Self-contained and non-perturbative definition of Collins-Soper kernel

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Introduction

Collins-Soper kernel is a part of TMD evolution construct
I will show that this is an self-contained and interesting function which
1) can be extracted from the experimental data
2) gives information about QCD vacuum

Plan of talk

- Review of TMD evolution
- What is known about Collins-Soper kernel
- Self-contained definition
- Applications

Many names – single object

- Collins-Soper kernel; K [Collins,Soper 1981]
- Rapidity anomalous dimension (RAD); $\mathcal{D}$, $\gamma_\nu^{f\perp}$
- Collinear anomaly; $F_{q\bar{q}}$
- ...

$$\mathcal{D} = -\frac{1}{2}K = \frac{1}{2}F_{q\bar{q}} = -\frac{1}{2}\gamma_\nu^{f\perp}$$

$$\begin{aligned}
\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) &= \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta) \\
\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) &= -\mathcal{D}^f(b,\mu) F_{f\leftarrow h}(x,b;\mu,\zeta)
\end{aligned}$$

RAD depends only on the color-representation (quark/gluon) I will assume quark everywhere.

Appearance

- Rapidity logarithms appears in TMD-like factorization is the soft-gluon exchanges between hadrons

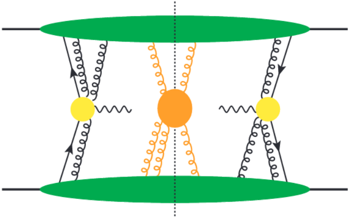
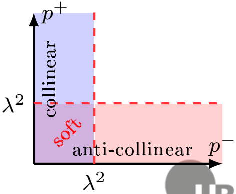


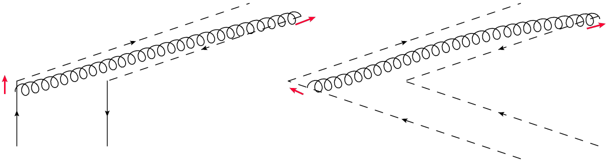
figure from [1111.0910]

$$\begin{aligned} \text{collinear} &\sim \{1, \lambda^2, \lambda\} \\ \text{anti-collinear} &\sim \{\lambda^2, 1, \lambda\} \\ \text{soft} &\sim \{\lambda^2, \lambda^2, \lambda\} \end{aligned}$$

$$d\sigma \sim \int dbe^{-i(bq)} H(Q/\mu_H)\tilde{F}_1(x_1, b; \mu_H) S^{-1}(b; \mu_H)\tilde{F}_2(x_2, b; \mu_H) H^\dagger(Q/\mu_H)$$

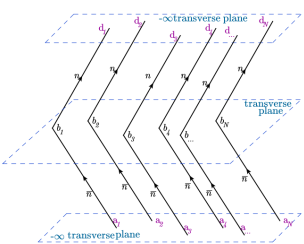
- Soft modes overlap with collinear modes and must be subtracted (to avoid double-counting)
- F_1, F_2 and S have rapidity divergences



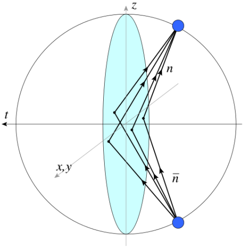


Rapidity divergences appears in the long-range ineraction
with the “far end” of light-like Wilson line

- (In conformal field theory) rapidity divergences are equivalent to ultraviolet divergences



$\Longleftrightarrow$



- Rapidity divergences are multiplicatively renormalizable [AV,1707.07606]

$$S(\mu,b)=\tilde{R}^\dagger(\mu,\nu_+)S_0(\mu,b;\nu^2)\tilde{R}(\mu,\nu_-)=R^\dagger(\mu,\nu_-,\nu^2)R(\mu,\nu_+,\nu^2) \tag{1}$$

- The renormalization scheme is defined by $S_0=1$
- This finalizes the factorization theorem

$$\begin{aligned}\tilde{F}_1(x_1,b;\mu)S^{-1}(b;\mu)\tilde{F}_2(x_2,b;\mu) &= \tilde{F}_1(x_1,b;\mu)R^{-1}(\mu,\nu_-,\nu^2)R^{-1}(\mu,\nu_+,\nu^2)\tilde{F}_2(x_2,b;\mu)\\ &= F_1(x_1,b;\mu,\zeta_1)F_2(x_2,b;\mu,\zeta_2)\end{aligned}$$

$$\zeta_1=2(p^+)^2\frac{\nu^-}{\nu^+},\qquad \zeta_2=2(p^-)^2\frac{\nu^+}{\nu^-},\qquad \zeta_1\zeta_2=(2p^+p^-)^2\simeq Q^4$$

Rapidity anomalous dimension

$$\mathcal{D}(b,\mu)=\frac{1}{2}R^{-1}(b,\mu;\nu)\frac{d}{d\ln\nu}R(b,\mu;\nu)$$

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Renormalization equation for CS kernel

- Rapidity logarithms × ordinary logarithms ⇒ double logarithms

$$\frac{d}{d\ln\mu}\mathcal{D}(b,\mu) = \Gamma_{cusp}(\mu) \tag{2}$$

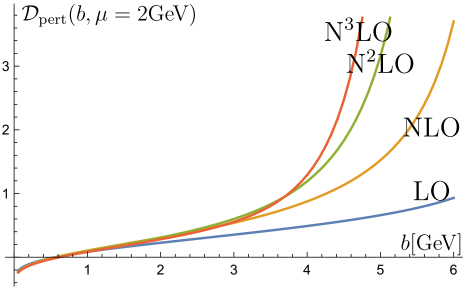
- RAD has non-trivial structure in $\ln(\mu b)$
- And non-trivial counting rules (typically order$_{cusp}$>order$_{\mathcal{D}}$)
- E.g. LO

$$\mathcal{D} = -\frac{\Gamma_0}{2\beta_0}\ln\left(1-\beta_0 a_s(\mu)\ln\left(\frac{\mu^2\mathbf{b}^2}{4e^{-2\gamma}}\right)\right) + a_s ...$$

Rapidity anomalous dimension in perturbation theory

- 2-loop [Echevarria,et al,1511.05590][many others]
- 3-loop [Li,Zhu,1604.01404]
- SAD/RAD correspondence [AV,1511.05590]
- 4-loop(?) [Zhu,et al, SCET 2020]

$$\gamma_s(\mu, v_{ij}) = 2\mathcal{D}(\mu, b^2(v); \epsilon^*)$$



Rapidity anomalous dimension is not an ordinary anomalous dimension

- RAD is non-perturbative function at large-b
- IR Renormalons at $n = 1, 2, 3...$ [Korchemsky,Tafat,2001;Scimemi,AV;2016]

$$\mathcal{D}(b,\mu) = \mathcal{D}_{\text{pert}}(\mu, b) + b^2 g_K + b^4 ... \tag{3}$$

Rapidity anomalous dimension must be determined from the data

Extraction of rapidity anomalous dimension is not trivial

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta)$$
$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}^f(b,\mu) F_{f\leftarrow h}(x,b;\mu,\zeta)$$

The **non-perturbative** TMD distribution is a function **non-perturbative** RAD
$F(x,b;\mu,\zeta;[\mathcal{D}(\mu,b)])$

Any variation in μ or ζ mixes TMD and $\mathcal{D}$
Especially horrible in CSS approach, where
$\mu \to \mu(b)$

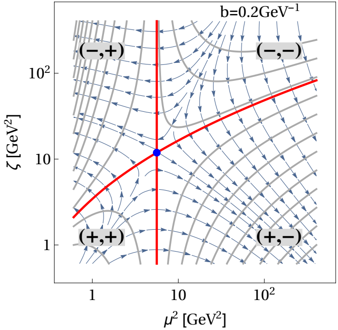
RAD cannot be imagined without corresponding TMDs?
Breakdown of universality?

Extraction of rapidity anomalous dimension is not trivial

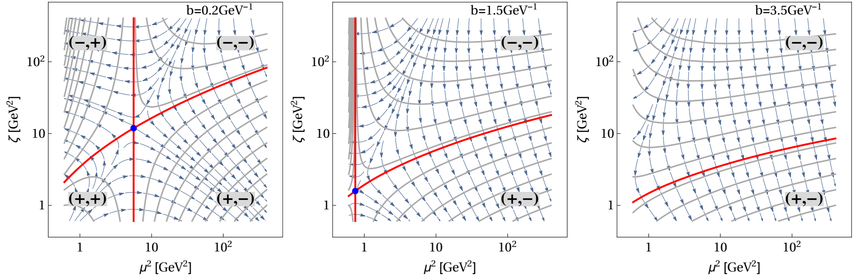
$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta)$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}^f(b,\mu) F_{f\leftarrow h}(x,b;\mu,\zeta)$$

Answer:
TMD evolution is 2D evolution and equi-evolutional curves corresponds to non-mixing change of scales.
The **saddle-point** and **special equi-evolution curves** corresponds to $F(x,b;\mu,\zeta;[\mathcal{D}(b)=0])$



The picture of evolution plane is different for different b and different model of RAD



It must be accounted in the solution [Scimemi,AV;1912.06532]

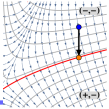
$$F(x,b;\mu,\zeta) = \left(\frac{\zeta}{\zeta_\mu[\mathcal{D}]}\right)^{-\mathcal{D}(\mu,b)} F(x,b) \tag{4}$$

$$\zeta_\mu[\mathcal{D}(b)] = \mu^2 e^{-\frac{g}{a_s(\mu)\mathcal{D}}}, \qquad g = \frac{\Gamma_0}{2\beta_0^2}\left(e^{-\mathcal{D}} + \mathcal{D} - 1\right) + a_s(...) + ...$$

This "exact" solution does not specify RAD.
It can be any function and the solution is valid at any b!

TMD factorization in ζ-prescription

$$\frac{d\sigma}{dxdzdQ^2d^2\mathbf{q}_T}=\sum_{ff'}H_{ff'}\left(\frac{Q}{\mu}\right)\int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)}\,F_{f\leftarrow p}(x_1,b,\mu,\zeta)F_{f'\leftarrow\pi}(x_2,b,\mu,\zeta)$$

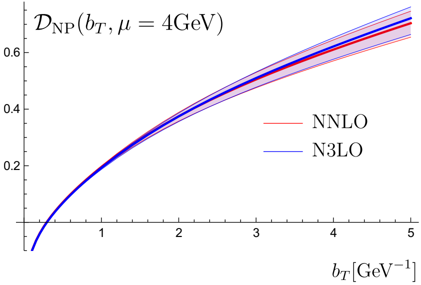


TMD evolution (optimal solution)
"ζ-prescription"

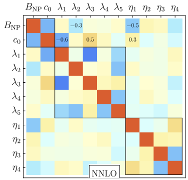
$$\frac{d\sigma}{dxdzdQ^2d^2\mathbf{q}_T}=\sum_{ff'}H_{ff'}\left(\frac{Q}{\mu}\right)\int d^2b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T)}\left(\frac{Q^2}{\zeta_\mu[\mathcal{D}]}\right)^{-2\mathcal{D}(b,\mu)}F_{f\leftarrow p}(x_1,b)F_{f'\leftarrow\pi}(x_2,b)$$

- $\mu\sim Q$ everywhere
- Simple and fast expression for TMD evolution factor (just an algebraic function)
- Simpler expression for perturbative matching for TMDs

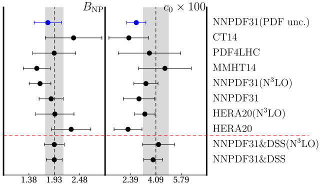
Universal TMD evolution kernel



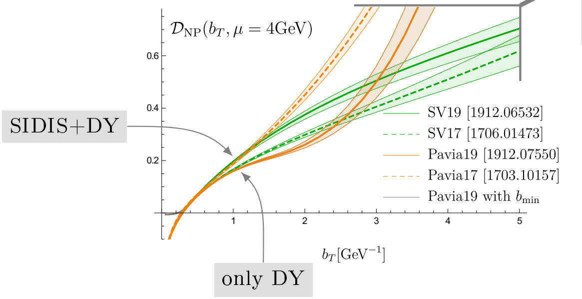
$$\mathcal{D}_{\rm NP}(b,\mu)=\mathcal{D}_{\rm perp}(b^*,\mu)+c_0 bb^*, \qquad b^*=b/\sqrt{1+b^2/B_{NP^2}}$$



- Extracted from **joined fit** of DY + SIDIS ($2<Q<150$GeV)
- Mild correlation with TMDs
- Stable with respect to collinear input



Universal TMD evolution kernel
Comparison



- SV19 $\sim b$
- SV17 $\sim b^2$
- Pavia19 $\sim b^4$
- Pavia17 $\sim b^2$

Intermediate conclusion

- RAD is an independent non-perturbative function
- RAD is independent on hadron!
- RAD is universal DY, SIDIS, jets, DPDs,...
- Small-b limit of RAD is perfectly fixed (N^3LO!)
- The large-b is **almost unknown**

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Many questions!

- If RAD is "an independent non-perturbative function" how to define in quantum field theory?
- Are there ways to compute the large-b for RAD?
- What it measures?

Despite RAD is known for $\sim 30-40$ years, there is no dedicated theoretical discussion in literature about these questions

Only a few papers which touch these questions [Tafat,010227],[Schweitzer,Strikman,Weiss,1210.1267],[Becher,Bell,1312.5327], [Collins,Rogers,1412.3820],[Scimemi,AV,1609.06047] **[AV,2003.02288]**

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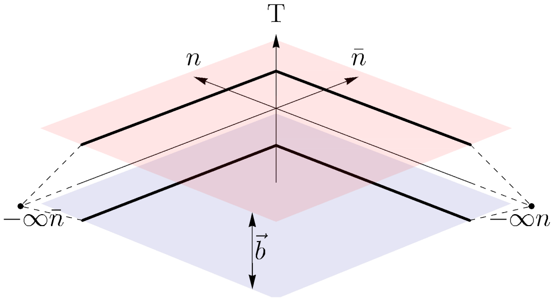
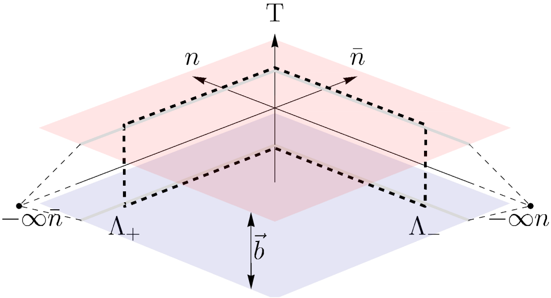
- RAD appears in TMD soft factor

$$S(b,\mu)=\frac{\text{Tr}}{N_c}\langle 0|W_C|0\rangle Z_S^2(\mu)$$

- In some proper regularization of rap.div.($\varrho\to 0$)

$$S(b,\mu)=\exp(2\mathcal{D}(b,\mu)\ln\varrho+B(b,\mu)+...)$$





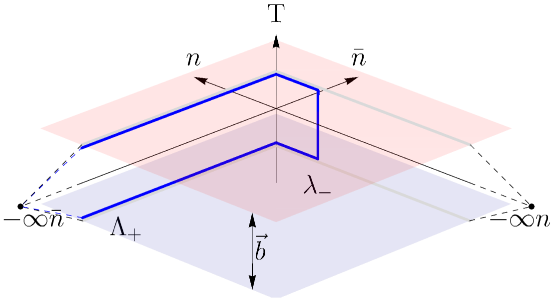
Suitable regulator

- Defined on the operator level
- Preserve gauge invariance
- Do not introduce extra divergences (nor mix with other divergences)

All "convenient" regulators fail...

It can be done by geometric deformation of WL's

$$\varrho = (\Lambda_+ \Lambda_-)^{-1}$$



Only one Λ is needed

$$S(b,\mu) = \frac{\text{Tr}}{N_c}\langle 0|W_{C'}(\Lambda_+\lambda_-)|0\rangle Z_S^2(\mu) = \exp\left(-2\mathcal{D}(b,\mu)\ln(\Lambda_+\lambda-) + B(\mu,b) + O\left(\frac{1}{\Lambda_+\lambda-}\right)\right)$$

$$\mathcal{D}(b,\mu) = \frac{1}{2}\lim_{\Lambda_+\to\infty}\frac{d\ln S_{C'}(b,\mu)}{d\ln\lambda_-}$$

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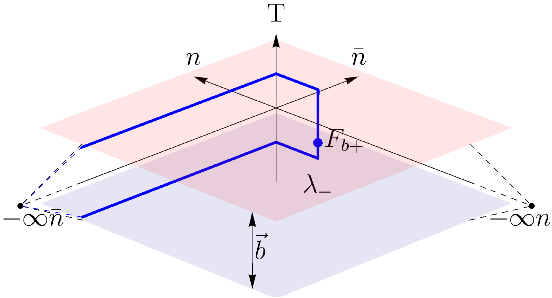

$$\mathcal{D}(b,\mu) = \lambda_- \frac{ig}{2} \frac{\mathrm{Tr} \int_0^1 d\beta \langle 0| F_{b+}(-\lambda_- n + b\beta) W_{C'} |0\rangle}{\mathrm{Tr}\langle 0| W_{C'} |0\rangle} + Z_{\mathcal{D}}(\mu)$$

- λ_- independent!
- Additive renormalization $\dfrac{dZ_{\mathcal{D}}}{d\ln\mu} = \Gamma_{cusp}(\mu)$

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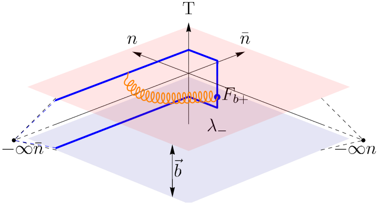
Perturbation

$$\text{LO-diagram} = -4g^2 C_F \frac{\Gamma(2-\epsilon)}{4\pi^{d/2}} \lambda_- \int_0^1 d\beta \int_{-\infty}^0 d\alpha \frac{b^2\beta}{[-2\alpha\lambda_- - \beta^2 b^2 + i0]^{2-\epsilon}}$$



- Exactly coincides with ordinary computation (as function of ϵ)
- Renormalization is indeed additive
- Structure of integrals here and in SF is different
- Some two-loop checks (cancellation of rap.divs.) also passes

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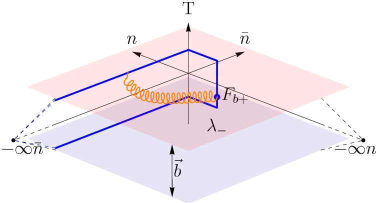

$$\mathcal{D}_{\text{LO}} = -2C_F a_s \left[\Gamma(-\epsilon)\left(\frac{\mathbf{b}^2\mu^2}{4e^{-\gamma_E}}\right)^{\epsilon} + \frac{1}{\epsilon}\right]$$

- Exactly coincides with ordinary computation (as function of ϵ)
- Renormalization is indeed additive
- Structure of integrals here and in SF is different
- Some two-loop checks (cancellation of rap.divs.) also passes

Small-b expansion

$$\mathcal{D}(b,\mu) = \mathcal{D}_0(b,\mu) + b^2\mathcal{D}_2(b) + b^4\mathcal{D}_4(b) + ...$$

- $\mathcal{D}_0$ starts from 1-loop, known up to 3-loop
- $\mathcal{D}_{2,4,..}$ starts from tree-orders, unknown

Using new definition they are straightforward to compute

- Tree order power corrections expresses via correlators of the type

$$\langle 0|F_{\mu_1 x_1}(x_1)[x_1,0]F_{\mu_2 x_2}(x_2)[x_2,0]...F_{\mu_n x_n}(x_n)[x_n,0]|0\rangle$$

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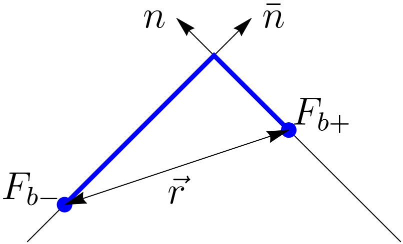
b^2-terms

$$\mathcal{D}_2(b) = \frac{1}{2}\int_0^\infty d\mathbf{r}^2 \frac{\varphi_1(\mathbf{r}^2, 0, 0)}{\mathbf{r}^2} + O(\alpha_s)$$

$$\Phi_{\mu\nu}(x,y) = g^2 \langle 0|F_{\mu x}(x)[x,0][0,y]F_{\nu y}(y)|0\rangle$$

$$\Phi_{\mu\nu}(x,y) = \left(g_{\mu\nu} - \frac{y_\mu x_\nu}{(xy)}\right)\varphi_1(r^2, x^2, y^2) \qquad (16)$$
$$+\frac{(x_\mu(xy) - y_\mu x^2)(y_\nu(xy) - x_\nu y^2)}{(xy)((xy)^2 - x^2y^2)}\varphi_2(r^2, x^2, y^2),$$

where $r^2 = (x-y)^2$. At $x^2 = y^2 = 0$, φ_2 vanishes



- First(and only) model-independent expression for power correction to CS kernel
- I have also checked it by direct computation of TMD SF in the back-ground (and extraction of rapidity divergence in δ-regulator)

$$\mathcal{D}_2(b) = \frac{1}{2}\int_0^\infty d\mathbf{r}^2 \frac{\varphi_1(\mathbf{r}^2,0,0)}{\mathbf{r}^2} + O(\alpha_s)$$

$$\lim_{\mathbf{r}^2\to 0} \frac{\varphi_1(\mathbf{r}^2,0,0)}{\mathbf{r}^2} = \frac{\pi^2}{36} G_2$$

Assuming that φ_1 has some effective radius $\sim \Lambda_{QCD}^{-1}$

$$\mathcal{D}_2 \sim \frac{\pi^2}{72}\frac{G_2}{\Lambda_{QCD}^2} \simeq (1.-5.)\times 10^{-2}\text{GeV}^{-2} \qquad (6)$$

	Pavia17	SV19	SV17	Pavia19	BLNY(03/14)
$\mathcal{D}_2 \times 10^2$	2.8 ± 0.5	2.9 ± 0.6	$0.7^{+1.2}_{-0.7}$	0.9 ± 0.2	$20 - 35$

The power correction is a notably small number.

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Interpretation can be made with model

Stochastic vacuum model

- Simplistic model of QCD vacuum
- Build-in confinement (area law)
- **Central assumption**: at large-distances Wilson lines can be ignored
- **Secondary assumption**: Multi-point correlators can be reduced to product of two-point correlators
- [Dosch,Shevchenko,Simonov,hep-ph-0212159]

$$\Delta_{\mu_1\nu_1;\mu_2\nu_2}(x_1,x_2;x_0) = g^2\frac{\mathrm{Tr}}{N_c}\langle 0|F_{\mu_1\nu_1}(x_1)[x_1,x_0][x_0,x_2]F_{\mu_2\nu_2}(x_2)|0\rangle.$$

$$\begin{aligned}\Delta_{\mu\nu;\lambda\rho}(u,v;x_0) &= \beta\Big\{(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda})\Delta((u-v)^2) \\ &+\frac{1}{2}\left[\frac{\partial}{\partial u_\mu}((u-v)_\lambda g_{\nu\rho}-(u-v)_\rho g_{\nu\lambda})+\frac{\partial}{\partial u_\nu}((u-v)_\rho g_{\mu\lambda}-(u-v)_\lambda g_{\mu\rho})\right]\Delta_1((u-v)^2)\Big\} \\ &= \beta\Big\{(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda})\left(\Delta(z^2)+\Delta_1(z^2)\right) \\ &+[g_{\mu\lambda}z_\nu z_\rho - g_{\nu\lambda}z_\mu z_\rho - g_{\mu\rho}z_\nu z_\lambda + g_{\nu\rho}z_\mu z_\lambda]\frac{\partial\Delta_1(z^2)}{\partial z^2}\Big\},\end{aligned} \tag{7.91}$$

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Computation of RAD in SVM

- Step 1: Apply Non-Abelian Stockes theorem
- Step 2: Rewrite all multi-point correlators $\langle F..F\rangle$ by products of $\langle FF\rangle^n$ (effectively replace non-Abelian Stockes theorem by Abelian one)
- Step 3: Make some change of variables and integration by parts

$$\mathcal{D} = \int_0^{\boldsymbol{b}^2} d\boldsymbol{y}^2 \left(\frac{\sqrt{\boldsymbol{b}^2}}{\sqrt{\boldsymbol{y}^2}} - 1\right)\left\{-\boldsymbol{y}^2\Delta_1(\boldsymbol{y}^2) + \int_0^\infty d\boldsymbol{r}^2 \left(\Delta(\boldsymbol{r}^2+\boldsymbol{y}^2) + \frac{\Delta_1(\boldsymbol{r}^2+\boldsymbol{y}^2)}{2}\right)\right\}.$$

Asymptotic behavior of RAD

- At large-b linear asymptotic

$$\lim_{\mathbf{b}^2\to\infty}\mathcal{D}(b)=\sqrt{\mathbf{b}^2}\underbrace{\int_0^\infty d\mathbf{y}^2 2\sqrt{y^2}\Delta(\mathbf{y}^2)}_{c_\infty},$$

- Lattice computations [Bali,Brambilla,Vairo,97; Meggiolaro,98]

$$c_\infty\simeq 0.01-0.4\text{GeV}\qquad\text{compare to}\qquad c_\infty^{\text{SV19}}\simeq 0.06\pm 0.01\text{GeV}$$

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$$\mathcal{D} = \int_0^{b^2} d\boldsymbol{y}^2 \left(\frac{\sqrt{\boldsymbol{b}^2}}{\sqrt{\boldsymbol{y}^2}} - 1 \right) \left\{ -\boldsymbol{y}^2 \Delta_1(\boldsymbol{y}^2) + \int_0^\infty d\boldsymbol{r}^2 \left(\Delta(\boldsymbol{r}^2 + \boldsymbol{y}^2) + \frac{\Delta_1(\boldsymbol{r}^2 + \boldsymbol{y}^2)}{2} \right) \right\}.$$

Common assumption $\Delta \gg \Delta_1$

$$\Delta(\mathbf{b}) = \frac{-1}{4b} \frac{\partial^3 \mathcal{D}(\mathbf{b})}{\partial \mathbf{b}^3}, \qquad \mathbf{b} = \sqrt{\mathbf{b}^2}$$

In this model, RAD is directly related to the non-perturbative gluon propagator

Interesting note: to solve the equation I must assume that (similar conclusion made in [Collins,Rogers,1412.3820])

$$\lim_{\mathbf{b}\to\infty} \mathcal{D}(\mathbf{b}) \sim (\mathbf{b}^2)^{1/2-\delta}, \qquad \delta \geqslant 0 \tag{7}$$

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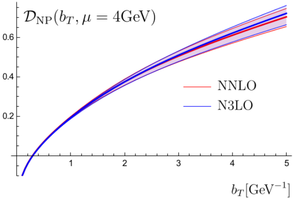
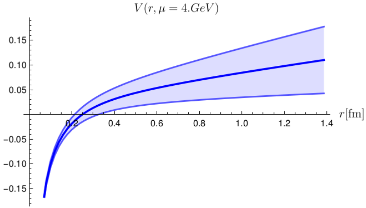
Relation to the static potential

In SVM the potential between two quark sources (confining potential) is [Brambilla,Vairo,hep-ph/9606344]

$$V(\boldsymbol{b}) = 2\int_0^{\boldsymbol{b}} d\boldsymbol{y}(\boldsymbol{b}-\boldsymbol{y})\int_0^\infty d\boldsymbol{r}\Delta(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}) + \int_0^{\boldsymbol{b}} d\boldsymbol{y}\boldsymbol{y}\int_0^\infty d\boldsymbol{r}\Delta_1(\sqrt{\boldsymbol{r}^2+\boldsymbol{y}^2}).$$





$$V(r) = r\frac{\pi}{4}\mathcal{D}''(0) + \frac{\mathcal{D}'(0)}{2} + \frac{r^2}{2}\int_r^\infty \frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}} + \dots$$

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Conclusion

Collins-Soper kernel is the way to study QCD vacuum with collider experiments

Self-contained definition of Collins-Soper kernel

- Definition without reference to any process/distribution
- A way to power-corrections,non-perturbative modeling and interpretation
- A peculiar quantum-field-theoretical construction: anomalous dimension as a matrix element
- Casimir scaling at power-correction level

Many thoughts...

- Iterative structure
- Soft/rapidity anomalous dimension correspondence
- ...

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