It gives

$$\mathcal{D}(b\to\infty) \sim 2\sqrt{\mathbf{b}^2}\int_0^\infty d\mathbf{y}^2 \sqrt{\mathbf{y}^2}\Delta(\mathbf{y}) + \text{const.} + \text{supressed terms}$$

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Estimation of parameters for Δ from lattice [Bali, et al,05;Dosch,et al,07;Smirnov,18] gives

$$\mathcal{D}(\mathbf{b} \to \infty) \sim (0.01 - 0.1\text{GeV})\sqrt{\mathbf{b}^2}$$

Compare to SV19

$$\mathcal{D}(\mathbf{b} \to \infty) = c_0 B_{NP} \sqrt{\mathbf{b}^2} = (0.08 \pm 0.01\text{GeV})\sqrt{\mathbf{b}^2}$$

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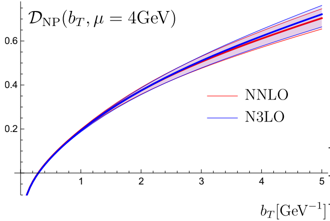
Example: **stochastic vacuum model**: Relation to static potential

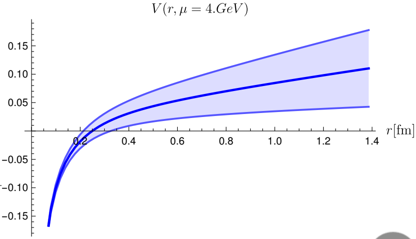
In terms of SVM the static potential reads [Brambilla,et al,9606344]

$$V(\mathbf{b}) = 2\int_0^{\mathbf{b}} d\mathbf{y}(\mathbf{b}-\mathbf{y})\int_0^{\infty} d\mathbf{r}\Delta(\sqrt{\mathbf{r}^2+\mathbf{y}^2}) + \int_0^{\mathbf{b}} d\mathbf{y}\mathbf{y}\int_0^{\infty} d\mathbf{r}\Delta_1(\sqrt{\mathbf{r}^2+\mathbf{y}^2}).$$

Neglecting Δ_1-part (numerically small)

$$V_\Delta(\mathbf{b}) = \mathbf{b}\frac{\pi}{4}\mathcal{D}''_\Delta(0) + \frac{\mathcal{D}'_\Delta(0)}{2} + \frac{\mathbf{b}^2}{2}\int_{\mathbf{b}}^{\infty} d\mathbf{x}\frac{\mathcal{D}'_\Delta(\mathbf{x})}{\mathbf{x}^2\sqrt{\mathbf{x}^2-\mathbf{b}^2}}.$$





String-tension from SV19 $\sigma = (0.06 \pm 0.01)\text{GeV}^2$ vs. $(0.2 \pm 0.02)\text{GeV}^2$ [lattice,96-98]

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Conclusion

RAD (CS-kernel) is a non-perturbative function
It is a unique continues function that
1) can be measured on experiment
2) depends entirely on properties of QCD vacuum

I have demonstrated

- Lattice approach to RAD
- Evaluation of a power correction within OPE (model independent)
- Non-perturbative definition of RAD
- Some checks and comparisons with models
- All in progress...

Some lessons

- RAD is a “slow” function (typical scales are “vacuum like”)
- Power correction $\sim (0.01 - 0.06)b^2$
- Asymptotic behavior $\sim \sqrt{\mathbf{b}^2}$