Extraction of transverse momentum dependent parton distributions

Alexey Vladimirov

based on [1902.08474]

Universität Regensburg

I will present (recent) extraction of
unpolarized transverse momentum dependent
parton distribution functions (TMDPDF)

The work is unique in several aspects

- The first full-NNLO extraction of uTMDPDF

NNLO attempt [I.Scimemi,AV,1707.07606]
NNLL/NLO [U.D'Alesio,et al,1407.3311]
plenty of NLL/LO e.g [A.Bacchetta,et al,1703.10157]

- Largest set of Drell-Yan data (including LHC data)
- Consistent theory treatment (based on ζ-prescription)

Outline of talk

- Some details about TMD factorization
- Some details about fitting procedure
- Results of extraction
- Future directions

Transverse momentum distributions of leading order

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

- + 8 gluon TMDs
- + 2 (or 8) TMD fragmentation function
- + non-perturbative evolution kernel

Universität Regensburg

Transverse momentum distributions of leading order

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

- 8 gluon TMDs
- 2 (or 8) TMD fragmentation function
- non-perturbative evolution kernel

Topic of talk:
Unpolarized TMDPDF

+

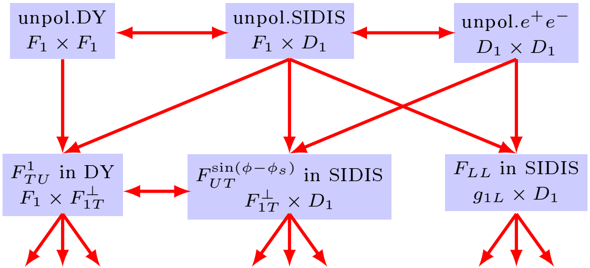
non-perturbative evolution kernel

Why unpolarized TMDPDF especially important?

Each TMD factorized cross-section has three NP functions

$$\frac{d\sigma}{dp_T^2 dQ} \simeq \sigma_0(Q) \int d^2\mathbf{b} e^{i\mathbf{b}\mathbf{p}_T} R[Q \to (\mu,\zeta)]^2 F_1(x_1,\mathbf{b};\mu,\zeta) F_2(x_2,\mathbf{b};\mu,\zeta)$$

- Two TMD distributions F_1 & F_2
- non-perturbative evolution $R \sim \exp(-\mathcal{D})$



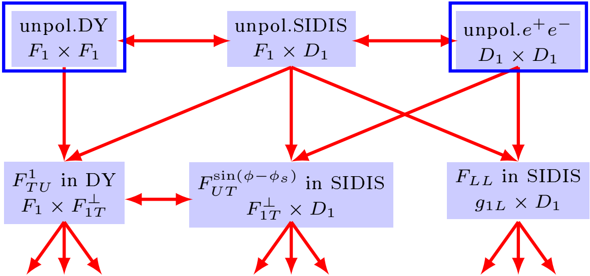
Universität Regensburg

Why unpolarized TMDPDF especially important?

Each TMD factorized cross-section has three NP functions

$$\frac{d\sigma}{dp_T^2 dQ} \simeq \sigma_0(Q) \int d^2\mathbf{b} e^{i\mathbf{b}\mathbf{p}_T} R[Q \to (\mu,\zeta)]^2 F_1(x_1,\mathbf{b};\mu,\zeta) F_2(x_2,\mathbf{b};\mu,\zeta)$$

- Two TMD distributions F_1 & F_2
- non-perturbative evolution $R \sim \exp(-\mathcal{D})$



Theory

Cross-section for Drell-Yan process (DY)

$$h_1 + h_2 \to \gamma^*/Z(\to ll') + X$$

$$\frac{d\sigma}{dy dQ^2 d^2\mathbf{q}_T} = \sigma_0 \int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'}(Q,\mu) F_{f\leftarrow h}(x_1, b; \mu, Q^2) F_{f'\leftarrow h}(x_2, b; \mu, Q^2) + \ldots$$

Kinematics

$$p_1^\mu = \bar{n}^\mu p_1^+ + \frac{M^2}{2p_1^+} n^\mu$$

$$p_2^\mu = n^\mu p_2^+ + \frac{M^2}{2p_2^+} \bar{n}^\mu$$

p_1, p_2 define scattering plane

$$(p_1 + p_2)^2 = s$$

$$q^2 = (l + l')^2 = Q^2$$

$$x_1 = \frac{q^+}{p_1^+} = \sqrt{\frac{Q^2 + \mathbf{q}_T^2}{s}} e^y$$

$$x_2 = \frac{q^+}{p_1^+} = \sqrt{\frac{Q^2 + \mathbf{q}_T^2}{s}} e^{-y}$$

$$q = [q^+, q^-, \mathbf{q}_T] = \left[\sqrt{\frac{Q^2 + \mathbf{q}_T^2}{2}} e^y, \sqrt{\frac{Q^2 + \mathbf{q}_T^2}{2}} e^{-y}, \mathbf{q}_T\right]$$

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T}=\sigma_0\int d^2b\,e^{i(\mathbf{b}\cdot\mathbf{q}_T)}H_{ff'}(Q,\mu)F_{f\leftarrow h}(x_1,b;\mu,Q^2)F_{f'\leftarrow h}(x_2,b;\mu,Q^2)+...$$

Evolution

TMD evolution is given by 2 equations

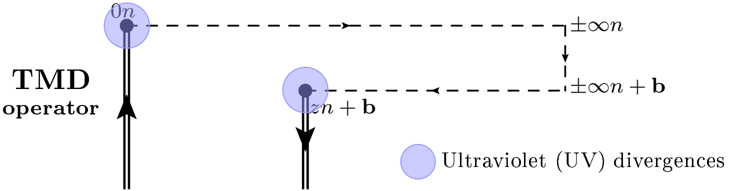
$$\mu^2\frac{dF(x,b;\mu,\zeta)}{d\mu^2}=\gamma_F(\mu,\zeta)F(x,b;\mu,\zeta),\qquad \zeta\frac{dF(x,b;\mu,\zeta)}{d\zeta}=-\mathcal{D}(\mu,b)F(x,b;\mu,\zeta)$$

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T} = \sigma_0 \int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'}(Q,\mu) F_{f\leftarrow h}(x_1,b;\mu,Q^2) F_{f'\leftarrow h}(x_2,b;\mu,Q^2) + \ldots$$

Evolution

TMD evolution is given by 2 equations

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \gamma_F(\mu,\zeta) F(x,b;\mu,\zeta), \qquad \zeta\frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,b)F(x,b;\mu,\zeta)$$

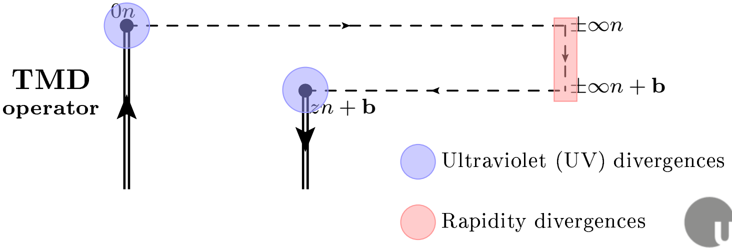


$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T} = \sigma_0 \int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'}(Q,\mu) F_{f\leftarrow h}(x_1,b;\mu,Q^2) F_{f'\leftarrow h}(x_2,b;\mu,Q^2) + \ldots$$

Evolution

TMD evolution is given by 2 equations

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \gamma_F(\mu,\zeta) F(x,b;\mu,\zeta), \qquad \zeta \frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,b) F(x,b;\mu,\zeta)$$



Universität Regensburg

Solution

$$F(x,\mathbf{b};\mu_1,\zeta_1) = R[\mathbf{b};(\mu_1,\zeta_1)\to(\mu_2,\zeta_2)]F(x,\mathbf{b};\mu_2,\zeta_2)$$

Initial scales:
$\mu_1 \simeq Q$
$\zeta_1 = Q^2$

→

Final scales:
$\mu_2 \sim$??
$\zeta_2 \sim$??

Solution

$$F(x,\mathbf{b};\mu_1,\zeta_1) = R[\mathbf{b};(\mu_1,\zeta_1)\to(\mu_2,\zeta_2)]F(x,\mathbf{b};\mu_2,\zeta_2)$$

Initial scales:
$\mu_1 \simeq Q$
$\zeta_1 = Q^2$

→

Final scales:
$\mu_2 \sim$??
$\zeta_2 \sim$??

Main complication:
$\mathbf{b}^2 \in (0,\infty)$
perturbative logarithms $\ln(\mathbf{b}^2\mu^2)$, $\ln(\mathbf{b}^2\zeta)$, $\ln(\mu^2/\zeta)$

Universität Regensburg

Solution

$$F(x,\mathbf{b};\mu_1,\zeta_1) = R[\mathbf{b};(\mu_1,\zeta_1)\to(\mu_2,\zeta_2)]F(x,\mathbf{b};\mu_2,\zeta_2)$$

Initial scales:
$\mu_1 \simeq Q$
$\zeta_1 = Q^2$

→

Final scales:
$\mu_2 \sim$??
$\zeta_2 \sim$??

Main complication:
$\mathbf{b}^2 \in (0,\infty)$
perturbative logarithms $\ln(\mathbf{b}^2\mu^2)$, $\ln(\mathbf{b}^2\zeta)$, $\ln(\mu^2/\zeta)$

Approaches

- CSS [Collins,Soper,Sterman,1984]

$$\mu = \mu^*(\mathbf{b}) \sim \begin{cases} 1/\mathbf{b} & \mathbf{b}\to 0, \\ \text{const} & \mathbf{b}\gg 0, \end{cases} \qquad \zeta = \mu^2$$

  - Formally, no large-logarithms in perturbative regeme
  - Extremely unstable perturbative expression
  - Commonly used LO approach, but useless at NNLO

Universität Regensburg

Solution
$$F(x,\mathbf{b};\mu_1,\zeta_1) = R[\mathbf{b};(\mu_1,\zeta_1)\to(\mu_2,\zeta_2)]F(x,\mathbf{b};\mu_2,\zeta_2)$$

Initial scales:
$\mu_1 \simeq Q$
$\zeta_1 = Q^2$

→

Final scales:
$\mu_2 \sim$??
$\zeta_2 \sim$??

Main complication:
$\mathbf{b}^2 \in (0,\infty)$
perturbative logarithms $\ln(\mathbf{b}^2\mu^2)$, $\ln(\mathbf{b}^2\zeta)$, $\ln(\mu^2/\zeta)$

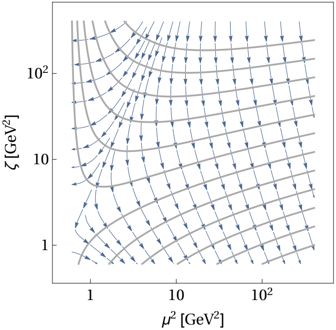
Approaches

- ζ-prescription [I.Scimemi,AV,1803.11089]

$$\mu = \textbf{doesn't matter}, \qquad \zeta = \zeta_\mu$$

  - Formally, no large-logarithms in perturbative regeme
  - Stable perturbative expression
  - Universal definition
  - Let $\mu \simeq Q$

ζ-prescription



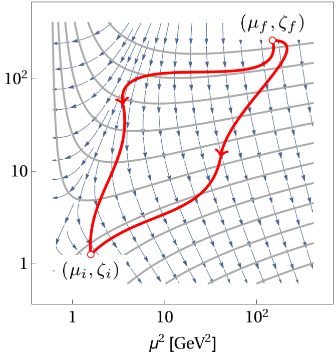
TMD evolution is 2D evolution

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \gamma_F(\mu,\zeta)F(x,b;\mu,\zeta)$$

$$\zeta \frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,b)F(x,b;\mu,\zeta)$$

Evolution field	$\mathbf{E} = \left(\frac{\gamma_F}{2}, -\mathcal{D}\right)$
is conservative	$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0$
Evol.potential:	$\mathbf{E} = \boldsymbol{\nabla} U$

ζ-prescription



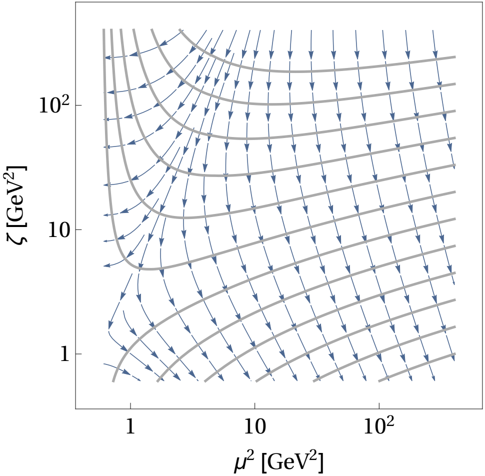
TMD evolution is 2D evolution

$$R[\mathbf{b}; i \to f] =$$
$$\exp \int_P d\boldsymbol{\nu} \cdot \mathbf{E} = \exp(U_f - U_i) =$$
$$\exp \left[\int_P \left(\gamma_F(\mu, \zeta) \frac{d\mu}{\mu} - \mathcal{D}(\mu, \mathbf{b}) \frac{d\zeta}{\zeta} \right) \right]$$

- Path independence
- Unified picture of various evolution scenarios

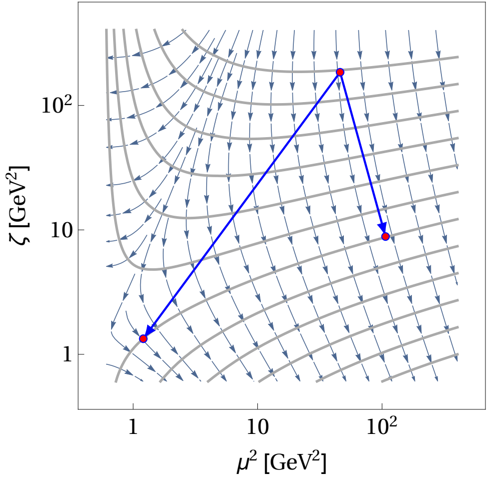
Evolution field	$\mathbf{E} = \left(\frac{\gamma_F}{2}, -\mathcal{D} \right)$
is conservative	$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0$
Evol.potential:	$\mathbf{E} = \boldsymbol{\nabla} U$

TMD distribution is not defined by a scale (μ, ζ)
It is defined by an equipotential line.



The scaling is defined by
~~a difference between scales~~
a difference between potentials

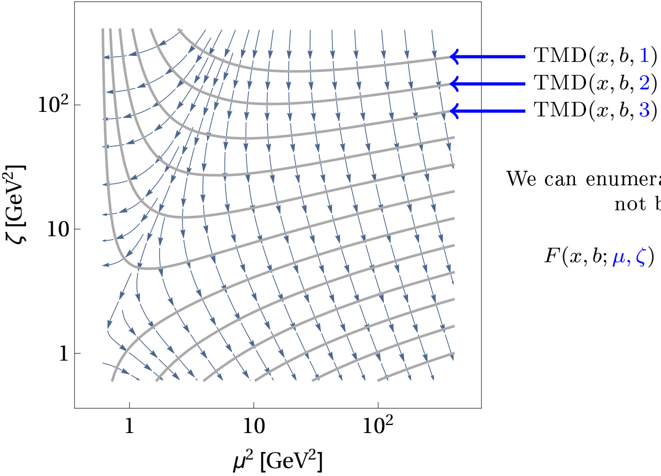
TMD distribution is not defined by a scale (μ, ζ)
It is defined by an equipotential line.



The scaling is defined by
~~a difference between scales~~
a difference between potentials

Evolution factor to both points
is the same
although the scales are
different by 10^2GeV^2

TMD distributions on the same equipotential line are equivalent.



We can enumerate them by a lines not by (μ, ζ)

$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

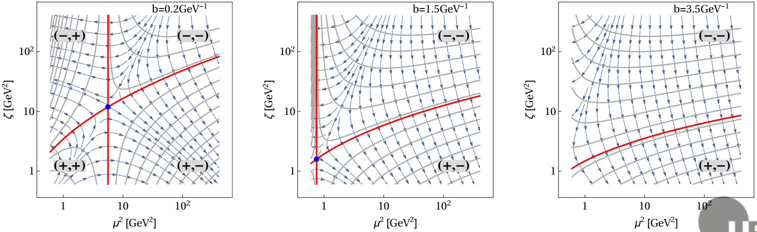
Universal scale-independent TMD

There is a unique line which passes though all μ's

The optimal TMD distribution

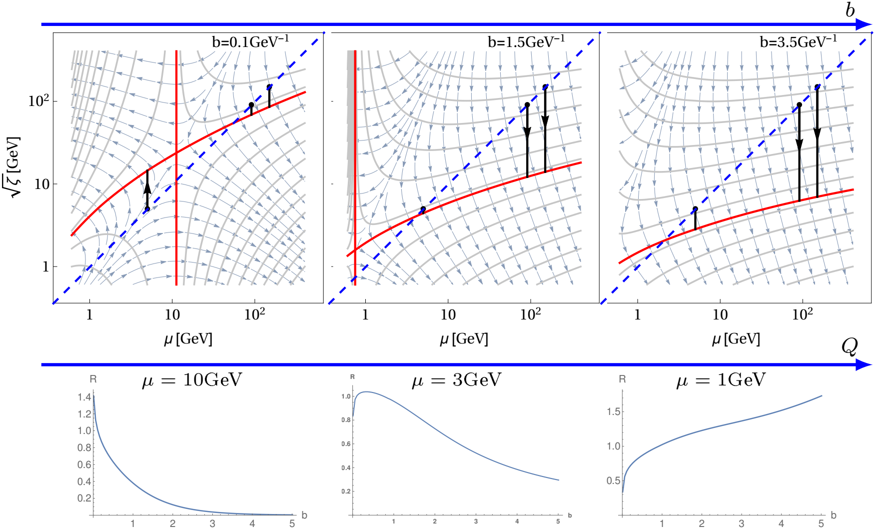
$$F(x,b)=F(x,b;\mu,\zeta_\mu)$$

where ζ_μ is the special line.

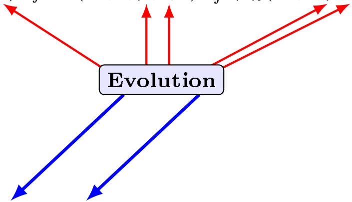


Universität Regensburg

The evolution potential depends on b.
Relative position of its elements (saddle-point, special lines) dictates the shape of evolution factor.



$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T}=\sigma_0\int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)}H_{ff'}(Q,\mu)F_{f\leftarrow h}(x_1,b;\mu,Q^2)F_{f'\leftarrow h}(x_2,b;\mu,Q^2)+...$$



$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T}=\sigma_0\int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)}H_{ff'}(Q,\mu)R[\mathbf{b};(\mu,Q^2)\to \text{s.l.}]^2F_{f\leftarrow h}(x_1,b)F_{f'\leftarrow h}(x_2,b)+...$$

Evolution factor has simple expression

$$R[\mathbf{b};(\mu,\zeta)\to \text{s.l.}]=\left(\frac{\zeta}{\zeta_\mu}\right)^{-\mathcal{D}(\mathbf{b},\mu)}$$

Good PT convergence

$\mu=Q$

Essential feature of TMD phenomenology:
perturbative and non-perturbative (NP) in the same expression

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T}=\sigma_0\int d^2b\,e^{i(\mathbf{b}\cdot\mathbf{q}_T)}H_{ff'}(Q,\mu)\underline{R[\mathbf{b};(\mu,Q^2)\to \text{s.l.}]}^2\,\underline{F_{f\leftarrow h}(x_1,b)}\,\underline{F_{f'\leftarrow h}(x_2,b)}+...$$

Rapidity AD is (generally) NP

TMD distributions are (generally) NP

Essential feature of TMD phenomenology:
perturbative and non-perturbative (NP) in the same expression

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T} = \sigma_0 \int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)} H_{ff'}(Q,\mu) R[\mathbf{b};(\mu,Q^2)\to \text{s.l.}]^2 F_{f\leftarrow h}(x_1,b) F_{f'\leftarrow h}(x_2,b) + \ldots$$

Rapidity AD is (generally) NP

TMD distributions are (generally) NP

General rule:

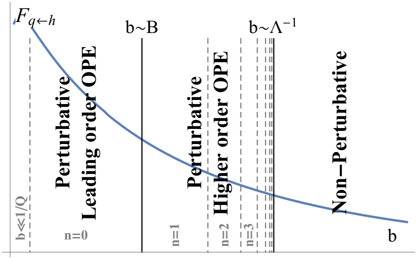
- small-b: $\mathbf{b} \ll B$ perturbative OPE
- intermediate-b: $\mathbf{b} \sim B$ "interpolation"
- large-b: $\mathbf{b} \gg B$ non-perturbative model

Universität Regensburg

Matching for TMDPDF

In order to reduce model-dependence one builds an anzats such that it respects OPE at small-b.
This is called "small-b **matching**"

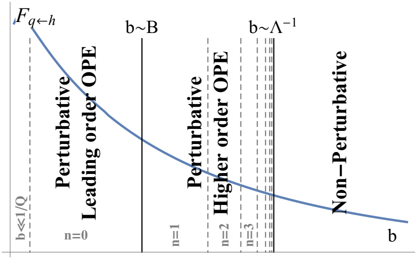
$$F_{f\leftarrow h}(x,\mathbf{b}) \simeq \sum_{f'}\int_x^1 \frac{dz}{z} C_{f\leftarrow f'}(z;\mu_{\text{OPE}},\ln[\mathbf{b}^2\mu_{\text{OPE}}^2]) f_{f'\to h}\left(\frac{x}{z},\mu_{\text{OPE}}\right) + \mathbf{b}^2[...] + ...$$



Matching for TMDPDF

In order to reduce model-dependence one builds an anzats such that it respects OPE at small-b.
This is called “small-b **matching**”

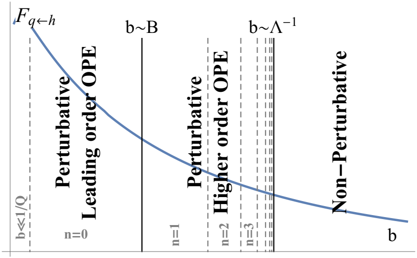
$$F_{f\leftarrow h}(x,\mathbf{b}) \simeq \sum_{f'} \int_x^1 \frac{dz}{z} C_{f\leftarrow f'}(z;\mu_{\text{OPE}},\ln[\mathbf{b}^2\mu^2_{\text{OPE}}]) f_{f'\to h}\left(\frac{x}{z},\mu_{\text{OPE}}\right) \left\{1+\mathbf{b}^2[...]+...\right\}$$



Matching for TMDPDF

In order to reduce model-dependence one builds an anzats such that it respects OPE at small-b.
This is called “small-b **matching**”

$$F_{f\leftarrow h}(x,\mathbf{b}) = \sum_{f'}\int_x^1 \frac{dz}{z} C_{f\leftarrow f'}(z;\mu_{\text{OPE}},\ln[\mathbf{b}^2\mu_{\text{OPE}}^2]) f_{f'\to h}\left(\frac{x}{z},\mu_{\text{OPE}}\right) f_{\text{NP}}^{f'\leftarrow h}(x,z,\mathbf{b}^2)$$



- $f_{\text{NP}}(x,z,\mathbf{b}^2\sim 0)\sim 1$
- In ζ-prescription all scaling properties are respected!

f_{NP} is subject of fitting

Matching for $\mathcal{D}$

Rapidity anomalous dimension can be expressed as vacuum matrix element of known operator

$$\mathcal{D}(\mu,\mathbf{b}) = \frac{ig\frac{\text{Tr}}{N_c}\int_0^1 d\beta \langle 0|F^{+\mathbf{b}}(-\Lambda_+ n + \mathbf{b}\beta)[\text{SF contour}]|0\rangle}{\langle 0|[\text{SF contour}]|0\rangle}$$

[A.Schafer,AV,in preparation]

- Can be also systematically computed by OPE at small-b

$$\mathcal{D}(\mu,\mathbf{b}) = d(a_s(\mu),\ln(\mu^2\mathbf{b}^2)) + \mathbf{b}^2(...) + ...$$

Different kind of evolution

$$\mu^2\frac{d\mathcal{D}(\mu,\mathbf{b})}{d\mu^2} = \frac{\Gamma_{\text{cusp}}(\mu)}{2}$$

Universität Regensburg

Matching for $\mathcal{D}$

Rapidity anomalous dimension can be expressed as vacuum matrix element of known operator

$$\mathcal{D}(\mu,\mathbf{b}) = \frac{ig\frac{\text{Tr}}{N_c}\int_0^1 d\beta \langle 0|F^{+\mathbf{b}}(-\Lambda_+ n + \mathbf{b}\beta)[\text{SF contour}]|0\rangle}{\langle 0|[\text{SF contour}]|0\rangle}$$

[A.Schafer,AV,in preparation]

- Can be also systematically computed by OPE at small-b

$$\mathcal{D}(\mu,\mathbf{b}) = \underbrace{d(a_s(\mu), \ln(\mu^2\mathbf{b}^2))}_{\text{has correct evol.}} + \mathbf{b}^2(...) + ...$$

Different kind of evolution

$$\mu^2 \frac{d\mathcal{D}(\mu,\mathbf{b})}{d\mu^2} = \frac{\Gamma_{\text{cusp}}(\mu)}{2}$$

Matching for $\mathcal{D}$

Rapidity anomalous dimension can be expressed as vacuum matrix element of known operator

$$\mathcal{D}(\mu,\mathbf{b})=\frac{ig\frac{\text{Tr}}{N_c}\int_0^1 d\beta\langle 0|F^{+\mathbf{b}}(-\Lambda_+ n+\mathbf{b}\beta)[\text{SF contour}]|0\rangle}{\langle 0|[\text{SF contour}]|0\rangle}$$

[A.Schafer,AV,in preparation]

- Can be also systematically computed by OPE at small-b

$$\mathcal{D}(\mu,\mathbf{b})=\underbrace{d(a_s(\mu),\ln(\mu^2\mathbf{b}_{\text{NP}}^2))}_{\text{has correct evol.}}+\left\{\mathbf{b}^2(...)+...\right\}$$

Different kind of evolution

$$\mu^2\frac{d\mathcal{D}(\mu,\mathbf{b})}{d\mu^2}=\frac{\Gamma_{\text{cusp}}(\mu)}{2}$$

- $\mathbf{b}_{\text{NP}}^2(b\sim 0)\sim\mathbf{b}^2$
- $\mathbf{b}_{\text{NP}}^2(b\gg\Lambda^{-2})\sim\text{const}$

b_{NP} and $g(\mathbf{b})$ are subjects of fitting

Matching for $\mathcal{D}$

Rapidity anomalous dimension can be expressed as vacuum matrix element of known operator

$$\mathcal{D}(\mu,\mathbf{b})=\frac{ig\frac{\text{Tr}}{N_c}\int_0^1 d\beta\langle 0|F^{+\mathbf{b}}(-\Lambda_+ n+\mathbf{b}\beta)[\text{SF contour}]|0\rangle}{\langle 0|[\text{SF contour}]|0\rangle}$$

[A.Schafer,AV,in preparation]

- Can be also systematically computed by OPE at small-b

$$\mathcal{D}(\mu,\mathbf{b})=d(a_s(\mu),\ln(\mu^2\mathbf{b}^2_{\text{NP}}))+g(\mathbf{b}^2,\mathbf{b}^2_{\text{NP}})$$

Different kind of evolution

$$\mu^2\frac{d\mathcal{D}(\mu,\mathbf{b})}{d\mu^2}=\frac{\Gamma_{\text{cusp}}(\mu)}{2}$$

- $\mathbf{b}^2_{\text{NP}}(b\sim 0)\sim\mathbf{b}^2$
- $\mathbf{b}^2_{\text{NP}}(b\gg\Lambda^{-2})\sim\text{const}$
- $g(\mathbf{b}^2\sim 0)\sim 0$

b_{NP} and $g(\mathbf{b})$ are subjects of fitting

Synopsis of PT & NP input

$$\frac{d\sigma}{dydQ^2d^2\mathbf{q}_T}=\sigma_0\int d^2b\,e^{i(\mathbf{b}\cdot\mathbf{q}_T)}H_{ff'}(Q,\mu)R[\mathbf{b};(\mu,Q^2)\to\text{s.l.}]^2F_{f\leftarrow h}(x_1,b)F_{f'\leftarrow h}(x_2,b)+...$$

- Hard part $H_{ff'}(Q,\mu)$
  - (perturbative) NNLO [Matsuura,et al,1989]
- TMD evolution
  - Γ-cusp N^{3}LO
  - γ_V NNLO [Gehrmann,et al,1209.06082]
  - Rapidity anomalous dimension $\mathcal{D}$ at small-b
    - NNLO (resummed!) [AV,1610.05791]
- Matching for unpolarized TMDPDF→unpolarized PDF
  - NNLO [M.Echevaria,AV,1604.07869]

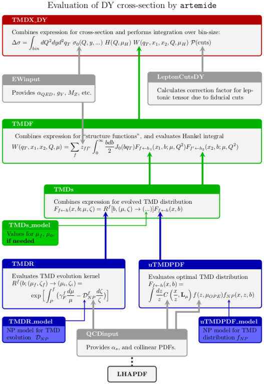
$$\mathcal{D}=\mathcal{D}_{\text{pert}}(\mu,\mathbf{b}^*)+c_0\mathbf{b}\cdot\mathbf{b}^*,\qquad \mathbf{b}^*=\mathbf{b}/\sqrt{1+\mathbf{b}^2/B_{\text{NP}}^2}$$

$$f_{NP}(x,\mathbf{b})=\exp\left(-\frac{(\lambda_1(1-x)+\lambda_2x+\lambda_3x(1-x))\mathbf{b}^2}{\sqrt{1+\lambda_4x^{\lambda_5}\mathbf{b}^2}}\right)$$

Universität Regensburg

Practice

Synopsis of numeric evaluation



picture from [artemide manual]

To reach the required data precision one has to include extra numerics:

- Finite bin-size effects
- Fiducial cuts

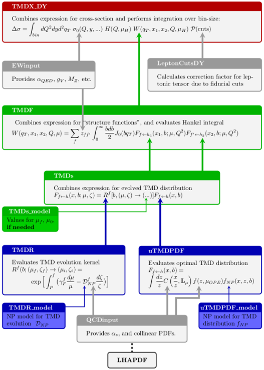
It make calculation very numerically intensive:

- 3 bin-size integrations
- + Hankel transform
- + 2 NNLO Mellin convolutions
- + ? integrations for evolution

$\sim 10^6 - 10^7$ calls of PDF for a single data point

Synopsis of numeric evaluation

Evaluation of DY cross-section by artemide



picture from [artemide manual]

ARTEMIDE

https://teorica.fis.ucm.es/artemide/

Package for TMD phenomenology

- Flexible definition of factorization scheme
  - Variety of evolutions (CSS,ζ-prescription, etc.)
  - All available PT: LO, NLO, NNLO,resummed
  - No restriction for NP models
- Fast code
- Constantly expanding
  - (unpol.)DY cross-sections
  - (unpol.)SIDIS cross-sections
- Various theory tools.

repository:

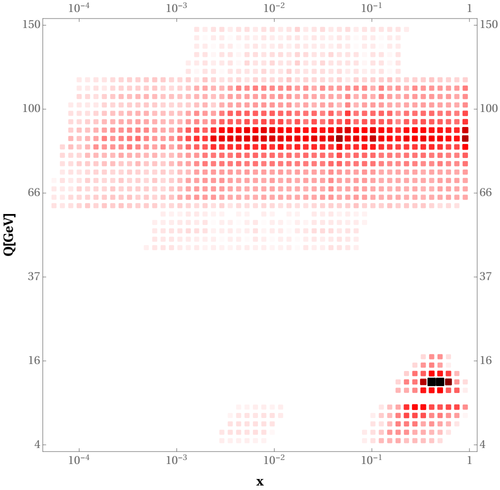
https://github.com/VladimirovAlexey/artemide-public

Universität Regensburg

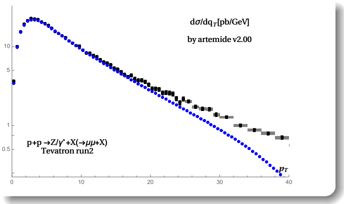
TMD factorization $\Rightarrow$ small-q_T/Q

$$\frac{q_T}{Q} < 0.25 \qquad \& \qquad \left(\frac{q_T}{Q}\right)^2 < \frac{\delta\sigma_{\text{uc}}}{\sigma}$$

dedicated study in [I.Scimemi,AV,1706.01473]



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y} \sqrt{1 + \frac{q_T^2}{Q^2}}$$

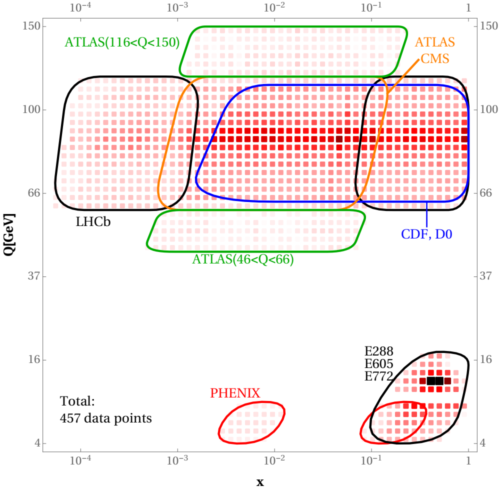


Universität Regensburg

TMD factorization $\Rightarrow$ small-q_T/Q

$$\frac{q_T}{Q} < 0.25 \qquad \& \qquad \left(\frac{q_T}{Q}\right)^2 < \frac{\delta\sigma_{uc}}{\sigma}$$

dedicated study in [I.Scimemi,AV,1706.01473]



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y} \sqrt{1 + \frac{q_T^2}{Q^2}}$$

High-energy: CDF, D0, ATLAS, CMS, LHCb
194 points

Low-energy: E288, E605, E772, PHENIX
263 points

Total: 457 points
$4 < Q < 150$GeV
$x > 10^{-4}$

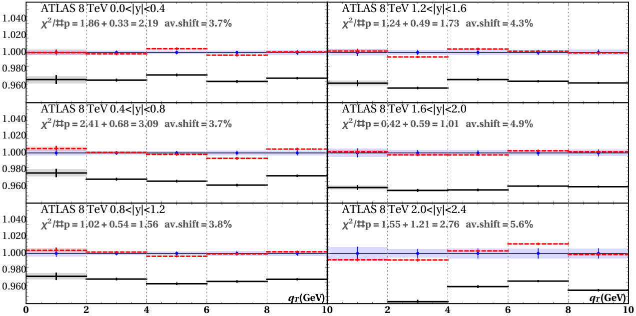
Results

Quality of the fit

$$\chi^2/N_{\text{pt}} = 1.17 \qquad \Leftarrow \begin{cases} \textbf{Low energy data:} & 0.90 \\ \textbf{High energy data:} & 1.55 \end{cases}$$

Systematically lower values!

nuisance parameters: $\chi^2 = \underbrace{\chi^2_D}_{shape} + \underbrace{\chi^2_\lambda}_{shift}$ $\Leftrightarrow$ $d =$ "systematic shift"



Quality of the fit

$$\chi^2/N_{pt} = 1.17 \;{=}1.05{+}0.12 \Leftarrow \begin{cases} \textbf{Low energy data:} & 0.90 \quad =0.86+0.04 \\ \textbf{High energy data:} & 1.55 \quad =1.30+\mathbf{0.25} \end{cases}$$

Systematically lower values!

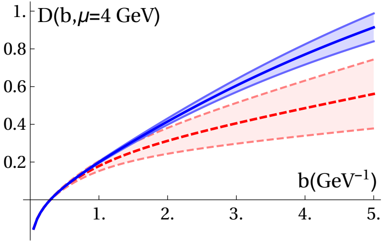
nuisance parameters: $\chi^2 = \underbrace{\chi^2_D}_{shape} + \underbrace{\chi^2_\lambda}_{shift}$ $\Leftrightarrow$ $d =$ "systematic shift"

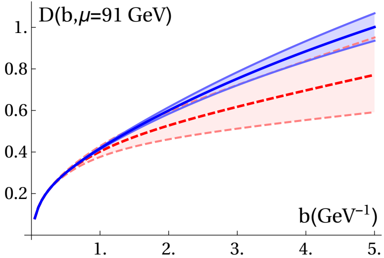
Data set	av.sys.	χ^2/N_{pt}	av.shift.
E288(200)	25%	0.86	41%
...	...	...	...
E772	10%	1.70	13%
ATLAS (8TeV) $\lvert y\rvert<0.4$	2.8%	2.19	3.7%
ATLAS (8TeV) $0.4<\lvert y\rvert<0.8$	2.8%	3.09	3.7%
ATLAS (8TeV) $0.8<\lvert y\rvert<1.2$	2.8%	1.56	3.8%
ATLAS (8TeV) $1.2<\lvert y\rvert<1.6$	2.8%	1.73	4.3%
ATLAS (8TeV) $1.6<\lvert y\rvert<2.0$	2.8%	1.01	4.9%
...	...	...	...
LHCb (7TeV)	1.7%	2.95	5.7%
LHCb (8TeV)	1.1%	5.54	5.7%
LHCb (13TeV)	3.9%	0.89	6.3%

The undershooting is mainly due to PDFs at large x

Results

Non-perturbative evolution kernel





Full data set — No LHC

Error band by replica method

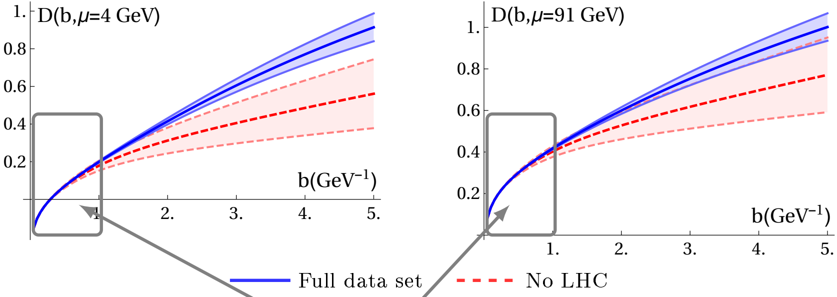
$$B_{\mathrm{NP}} = 3.31 \pm 0.28$$
$$c_0 = 0.024 \pm 0.006$$

$$B_{\mathrm{NP}} = 1.21 \pm 0.50$$
$$c_0 = 0.057 \pm 0.038$$

Universität Regensburg

Results

Non-perturbative evolution kernel



LHC
sensitive region

eplica method

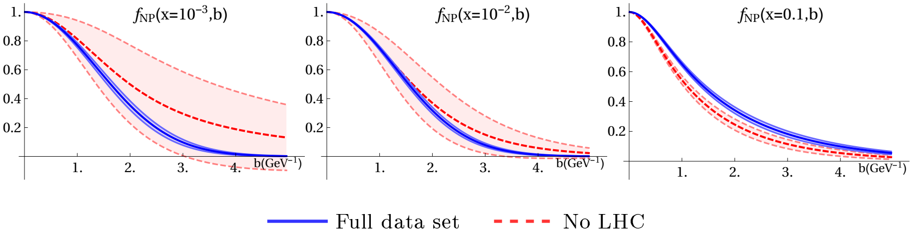
$$B_{\text{NP}} = 3.31 \pm 0.28$$
$$c_0 = 0.024 \pm 0.006$$

$$B_{\text{NP}} = 1.21 \pm 0.50$$
$$c_0 = 0.057 \pm 0.038$$

Universität Regensburg

Results

unpolarized TMD PDF



Error band by replica method

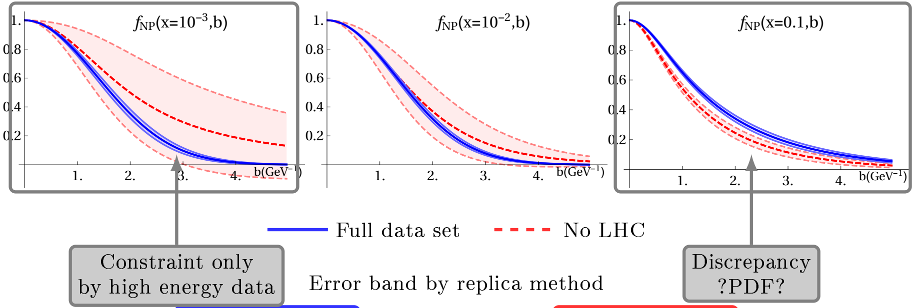
$\lambda_1 = 0.26 \pm 0.02$
$\lambda_2 = 8.18 \pm 1.00$
$\lambda_3 = -4.7 \pm 1.37$
$\lambda_4 = 300 \pm 89.0$
$\lambda_3 = 2.44 \pm 0.12$

$\lambda_1 = 0.21 \pm 0.17$
$\lambda_2 = 12.1 \pm 4.40$
$\lambda_3 = -3.5 \pm 5.41$
$\lambda_4 = 316 \pm 196.$
$\lambda_3 = 2.11 \pm 0.28$

Universität Regensburg

Results

unpolarized TMD PDF



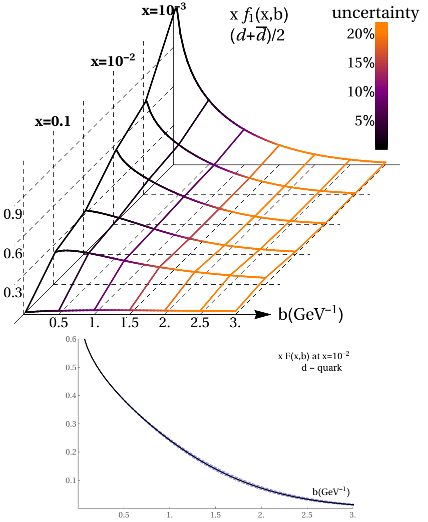
Error band by replica method

$$\lambda_1 = 0.26 \pm 0.02$$
$$\lambda_2 = 8.18 \pm 1.00$$
$$\lambda_3 = -4.7 \pm 1.37$$
$$\lambda_4 = 300 \pm 89.0$$
$$\lambda_3 = 2.44 \pm 0.12$$

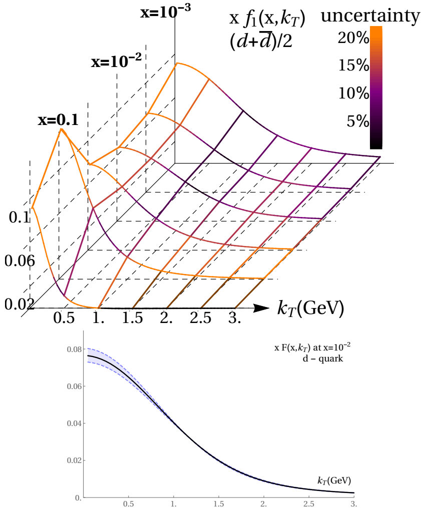
$$\lambda_1 = 0.21 \pm 0.17$$
$$\lambda_2 = 12.1 \pm 4.40$$
$$\lambda_3 = -3.5 \pm 5.41$$
$$\lambda_4 = 316 \pm 196.$$
$$\lambda_3 = 2.11 \pm 0.28$$

Universität Regensburg

uTMDPDF in b-space



uTMDPDF in k_T-space



Future studies (conclusion)

What I have:

- unpolarized TMD PDF
- non-perturbative TMD evolution (in strict universal definition)
- tuned code for low-q_T Drell-Yan process

Two main directions to continue

TMD phenomenology

EIC-like

High-energy phenomenology

LHC-like

What I have:

- unpolarized TMD PDF
- non-perturbative TMD evolution (in strict universal definition)
- tuned code for low-q_T Drell-Yan process

Two main directions to continue

TMD phenomenology

EIC-like

- SIDIS/TMDFF analysis (on-going)
- Single-spin asymmetries
- etc.

High-energy phenomenology

LHC-like

What I have:

- unpolarized TMD PDF
- non-perturbative TMD evolution (in strict universal definition)
- tuned code for low-q_T Drell-Yan process

Two main directions to continue

TMD phenomenology

EIC-like

- SIDIS/TMDFF analysis (on-going)
- Single-spin asymmetries
- etc.

High-energy phenomenology

LHC-like

- Jet evolution
  [W.Waalewijn,et al,1904.04259]
- Determination of EW parameters (W-mass)
- See next page

In modern high-energy phenomenology there is a standard cut on data:
$q_T > 30\text{GeV}$
Reason: non-perturbative TMD effects

There are plenty public codes which work at lower q_T (non-MC)

q_T-resummation

- `DYqT` [Bozzi,Catani,Ferrara,et al]
- `DYRes` [Catani,deFlorian,Ferrara,et al]
- `NNLOjet` [Glover,Gehrmann-De Ridder,Gehrmann,et al]
- `ResBos(-C,-P,-A,-2)` [G.Nadolsky,C.P.Yuan]
- **many of them**

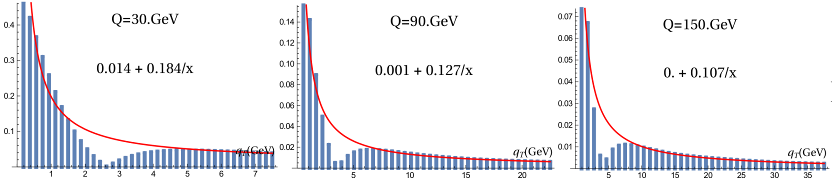
TMD factorization

- `artemide` [AV,I.Scimemi]

Remark:
at $q_T \gg \Lambda$ q_T-resummation and TMD-factorization with matching coincide.

(typically) q_T-resummation codes introduces only single NP-parameter
"non-perturbative Sudakov"
which mimics non-perturbative part of TMD evolution, and effectively cuts the Fourier integral
It allows to go lower in $q_T > 5 - 10$ GeV, for lower values prediction is unstable

Non-perturbative TMD effects
are very important at $q_T < 5 - 10$ GeV (even at very high energy)



[F.Hautmann,AV,in preparation]

Inclusion of low-q_T data can significantly effect the high-energy physics.

TMD fit is very sensitive to input PDF

- `HERAPDF20` $\chi^2/N = 0.95$
- `NNPDF3.1` $\chi^2/N = 1.17$
- `MMHT14` $\chi^2/N = 1.28$
- `PDF4LHC_5` $\chi^2/N = 1.49$
- `ABMP16_5` $\chi^2/N = 3.07$

Low-q_T data could be added to set of data for determination of PDFs

Inclusion of low-q_T data can significantly effect the high-energy physics.

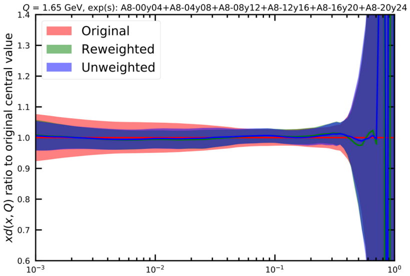
TMD fit is very sensitive to input PDF

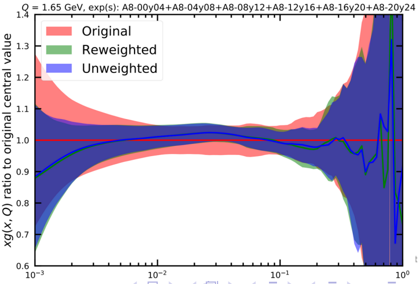
- HERAPDF20 $\chi^2/N = 0.95$
- NNPDF3.1 $\chi^2/N = 1.17$
- MMHT14 $\chi^2/N = 1.28$
- PDF4LHC_5 $\chi^2/N = 1.49$
- ABMP16_5 $\chi^2/N = 3.07$

Low-q_T data could be added to set of data for determination of PDFs

Effect: $\sim 40\%$ reduction of uncertainty band

[V.Bertone,A.Glasov,AV,in preparation]





Conclusion:

- unpolarized TMD PDF ✓
- non-perturbative TMD evolution ✓
- tuned code for low-q_T Drell-Yan process ✓

Thank you for attention!

TMD phenomenology

EIC-like

- SIDIS/TMDFF analysis (on-going)
- Single-spin asymmetries
- etc.

High-energy phenomenology

LHC-like

- Jet evolution
- Inclusion of lower-q_T data points in HEP/BSM studies
  - Determination of EW parameters (W-mass)
  - Determination of PDFs, $\alpha_s(M_Z)$