Unpolarized TMD distributions and their evolution from DY and SIDIS data

Alexey Vladimirov

Regensburg University

Introduction

I am presenting a (global) fit of the DY and SIDIS data and extraction non-perturbative functions using TMD-factorization approach

Theory details

- "CSS-like": same equations, but different solution – ζ-prescription
- Optimal TMD distribution as the boundary
- NNLO (or N^{3}LO) evolution+coefficient functions.
- NNLO matching to collinear PDF at $b \to 0$

`artemide`

- Complete freedom in definition of NP-input
- Variety of implementation of TMD evolution (“classic” CSS, γ-improved, resummed,...)
- Tools for theoretical studies (variation bands, cuts, ...)
- Modular structure
- FORTRAN 95 + python interface (**`harpy`**)
- "Mostly" documented

github.com/VladimirovAlexey/artemide-public

TMD factorization formula in ζ-prescription

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T} = \sigma_0 \sum_{ff'} \int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'}\left(\frac{Q}{\mu}\right) \left(\frac{Q^2}{\zeta_Q}\right)^{-2\mathcal{D}(Q,b)} F_{f\leftarrow h}(x_1,b) F_{f'\leftarrow h}(x_2,b)$$

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N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Rapidity anomalous dimension (universal)

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T} = \sigma_0 \sum_{ff'} \int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'}\left(\frac{Q}{\mu}\right) \left(\frac{Q^2}{\zeta_Q}\right)^{-2\mathcal{D}(Q,b)} F_{f\leftarrow h}(x_1,b) F_{f'\leftarrow h}(x_2,b)$$

- TMD distributions and evolution are independent
- ζ_Q is the (known) function of $\mathcal{D}$
- TMD distributions could be (or not) matched to collinear PDFs – it is a part of the modeling
- Q-dynamics split from q_T-dynamics

TMD factorization formula in ζ-prescription

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

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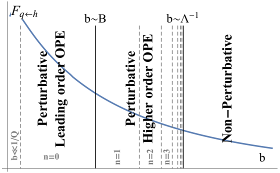
N^3LO (universal)

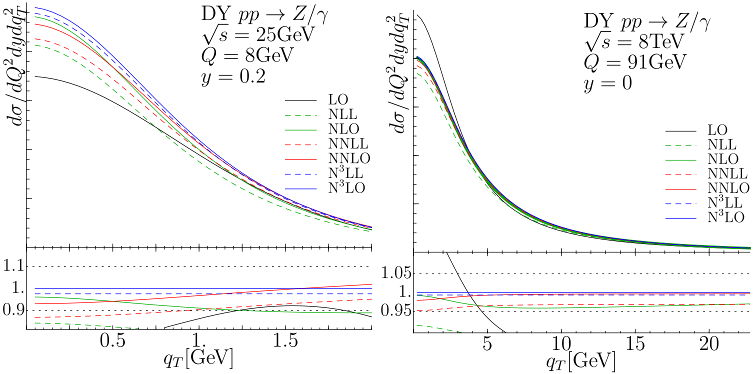
small-b N^3LO

small-b NNLO, NLO, LO depending on TMD



Small-b $\leftrightarrow$ large q_T
At small-b everything is perturbative
turns to resummation

All scales are at $\mu \sim Q$
Perturbative series is saturated



Difference between NNLO and N^{3}LO is marginal

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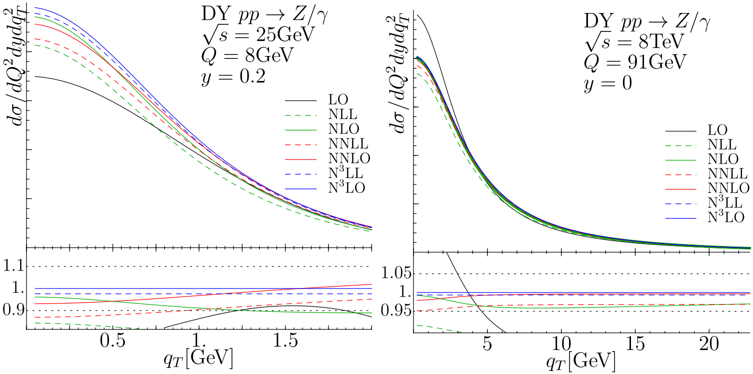
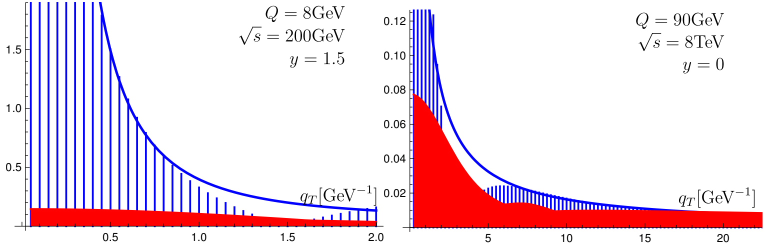


Difference between NNLO and N^{3}LO is marginal

All that follows is done in
NNLO or N^{3}LO of TMD factorization
+ NNLO matching of TMD model to collinear PDF

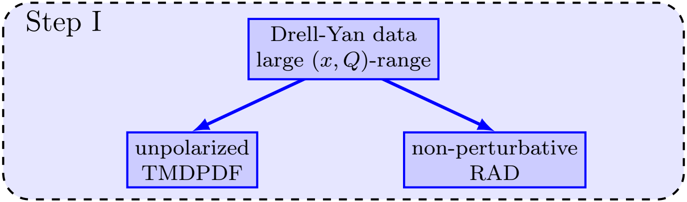
How large is non-perturbative component?

DY(resummation)/DY(TMD)

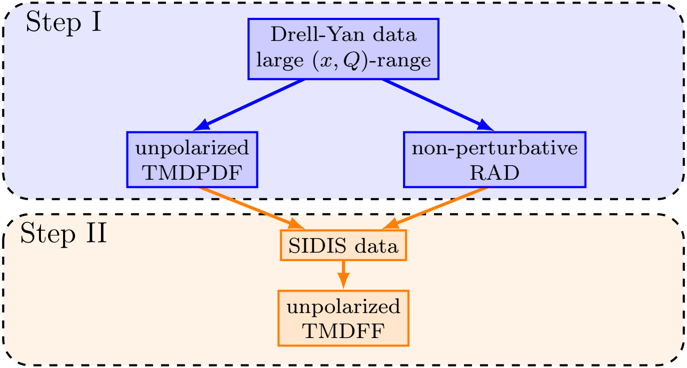


At $q_T \lesssim 10\text{GeV}$ non-perturbative corrections are essential and are to be extracted from the experimental data

Fit strategy and the test of universality



Fit strategy and the test of universality



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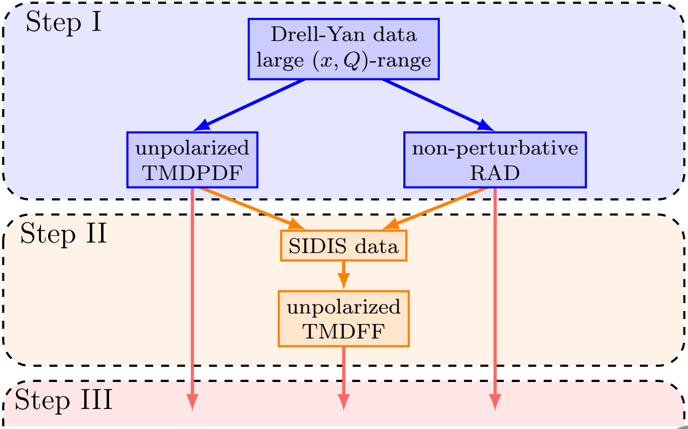
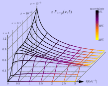


Check of universality

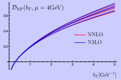
Step I

Low-energy: E228, E605,... PHENIX → **Drell-Yan fit** $\chi^2/N_{pt} = 1.2$ ← High-energy: ATLAS, LHCb,... CDF, D0

uTMDPDF (5 params +PDF)



RAD (2 params)



[V.Bertone,I.Scimemi,AV;1902.08474]

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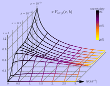
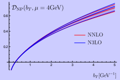


Drell-Yan fit → RAD (2 params)

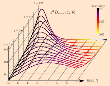


[V.Bertone,I.Scimemi,AV;1902.08474]

Step II

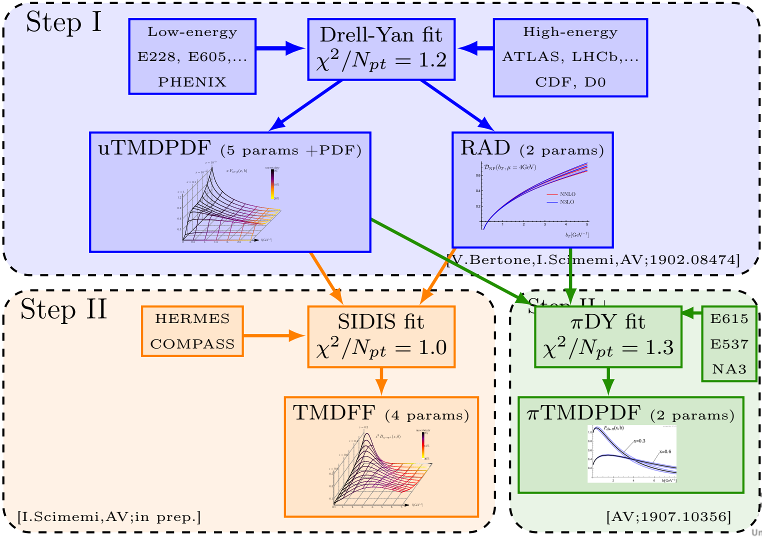
uTMDPDF, RAD, HERMES COMPASS → **SIDIS fit** $\chi^2/N_{pt} = 1.0$

SIDIS fit → TMDFF (4 params)

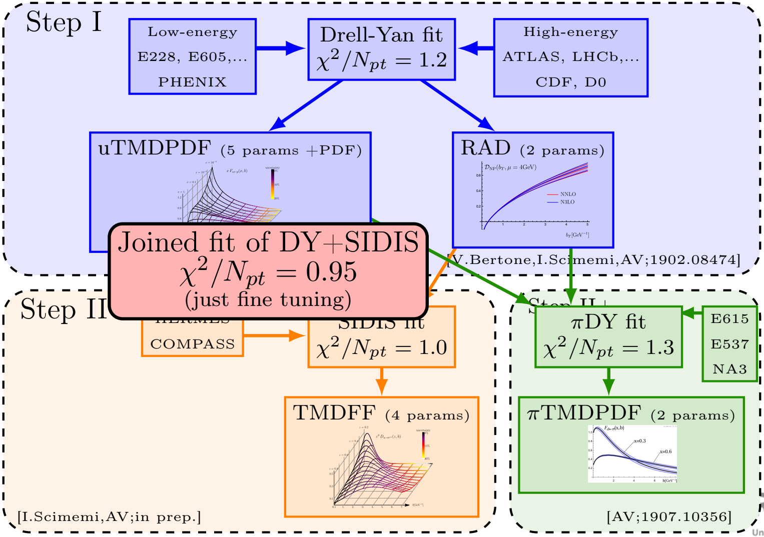


[I.Scimemi,AV;in prep.]

Check of universality



Check of universality



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There are plenty of details/questions

- How to cut the data?
- Where is the limit of TMD factorization?
- Power corrections:
  - Induced (ATLAS, LHCb) (linear in q_T)
  - Kinematic
  - In the definition of collinear frame
  - Target mass and produced mass
- Universality and correlations
- ... many others ...
- **What do we learn from it?**

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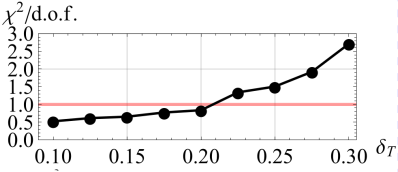
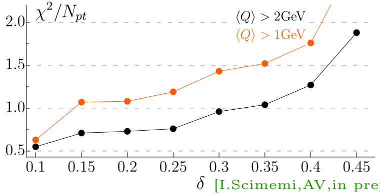
Test by inclusion of the data with
$q_T < \delta \cdot Q$

unpolarized DY



[I.Scimemi,AV,1706.01473]

unpolarized SIDIS



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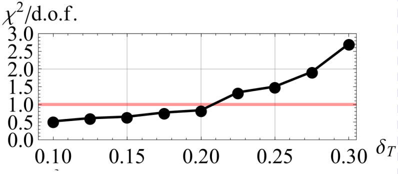
Test by inclusion of the data with

$$q_T < \delta \cdot Q$$

unpolarized DY



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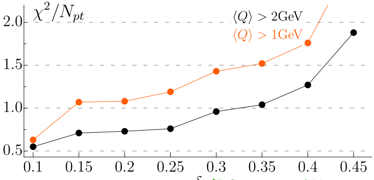
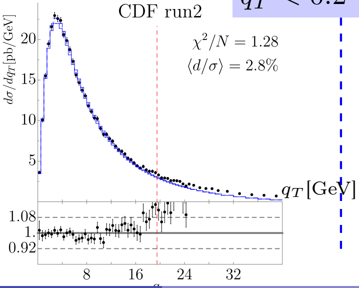
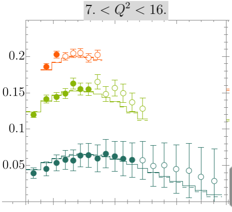
unpolarized SIDIS



[I.Scimemi,AV,in prep.]

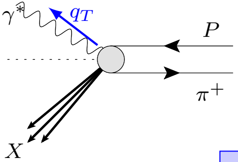
$$q_T < 0.2 - 0.25Q$$



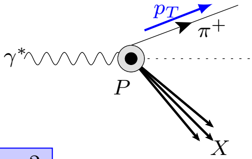


Proof of factorization is done in the Breit frame

Breit frame



Lab frame



$$q_T^2 = \frac{p_T^2}{z^2}\frac{1+\gamma^2}{1-\varsigma^2}$$

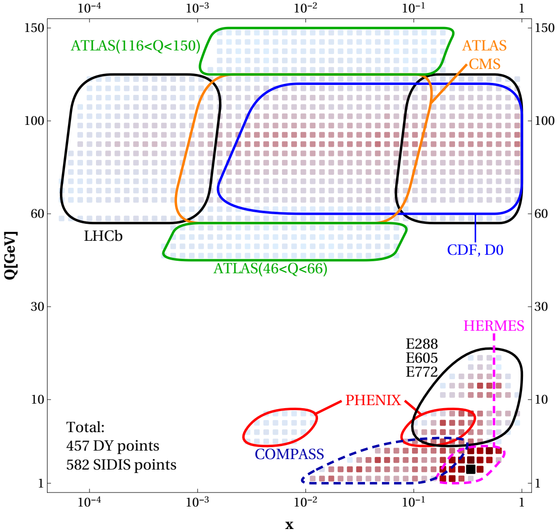
$$\gamma = \frac{2Mx}{Q}$$

$$\varsigma = \gamma\frac{m}{zQ}$$

In practice: $q_T < 0.25Q$

- Most part of data is not TMD factorisable.
- Low z's are not accessible
- H1, ZEUS data have no TMD points, too low z.

Data survived after the cut



High energy DY: 194 points
Low energy DY: 263 points
(Low energy) SIDIS: 582 points

NNLO: $\chi^2_{global}/N_{pt} = 0.95$
N^3LO: $\chi^2_{global}/N_{pt} = 1.05$

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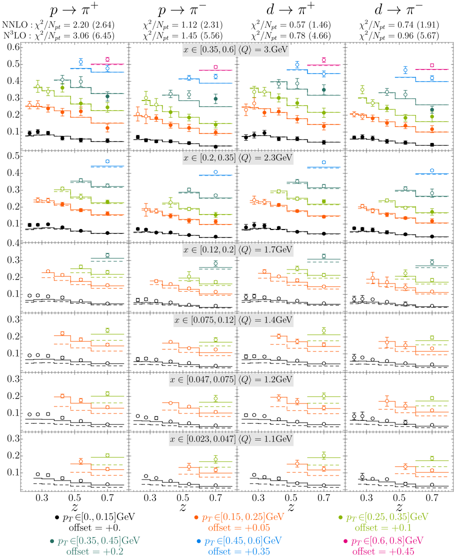
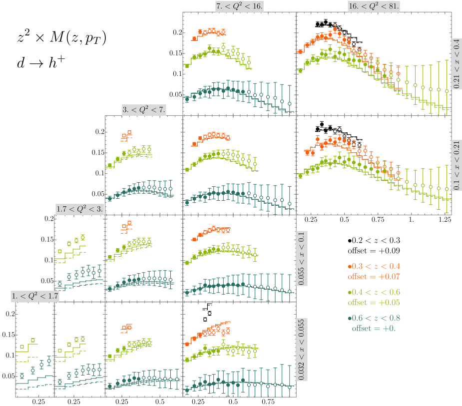


Example of SIDIS data



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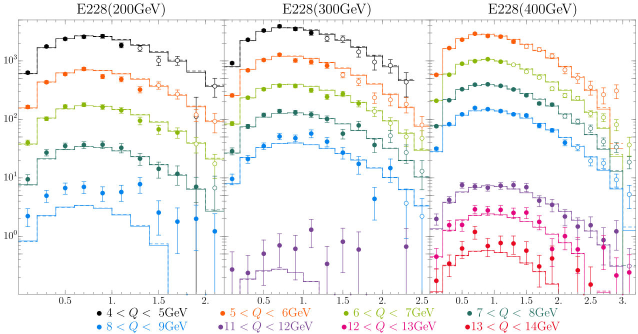
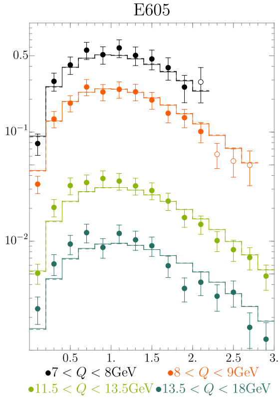
Example of low-energy DY



+35%   +28%   +19%

Norm.error of data 25%



+20%

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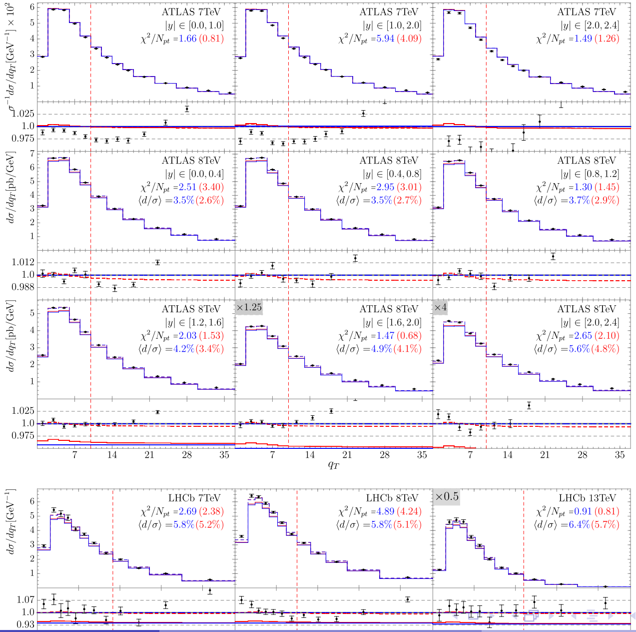
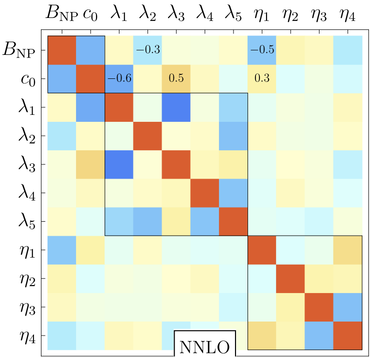
Example of Z-boson data





Evolution : 2 parameters
TMDPDF : 5 parameters + PDF
TMDFF : 4 parameters + NNFF

Different NP functions are almost decorrelated

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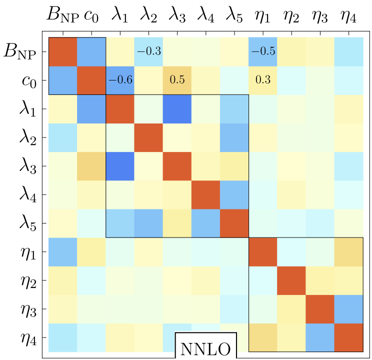


Evolution : 2 parameters
TMDPDF : 5 parameters + PDF
TMDFF : 4 parameters + NNFF

Different NP functions are almost decorrelated

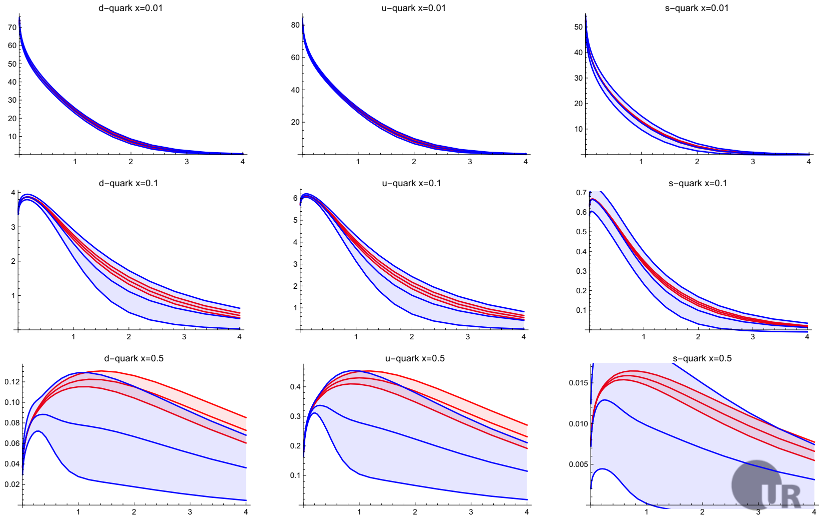
Fit quality essentially depends on the collinear input.

Vary NNPDF within the 1σ band

$$\chi^2/N_{pt} \in [0.8, 6.]$$

We cannot estimate accurately the PDF uncertainty.

$$F(x,b) = C(x,b,\mu_{OPE}) \otimes f_1(x,\mu_{OPE}) f_{NP}(x,b)$$



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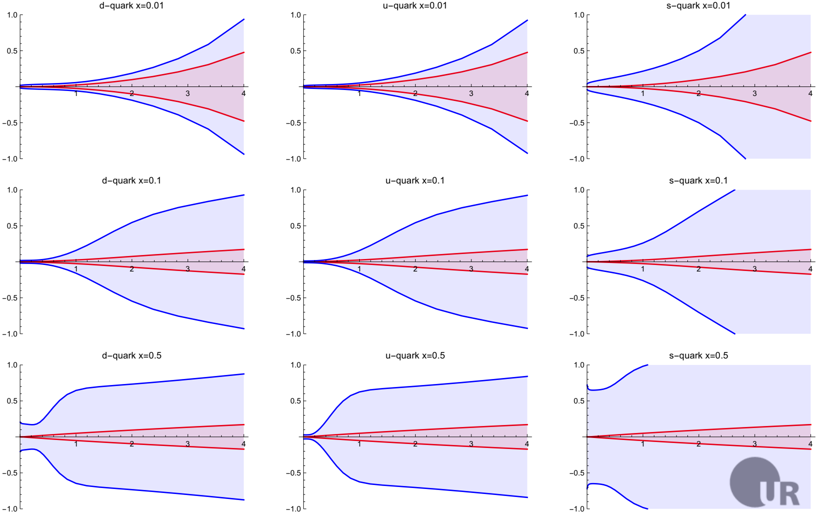

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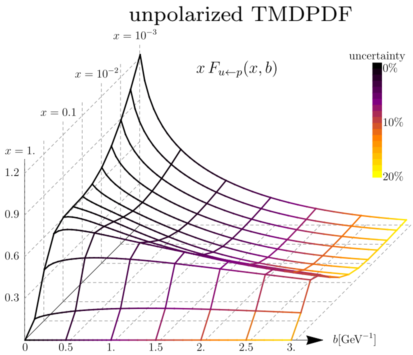
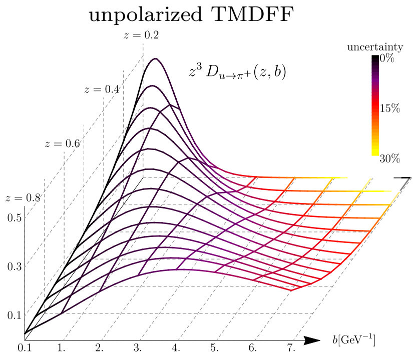
unpolarized TMD-distributions





... systematic error 10-20%

... evolution does not have this systematics

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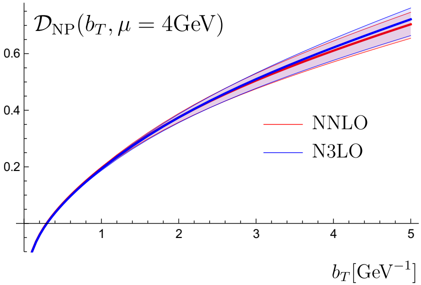
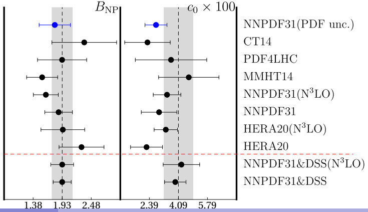
Universal TMD evolution kernel



$$\mathcal{D}_{\rm NP}(b,\mu) = \mathcal{D}_{\rm perp}(b^*,\mu) + c_0 b b^*, \qquad b^* = b/\sqrt{1 + b^2/B_{NP^2}}$$

- Linear asymptotic at $b \to \infty$ (ed.assumption)
- RAD is independent on PDF set



What could be learned
about QCD from TMD physics?

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TMD distributions

- Distribution of spin, momentum inside of hadron
- Hadronization properties

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What could be learned about QCD from TMD physics?

TMD distributions

- Distribution of spin, momentum inside of hadron
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Non-perturbative TMD evolution kernel (RAD, CSS-kernel, non-perturbative Sudakov)

- ???

What could be learned about QCD from TMD physics?

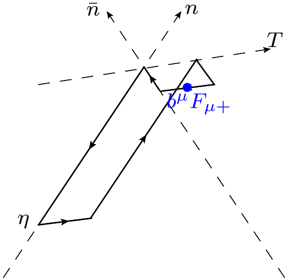
TMD distributions

- Distribution of spin, momentum inside of hadron
- Hadronization properties

Non-perturbative TMD evolution kernel (RAD, CSS-kernel, non-perturbative Sudakov)

- Properties of QCD vacuum

Rapidity anomalous dimension can be defined as a primary object.



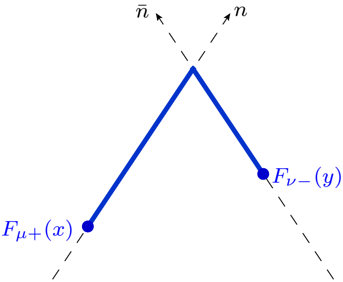
$$\mathcal{D}(b) = \frac{1}{2}\lim_{\eta\to\infty}\frac{S'(b;\eta\lambda)}{S(b)}$$

$$S'(b) = ig\int_0^b dr \frac{\text{Tr}}{N_c}\langle 0|F_{b+}(-\lambda n + r) P\exp\left(-ig\int_{C'} dx^\mu A_\mu(x)\right)|0\rangle$$

- Route to non-perturbative calculation and modeling.

Power correction to $\mathcal{D}$

$$\mathcal{D}(\mathbf{b}) = \underbrace{\mathcal{D}_{\text{pert}}(\ln(\mu^2\mathbf{b}^2))}_{\text{known at N}^3\text{LO}} + \mathbf{b}^2\mathcal{D}_1(\ln(\mu^2\mathbf{b}^2)) + \mathbf{b}^4\mathcal{D}_2(\ln(\mu^2\mathbf{b}^2)) + ...$$



$\mathcal{D}_1$ is expressed via 2-point correlators connected by "a minimal distance link"

$$g^2\frac{\text{Tr}}{N_c}\langle 0|F_{\mu x}(x)[x,0][0,y]F_{\nu y}(y)|0\rangle =$$
$$\left(g^{\mu\nu} - \frac{y^\mu x^\nu}{(xy)}\right)\varphi_1(x,y) + (...)^{\mu\nu}\varphi_2$$

At LO $x^2 = y^2 = 0$
$\varphi_1(x,y) = \varphi_1(r^2)$
$\varphi_2(x,y) = 0$

$$\mathcal{D}_1(\mathbf{b}) = \frac{1}{2}\int_0^\infty \frac{d\mathbf{r}^2}{\mathbf{r}^2}\varphi_1(\mathbf{r}^2)$$

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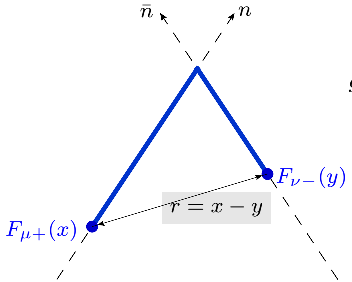

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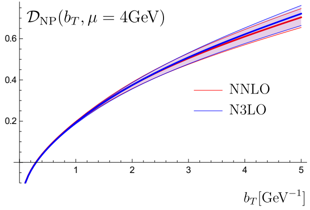
$$\mathcal{D}_1(\mathbf{b}) = \frac{1}{2}\int_0^\infty \frac{d\mathbf{r}^2}{\mathbf{r}^2}\varphi_1(\mathbf{r}^2) \lesssim \frac{\pi^2}{72}\frac{G_2}{\Lambda^2_{\text{QCD}}} \approx (0.01 - 0.05)\text{GeV}^2$$

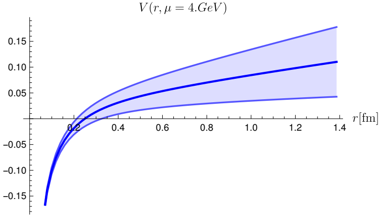
Extracted value: $\mathcal{D}_1 = 0.022 \pm 0.009\text{GeV}^2$

To get interpretation – apply a model

Example

In stochastic vacuum model





$$V(r)=r\frac{\pi}{4}\mathcal{D}''(0)+\frac{\mathcal{D}'(0)}{2}+\frac{r^2}{2}\int_r^\infty\frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}}+...$$

QCD string tension: $\sigma=0.014\pm0.01\text{GeV}^2$ vs. $\sim 0.1-0.2\text{GeV}^2$

Conclusion

TMD factorization is consistent and universal approach

I have demonstrated

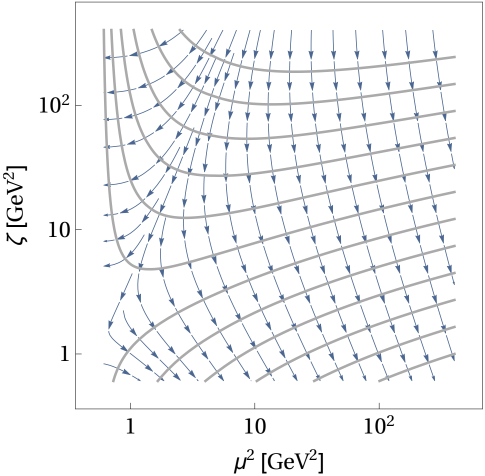
- Large bulk of data (DY+SIDIS) supports it
- Extracted NP-functions are (almost) uncorrelated
- Previous estimation of Limits for TMD factorization confirmed $q_T/Q < 0.25$
- Perfect perturbative stability
- Target mass corrections and proper definition of the kinematic variables helps

Plenty of points to improve

- Power corrections and higher q_T
- Realistic uncertainties and mode-bias
- More processes
- ...

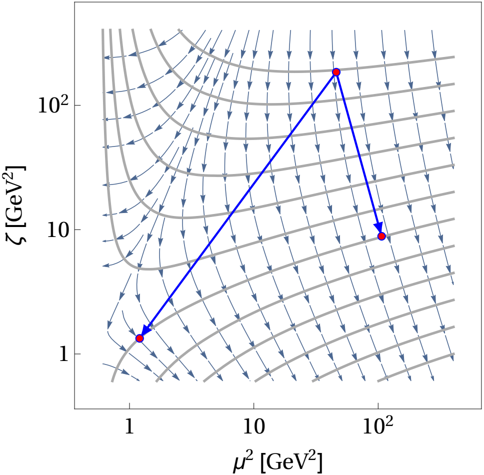
Backup slides

TMD distribution is not defined by a scale (μ, ζ)
It is defined by an equipotential line.



The scaling is defined by
~~a difference between scales~~
a difference between potentials

TMD distribution is not defined by a scale (μ, ζ)
It is defined by an equipotential line.



The scaling is defined by
~~a difference between scales~~
a difference between potentials

Evolution factor to both points
is the same
although the scales are
different by 10^2GeV^2

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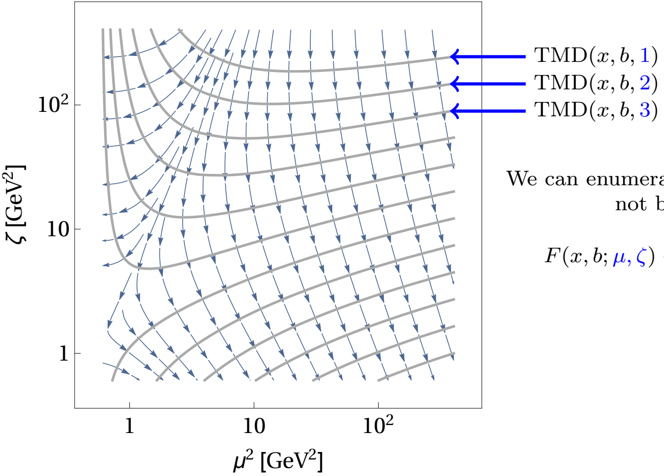
TMD distributions on the same equipotential line are equivalent.



We can enumerate them by a lines not by (μ, ζ)

$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

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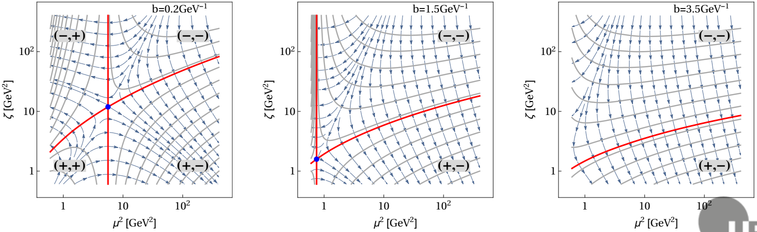
There is a unique line which passes though all μ's

The optimal TMD distribution

$$F(x,b) = F(x,b;\mu,\zeta_\mu)$$

where ζ_μ is the special line.



Comparison with other evolutions