

Limits and uncertainties of TMD factorization

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in collaboration with I.Scimemi
based on [1706.01473]

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Motivation

- The theory of TMD made huge progress in recent years
 - Evolution at $N^3\text{LO}$ [Li & Zhu,1604.01404; AV,1610.05791]
 - Coefficient function at NNLO [many]
 - Matching coefficients at NNLO [many]
 - Structure of power suppressed terms on small- b OPE [Scimemi & AV,1609.06047]
- Not widely used in phenomenology!

Talk synopsis

We have made the global fit of Drell-Yan data, that include high and low energy measurements, and the extraction of unpolarized TMDPDF with the use of latest theory achievements.

additionally we made

- Study of (perturbative) theory converge.
- Study of limits of TMD factorization (Y -term).
- Evaluate theoretical uncertainties.
- Consider multiple NP inputs, and favour/disfavour particular structures.

High- & low-energy data are used

- High-energy \Rightarrow precise fixation of asymptotic
- Low-energy \Rightarrow better access to NP structure
- To start with we considered only "well-established" data
- In the final fit $309 = \underbrace{163}_{\text{high}} + \underbrace{146}_{\text{low}}$ points used.

Included data

	reaction	\sqrt{s}	Q	comment	points
E288	$p + Cu \rightarrow \gamma^* \rightarrow \mu\mu$	19.4 GeV	4-9 GeV	norm=0.8	35
E288	$p + Cu \rightarrow \gamma^* \rightarrow \mu\mu$	23.8 GeV	4-9 GeV	norm=0.8	45
E288	$p + Cu \rightarrow \gamma^* \rightarrow \mu\mu$	27.4 GeV	4-9 & 11-14 GeV	norm=0.8	66
CDF+D0	$p + \bar{p} \rightarrow Z \rightarrow ee$	1.8 TeV	66-116 GeV		44
CDF+D0	$p + \bar{p} \rightarrow Z \rightarrow ee$	1.96 TeV	66-116 GeV		43
ATLAS	$p + p \rightarrow Z \rightarrow \mu\mu$	7 & 8 TeV	66-116 GeV	tiny errors!	18
CMS	$p + p \rightarrow Z \rightarrow \mu\mu$	7 & 8 TeV	60-120 GeV		14
LHCb	$p + p \rightarrow Z \rightarrow \mu\mu$	7 & 8 & 13 TeV	60-120 GeV		30
ATLAS	$p + p \rightarrow Z/\gamma^* \rightarrow \mu\mu$	8 TeV	46-66 GeV		5
ATLAS	$p + p \rightarrow Z/\gamma^* \rightarrow \mu\mu$	8 TeV	116-150 GeV		9



unpolarized Drell-Yan \Rightarrow unpolarized TMDPDF

Theory input

$$\frac{d\sigma}{dQdy d^2q_T} = H(Q, \mu) \int \frac{d^2b}{(2\pi)^2} e^{-ibq_T} F(x_A, b; \mu, \zeta) F(x_B, b; \mu, \zeta) + Y$$

$$F(x, \mathbf{b}; \mu, \zeta) = R[\mathbf{b}; (\mu, \zeta) \rightarrow (\mu_{\text{low}}, \zeta_\mu)] F^{\text{low}}(x; \mathbf{b})$$

$$F_k^{\text{low}}(x, \mathbf{b}) = \int_x^1 \frac{dy}{y} C_{k \leftarrow l}(y, \mathbf{b}; \mu) f_l\left(\frac{x}{y}, \mu\right) f_{NP}(y; \mathbf{b})$$



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hard c.
LO
NLO
NNLO

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evolution kernel
cusp ADs
NLO LO
NNLO NLO
N³LO NNLO

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small-*b* mach.
LO
NLO
NNLO

[MHHT2014](#)

We can define four successive orders

Name	$ C_V ^2$	$C_{f \leftarrow f'}$	Γ	γ_V	\mathcal{D}	PDF set	$a_s(\text{run})$	ζ_μ
NLL	a_s^0	a_s^0	a_s^2	a_s^1	a_s^1	nlo	nlo	NLL
NLO	a_s^1	a_s^1	a_s^2	a_s^1	a_s^1	nlo	nlo	NLO
NNLL	a_s^1	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLL
NNLO	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

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pure TMD factorization
 \Rightarrow small q_T

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object of the fit
minority restricted

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NNLL	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLL
NNLO	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

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ζ -prescription
to handle
large logs

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NNLL	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLL
NNLO	a_s^2	a_s^2	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

ζ -prescription

$$\ln(\mu^2 \mathbf{b}^2),$$

- There are (potentially large) logs of \mathbf{b} . Some prescription is needed to handle it.
- Typically, b^* -prescription used \Rightarrow induces power corrections (difficult to control)
- ζ -prescription does not introduce any artificial dependence

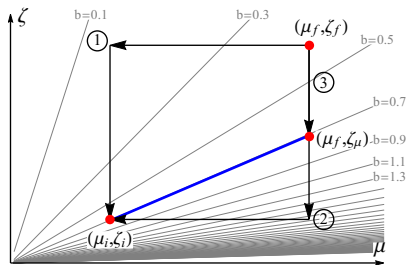


ζ -prescription

$$\ln(\mu^2 \mathbf{b}^2), \quad \ln(\zeta \mathbf{b}^2)$$

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ζ -prescription in a nutshell



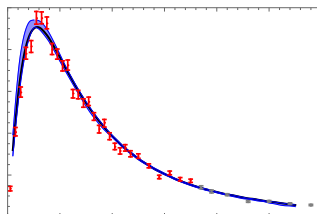
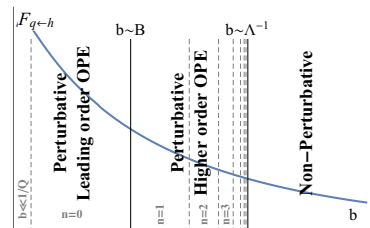
TMD evolves by a pair of equations.
Let logs cancel each other exactly.

$$\mu^2 \frac{d}{d\mu^2} F(x, \mathbf{b}; \mu, \zeta_\mu) = 0.$$

ζ_μ defined perturbatively

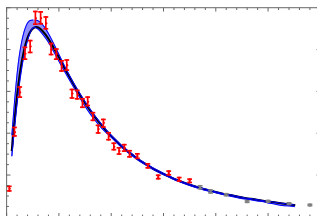
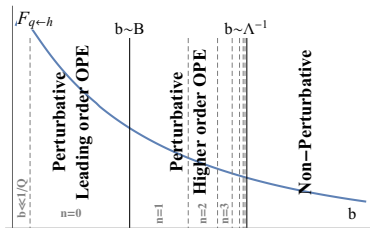
$$\text{NLO: } \zeta_\mu = \frac{2\mu}{|\mathbf{b}|} e^{-\gamma_E} e^{3/2+\dots}$$

Non-perturbative input



$b \ll Q^{-1}$	Perturbative	Not observable, deeply in Y -term dominated region
$b \ll B$	Perturbative	Leading twist contribution $F(x, b) \sim C(x, b) \otimes f(x)$
$b \sim B$	Perturbative but not calculable	Higher twist $F(x, b) \sim \sum_n \left(\frac{xb^2}{B^2}\right)^n C_n(x, b) \otimes f_n(x)$ the main scale parameter is xb^2 [I.Scimemi,AV, 1609.06047] $n = 1$ term can be estimated.
$b > \Lambda^{-1}$	Non-perturbative	Nothing is know. Exponential? Gaussian?

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Additionally, there can be non-perturbative contribution to the rapidity evolution
only even powers can appear

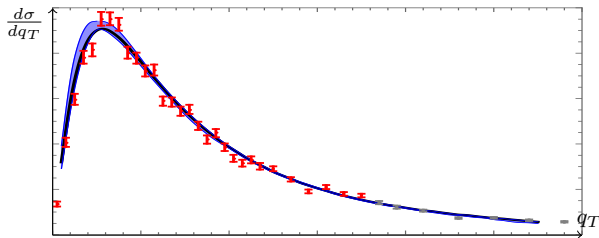
$$\mathcal{D}(b) = \mathcal{D}^{\text{perp}}(b) + g_K b^2 + \dots$$

Theory prediction: very small or zero $g_K = 0.01 \pm 0.03 \text{ GeV}^2$ [I.Scimemi,AV, 1609.06047]

Limits of application of TMD factorization \leftrightarrow size of Y-term

$$\frac{d\sigma}{dQ dy d^2q_T} = H(Q, \mu) \int \frac{d^2b}{(2\pi)^2} e^{-ibq_T} F(x_A, b; \mu, \zeta) F(x_B, b; \mu, \zeta) + Y$$

TMD factorization derived at small q_T
 the leading correction $\sim q_T^2/Q^2$



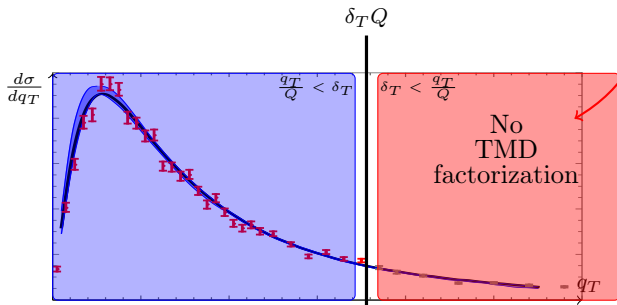
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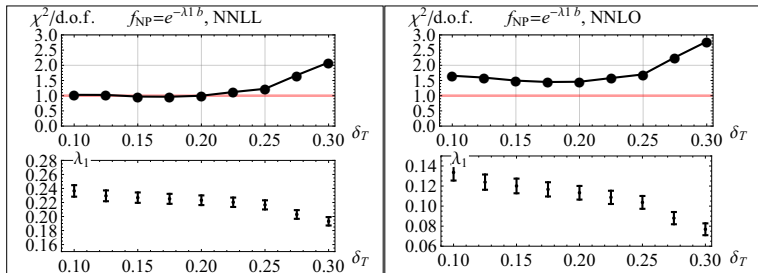
We include all points with $q_T < \delta_T Q$

To find the value of δ_T , we check **the stability of the fit**

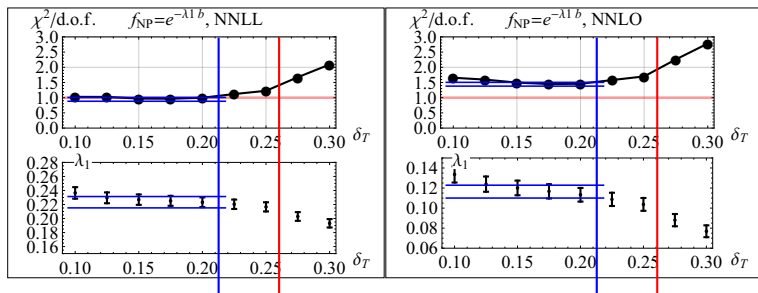
- Make fits with increasing δ_T (0.1 \rightarrow 0.3) (165 \rightarrow 399 points)
- The value of $\chi^2/d.o.f.$ should be independent on δ_T in allowed region



Example: $f_{NP} = e^{-\lambda b}$, no E288



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- $\delta_T < 0.2$ save region,
- $\delta_T < 0.25$ un-save region,
- $\delta_T > 0.25$ TMD factorization does not work.

To be on the save side we used $\delta_T = 0.2$
 There are 309 data points



Asymptotic of f_{NP}

Large- $b \Leftrightarrow$ small- q_T

The δ_T scans are very instructive!

The smaller- δ_T the better (not worse!) the fit should be. Unless f_{NP} is wrong.

Asymptotic of f_{NP}

Domination of $f_{NP} \rightarrow$ Large- $b \Leftrightarrow$ small- $q_T \leftarrow$ Very precise at HE

f_{NP} at large- b is nicely fixed from high-energy measurements.

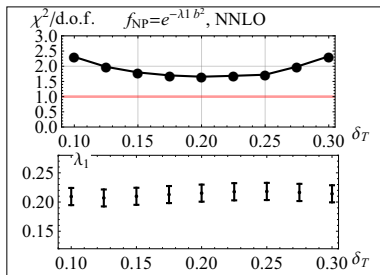
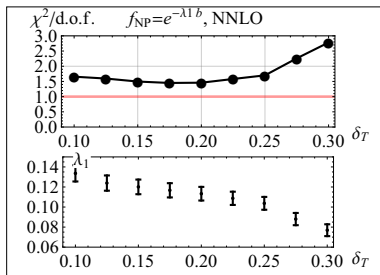
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Exp: 1.4 \rightarrow 1.6

vs.

Gauss: 1.7 \rightarrow 2.3



Perturbative uncertainties

$c_1 \rightarrow$ uncertainty of
RAD definition

$c_2 \rightarrow$ uncertainty of
hard matching

$c_3 \rightarrow$ uncertainty of
small-b matching

Total uncertainty is
the maximum of three

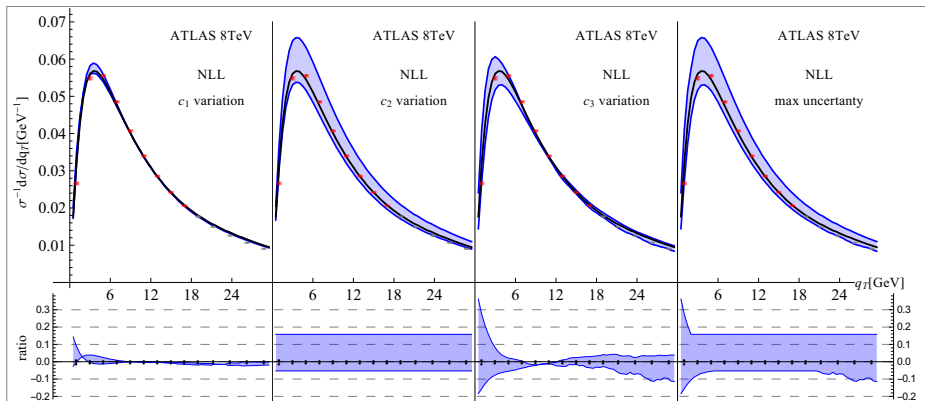
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$$H(c_2 \mu) F(c_2 \mu) F(c_2 \mu)$$

$$C(c_3 \mu_{\text{low}}) \otimes f(c_3 \mu_{\text{low}})$$

$$c_i \in (0.5, 2)$$

High-energy example: ATLAS 8 TeV (best precision)



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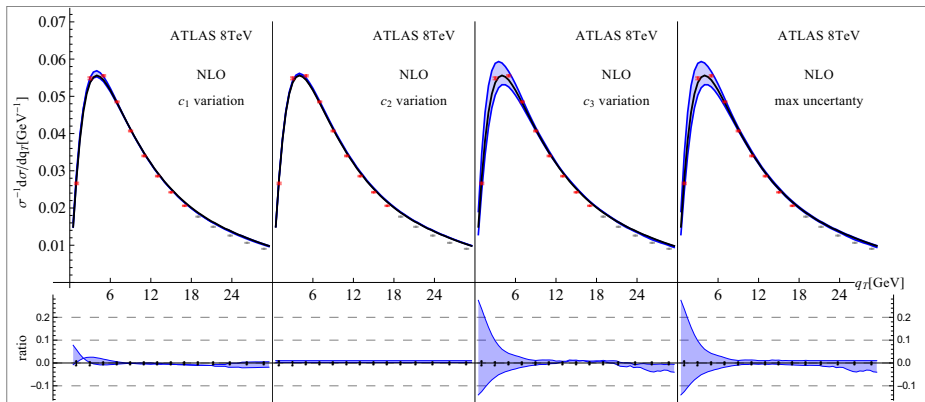
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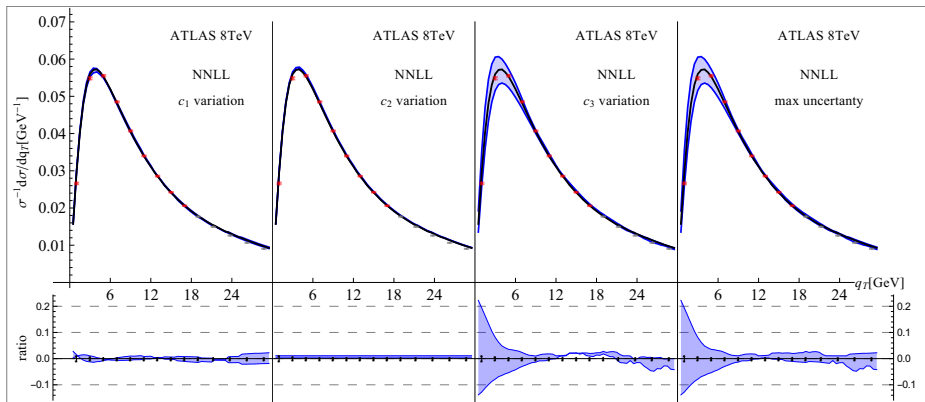
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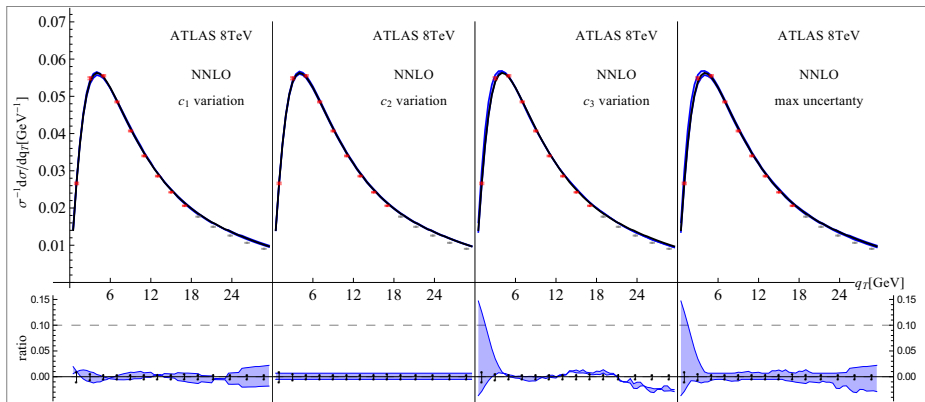
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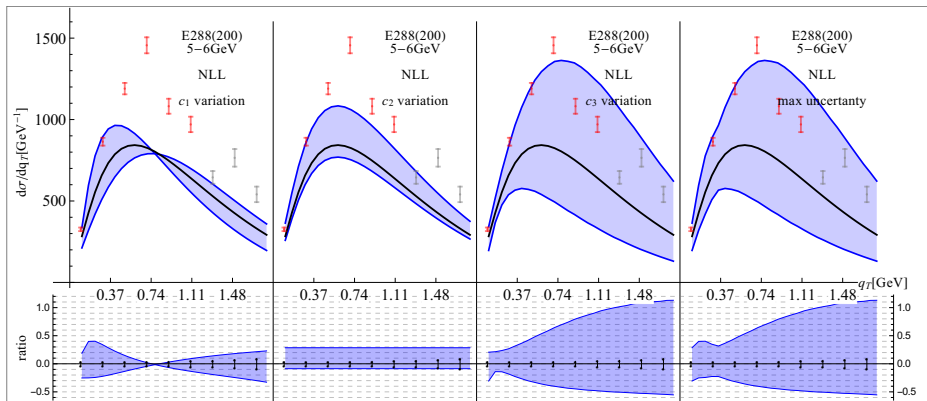
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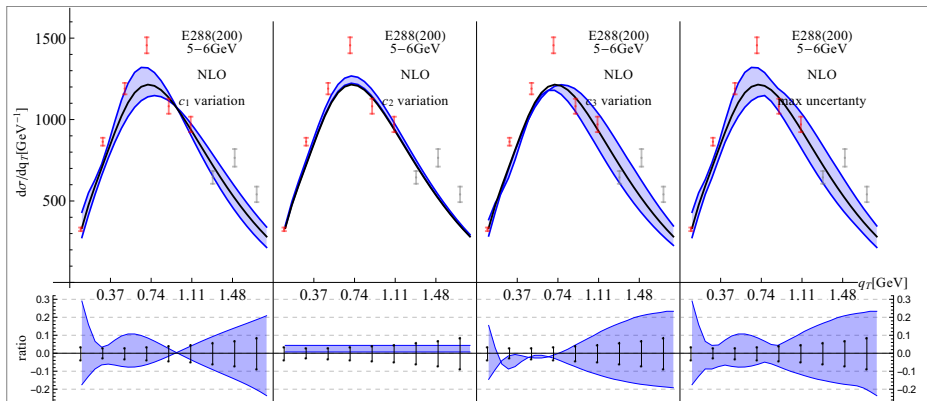
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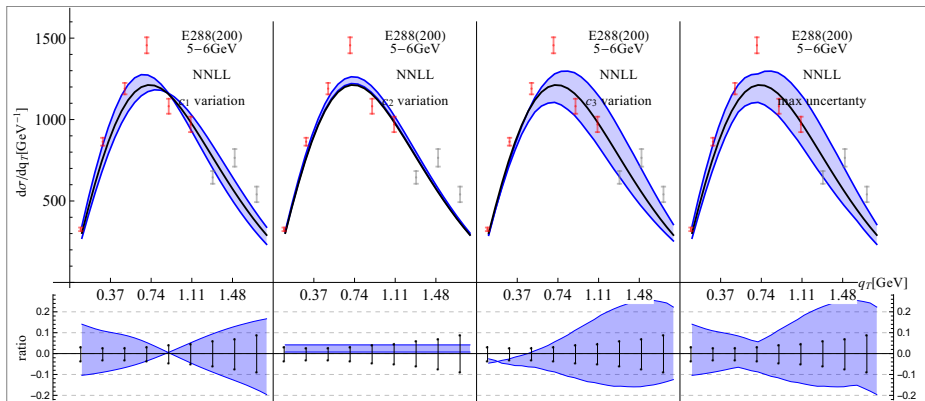
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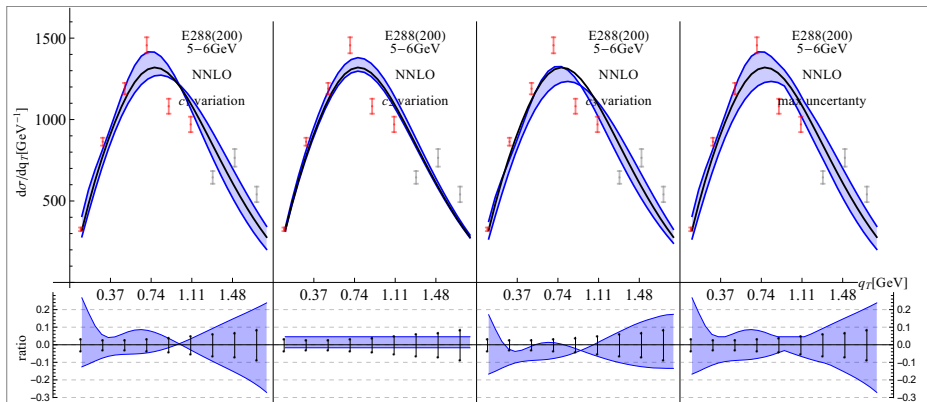
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Convergence of theory

We have selected 3 main models (all with 2 parameters)

Model 1

$$f_{NP} = e^{-\lambda_1 b} (1 + \lambda_2 b^2)$$

[D'Alesio, et al,1407.3311]

Model 2

$$f_{NP} = \exp\left(\frac{-\lambda_q z b}{\sqrt{1 + \frac{\lambda_q z^2 b^2}{\lambda_1^2}}}\right)$$

+ renormalon correction $\times \lambda_2$

[Scimemi,AV,1609.06047]

Model 3

$$f_{NP} = e^{-\lambda_1 b}$$
$$\mathcal{D}^{NP} = -g_K \mathbf{b}^2$$



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$$\mathcal{D}^{\text{NP}} = -g_K \mathbf{b}^2$$

All models show similar behaviour

$$\lim_{b \rightarrow \infty} f_{NP}(x, b) \sim e^{-\lambda_1 b}$$

$$f_{NP}(x, b) \sim 1 + \lambda_2 b^2 f'(x, 0) + \dots$$

$$\lambda_1 = 0.156(6) \text{ GeV}$$

$$\lambda_2 = 3.7(2) \times 10^{-2} \text{ GeV}^2$$

$$\lambda_1 = 0.162(5) \text{ GeV}$$

$$\lambda_2 = 1.3(8) \times 10^{-2} \text{ GeV}^2$$

$$\lambda_1 = 0.174(6) \text{ GeV}$$

$$g_K = 1.5(2) \times 10^{-2} \text{ GeV}^2$$

We cannot distinguish NP evolution from a power correction.

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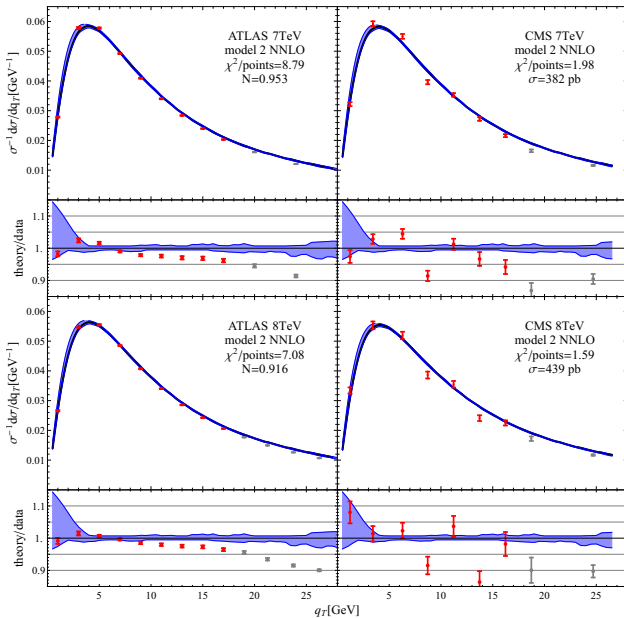
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$$\mathcal{D}^{NP} = -g_K \mathbf{b}^2$$

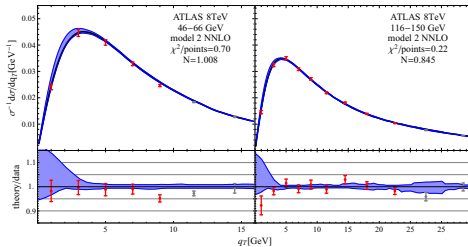
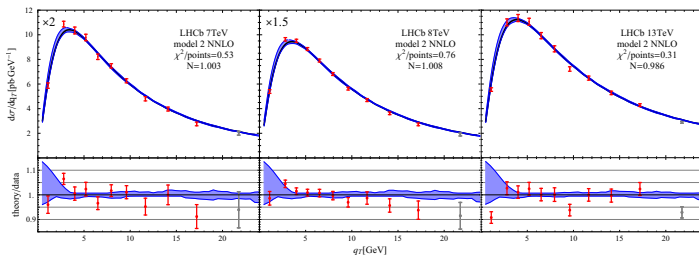
$\chi^2/d.o.f$			
	Model 1	Model 2	Model 3
NLL	9.21	9.07	8.90
NLO	2.62	2.64	1.90
NNLL	1.43	1.46	1.16
NNLO	1.84 (1.40)	1.79 (1.39)	1.94 (1.42)

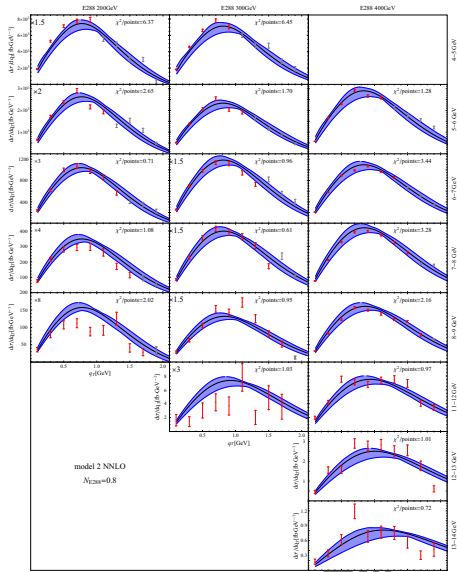
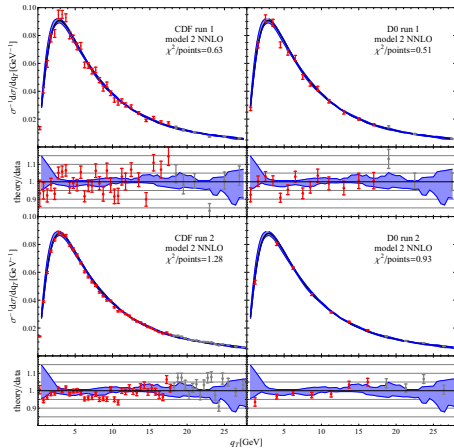
Main source of "problems" is ATLAS Z-boson



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Conclusion

arTeMiDe

- Code available : <https://teorica.fis.ucm.es/artemide/>
- Optimized (but not limited to) ζ -prescription.
- Evolution (up to N³LO), unpolarized TMDPDF (with up to NNLO matching), DY cross-sections.
- Flexible structure: input PDFs, f_{NP} .
- More to come...

Results of global fit

- Exponential TMD is preferable (ζ -prescription) $f_{NP} \sim \exp(-\lambda_1 b)$
- Natural large-distance scale $\lambda_1 \simeq 0.14 - 0.17 \text{ GeV}$ ($\sim m_\pi$).
- The TMD factorization works at $q_T \lesssim 0.2Q$
- Theory uncertainty is significant at small- Q (10-20% for $Q \sim 4 - 9\text{GeV}$)
- We cannot distinguish NP evolution from NP corrections.
- NNLO ingredients significantly reduce the theory uncertainties and prediction power.

LHC normalization

order	ATLAS Z-boson 7TeV	ATLAS Z-boson 8TeV	ATLAS 46-66 8TeV	ATLAS 116-150 8TeV	CMS 7TeV	CMS 8TeV	LHCb 7TeV	LHCb 8TeV	LHCb 8TeV
NLL	<i>0.80(1)</i>	0.79(1)	0.86(1)	0.74(0)	<i>0.83(1)</i>	<i>0.84(1)</i>	0.86(1)	0.87(1)	0.87(1)
NLO	<i>0.89(1)</i>	0.87(1)	0.96(3)	0.81(1)	<i>0.91(2)</i>	<i>0.93(2)</i>	0.95(2)	0.95(1)	0.95(1)
NNLL	<i>0.95(2)</i>	0.93(2)	1.04(4)	0.86(1)	<i>0.98(2)</i>	<i>1.00(2)</i>	1.01(2)	1.01(2)	1.01(2)
NNLO	<i>0.94(1)</i>	0.92(1)	1.01(2)	0.85(1)	<i>0.97(1)</i>	<i>0.99(1)</i>	1.00(1)	1.01(1)	1.01(1)

