Extraction of unpolarized TMD PDFs at NNLO: analysis and result

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in collaboration with I.Scimemi
based on [1706.01473]**(ver 2)**

Resummation, Evolution, Factorization
Madrid
Nov.2017

Motivation

Theory of TMDs

Huge progress in recent years

- Proof of collinear and rapidity factorization parts of TMD factorization
- NLO, NNLO, N^3LO pertubative calculations
- Various W to Y matching

Phenomenology of TMDs

A lot of data & a lot of fits

- Plentiful asymmetries from HERMES, COMPASS
- Many individual fits
- First global fits (SIDIS+DY)[A.Bacchetta,*at al*, 1703.10157]

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Practically do not use each other

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Efforts needed to bring together

Needed update for the theory

- Still a lot to calculate (polarized TMDs!)
- W-to-Y, power corrections,...
- Alternative NP inputs (lattice?)

Needed update for the phenomenology

- Critical, theory motivated reanalysis of data
- New codes for TMD with loops
- Any codes for twist-3 inputs (with evolution)

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That's what we do!

That's what we do!

But we just start.

Status

Theory state	
Universal	
Hard part	N^{3}LO
Evolution	N^{3}LO
Matching	
f_1	NNLO
g_1	NLO
h_1	**(N)NLO**
$h_{1T}^{\perp}$	**(N)NLO**($\neq 0$!)
$f_{1T}^{\perp}$	NLO???
$h_1^{\perp}$	–
$h_{1L}^{\perp}$	–
g_{1T}	–

- **`arTeMiDe`** package for evaluation of TMDs and related cross-sections.
  Ver.1.1 includes f_1
  (https://teorica.fis.ucm.es/artemide)
- We have extracted f_1 from DY and Z-boson production. (Presented here)

That's what we do!

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Extracted from jet evolution
talk tommorow 18:00

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$h_1^{\perp}$	–

Daniel Gutierrez Reyes
talk tommorow 12:30

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Theory input

$$\frac{d\sigma}{dQdy\,d^2q_T} = H(Q,\mu)\int \frac{d^2b}{(2\pi)^2}\, e^{-ibq_T}\, F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta) + Y$$

$$F(x,\mathbf{b};\mu,\zeta) = R[\mathbf{b};(\mu,\zeta)\to(\mu_{\text{low}},\zeta_\mu)]F^{\text{low}}(x;\mathbf{b})$$

$$F_k^{\text{low}}(x,\mathbf{b}) = \int_x^1 \frac{dy}{y} C_{k\leftarrow l}(y,\mathbf{b};\mu) f_l\left(\frac{x}{y},\mu\right) f_{NP}(y;\mathbf{b})$$

unpolarized Drell-Yan⇒ unpolarized TMDPDF

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hard c.
LO
NLO
NNLO

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evolution kernel

cusp	ADs
NLO	LO
NNLO	NLO
N^{3}LO	**NNLO**

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small-b mach.
LO
NLO
NNLO

MHHT2014

We can define four successive orders

Name	$\lvert C_V\rvert^2$	$C_{f\leftarrow f'}$	Γ	γ_V	$\mathcal{D}$	PDF set	a_s(run)	ζ_μ
NLL/LO	a_s^0	a_s^0	a_s^2	a_s^1	a_s^2	nlo	nlo	NLL
NLL/NLO	a_s^1	a_s^1	a_s^2	a_s^1	a_s^2	nlo	nlo	NLO
NNLL/NLO	a_s^1	a_s^1	a_s^3	a_s^2	a_s^3	nnlo	nnlo	NNLL
NNLL/NNLO	a_s^2	a_s^3	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

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pure TMD factorization ⇒ small q_T

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minory restricted

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NNLL/NLO	a_s^1	a_s^1	a_s^3	a_s^2	a_s^3	nnlo	nnlo	NNLL
NNLL/NNLO	a_s^2	a_s^3	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

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Theory input

$$\frac{d\sigma}{dQdy\,d^2q_T} = H(Q,\mu)\int \frac{d^2b}{(2\pi)^2}\, e^{-ibq_T}\, F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta) + Y$$

hard c.: LO, NLO, **NNLO**

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$$F(x,\mathbf{b};\mu,\zeta) = R[\mathbf{b};(\mu,\zeta)\to(\mu_{\text{low}},\zeta_\mu)]F^{\text{low}}(x;\mathbf{b})$$

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ζ-prescription to handle large logs

$$F_k^{\text{low}}(x,\mathbf{b}) = \int_x^1 \frac{dy}{y} C_{k\leftarrow l}(y,\mathbf{b};\mu) f_l\left(\frac{x}{y},\mu\right) f_{NP}(y;\mathbf{b})$$

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NNLL/NLO	a_s^1	a_s^1	a_s^3	a_s^2	a_s^3	nnlo	nnlo	NNLL
NNLL/NNLO	a_s^2	a_s^3	a_s^3	a_s^2	a_s^2	nnlo	nnlo	NNLO

ζ-prescription

$$\ln(\mu^2 \mathbf{b}^2), \qquad \ln(\zeta\, \mathbf{b}^2)$$

- There are (potentially large) logs of **b**. Some prescription is needed to handle it.
- Typically, b^*-prescription used $\Rightarrow$ induces power corrections and new parameters
- ζ-prescription does not introduce any artificial dependence

ζ-prescription uses the freedom
(granted to us by factorization theorem)
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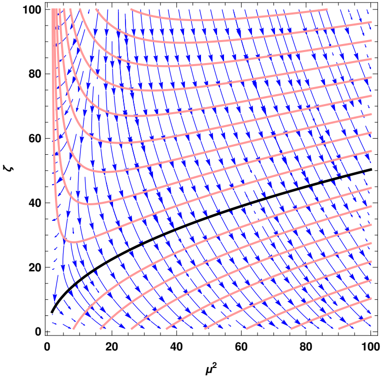
(granted to us by factorization theorem)
to choose the scale in any convenient way.

This freedom has not been used yet.

TMD evolution is multi-scale evolution

$$\mu^2 \frac{dF(x,\mathbf{b};\mu,\zeta)}{d\mu^2} = \frac{1}{2}\gamma_K(\mu,\zeta)F(x,\mathbf{b};\mu,\zeta)$$
$$\zeta \frac{dF(x,\mathbf{b};\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,\mathbf{b})F(x,\mathbf{b};\mu,\zeta)$$

The (μ,ζ)-plane has a rich structure



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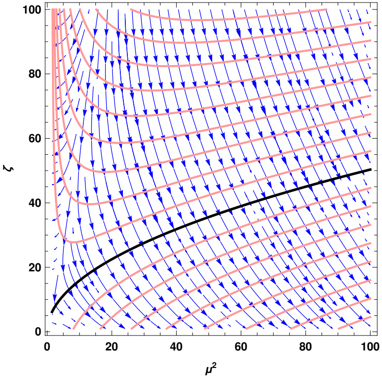

There are equi-evolution lines in the (μ,ζ)-plane

$$\mu^2 \frac{dF(x,\mathbf{b};\mu,\zeta(\mu))}{d\mu^2} = 0.$$

ζ-prescription in practice

In the ζ-prescription a TMD is EVOLUTIONless

$$\mu^2 \frac{d}{d\mu^2} F(x, \mathbf{b}; \mu, \zeta_\mu) = 0 \qquad \Leftrightarrow \qquad \zeta_\mu = \frac{2\mu}{|\mathbf{b}|} e^{-\gamma_E} \overbrace{e^{3/2+\dots}}^{\text{PT-calculable here LO}} .$$

ζ-prescription eliminates large logs from the expressions. Good example is the coefficient function:

$$F(x, \mathbf{b}; \mu, \zeta) = C(x, \mathbf{b}; \mu, \zeta) \otimes f(x, \mu)$$

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charge conservation: $\int_0^1 dx C(x,b) \otimes f(x) = \text{const}$

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$$F(x, \mathbf{b}; \mu, \zeta) = C(x, \mathbf{b}; \mu, \zeta) \otimes f(x, \mu)$$

No need for b^*, since there are no large logs.
But b^* can be used, (we are not).
Our choose the simplest function: $\mu_{\text{low}} = \mu_{\text{OPE}} = \frac{C_0}{b} + 2$

charge conservation: $\int_0^1 dx C(x, b) \otimes f(x) = \text{const}$

Perturbative uncertainties with in TMD cross-section

There are four perturbative scale entries ⇒ four constants to vary $\{c_1, c_2, c_3, c_4\}$.

$$\frac{d\sigma}{dX} = H(c_2\mu_{\text{hard}})\Big[R(c_2\mu_{\text{hard}} \to (c_3\mu_{\text{low}}, \zeta_{c_3\mu}; c_1\mu_0)F(c_4\mu_{\text{OPE}}, \zeta_{c_4\mu})\Big]^2$$

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Hard matching (c_2 in H and R)

Rapidity evolution matching ($c_1\mu_0$)

Low scale matching ($c_3\mu_{\text{low}}$, $\zeta_{c_3\mu}$)

Low scale matching ($c_4\mu_{\text{OPE}}$, $\zeta_{c_4\mu}$)

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Hard matching

Rapidity evolution matching

Evolution is disentangeled from TMD; thanks to ζ-prescription

Low scale matching

Low scale matching

Variation of c_i also leads to uncertanty in determination of NP parameters

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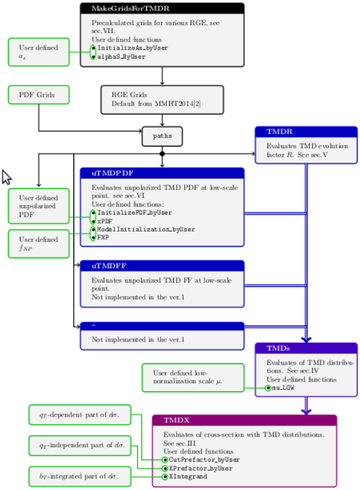

- FORTRAN 90 code
- Module structure
- Convolutions, evolution (LO,NLO,NNLO)
- Fourier to q_T-space, integrations over phase space
- Scale-variation (ζ-prescription)
- User defined PDFs, scales, f_{NP}
- Efficient code ($\sim 10^9$ TMDs $\sim$ 6. min at NNLO)

Currently ver 1.1

Available at: https://teorica.fis.ucm.es/artemide

Future plans: add modules for fragmentations, and polarized TMDs

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High- & low-energy data are used

- High-energy ⇒ precise fixation of asymptotic
- Low-energy ⇒ better access to NP structure
- To start with we considered only "well-established" data
- In the final fit $309 = \underbrace{163}_{\text{high}} + \underbrace{146}_{\text{low}}$ points used.

Included data (at $q_T < 0.2Q$)

	reaction	$\sqrt{s}$	Q	comment	points
E288	$p + Cu \to \gamma^* \to \mu\mu$	19.4 GeV	4-9 GeV	norm=0.8	35
E288	$p + Cu \to \gamma^* \to \mu\mu$	23.8 GeV	4-9 GeV	norm=0.8	45
E288	$p + Cu \to \gamma^* \to \mu\mu$	27.4 GeV	4-9 & 11-14 GeV	norm=0.8	66
CDF+D0	$p + \bar{p} \to Z \to ee$	1.8 TeV	66-116 GeV		44
CDF+D0	$p + \bar{p} \to Z \to ee$	1.96 TeV	66-116 GeV		43
ATLAS	$p + p \to Z \to \mu\mu$	7 & 8 TeV	66-116 GeV	tiny errors!	18
CMS	$p + p \to Z \to \mu\mu$	7 & 8 TeV	60-120 GeV		14
LHCb	$p + p \to Z \to \mu\mu$	7 & 8 & 13 TeV	60-120 GeV		30
ATLAS	$p + p \to Z/\gamma^* \to \mu\mu$	8 TeV	46-66 GeV		5
ATLAS	$p + p \to Z/\gamma^* \to \mu\mu$	8 TeV	116-150 GeV		9
				Total	309

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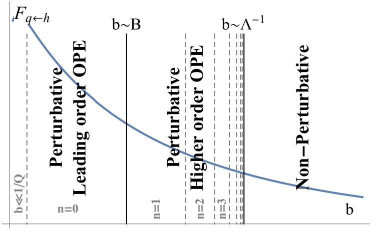
Non-perturbative input $F = C \otimes f \times f_{NP}$



$b \ll Q^{-1}$	Perturbative	Not observable, deeply in Y-term dominated region
$b \ll B$	Perturbative	Leading twist contribution $F(x,b) \sim C(x,b) \otimes f(x)$
$b \sim B$	Perturbative but not calculable	Higher twist $F(x,b) \sim \sum_n \left(\frac{xb^2}{B^2}\right)^n C_n(x,b) \otimes f_n(x)$ the main scale parameter is xb^2 [I.Scimemi,AV, 1609.06047] $n=1$ term can be estimated.
$b > \Lambda^{-1}$	Non-perturbative	Nothing is know. Presumably exponential.

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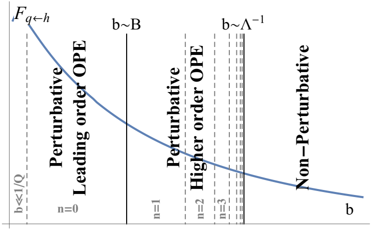

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Additionally, there can be non-perturbative contribution to the rapidity evolution

only even powers can appear

$$\mathcal{D}(b) = \mathcal{D}^{\text{perp}}(b) + g_K b^2 + \ldots$$

Theory prediction: very small or zero $g_K = 0.01 \pm 0.03 \text{GeV}^2$ [I.Scimemi,AV, 1609.06047]

Selection of f_{NP}

We have tested many models with different behaviour.

Lessons

- Test at NNLO. Since at NLO (or NLL) all models are equally good/bad.
- High-energy experiments favour Gaussian-like
- Low-energy experiment favour exponent-like
- Need at least 2 parameters (to control b^2 correction and the tail)

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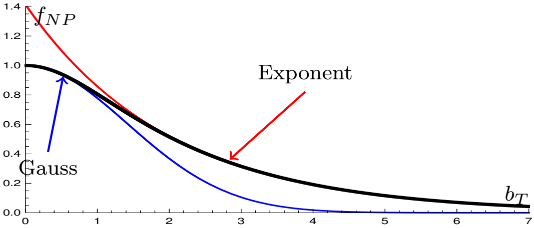


Best models (2 parameters)+g_K

$$f_{NP} = \frac{\cosh(\lambda_2 b)}{\cosh(\lambda_1 b)}$$

$$f_{NP} = \exp\left[-\frac{z\lambda_2 b^2}{\sqrt{1+\left(zb\frac{\lambda_2}{\lambda_1}\right)^2}}\right]$$

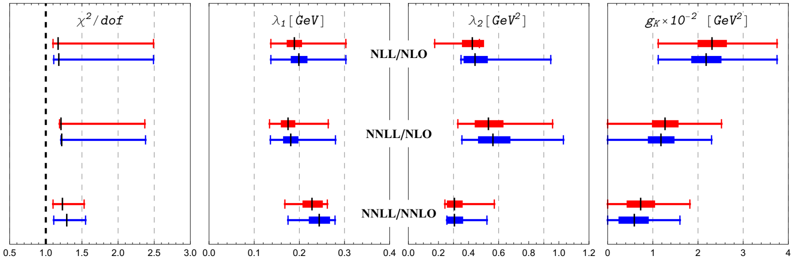
both

$$\frac{\chi^2}{dof} \simeq 1.2$$

Just as expected from theory!

Results

- For both models result practically the same
- NLL/LO has tremendously large error-bars → **excluded**
- $\chi^2/dof \sim 1.2$ with 2+1 parameters
- Theoretical error-bands significantly decrease at NNLL/NNLO
- g_K is very important at lower orders, but tends to zero with order increase



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Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition

$$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$$

$c_2 \to$ uncertainty of hard matching

$$H(c_2\mu)F(c_2\mu)F(c_2\mu)$$

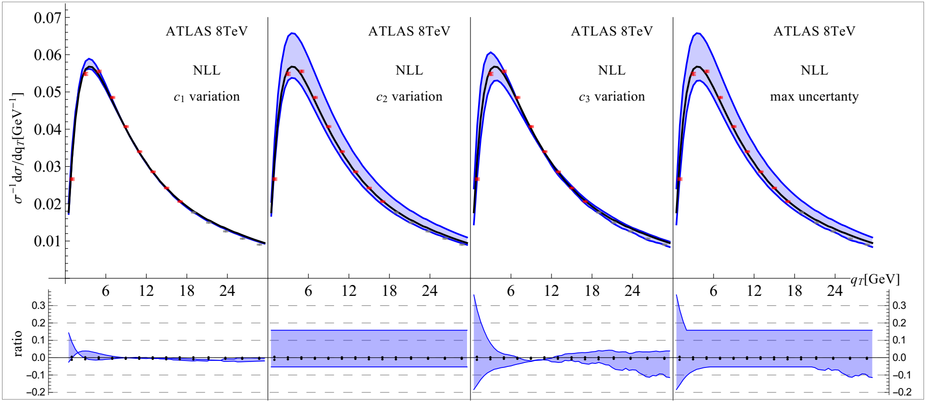
$c_3 \to$ uncertainty of small-b matching

$$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$$

Total uncertainty is the maximum of three

$$c_i \in (0.5, 2)$$

High-energy example: ATLAS 8 TeV (best precision)



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$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

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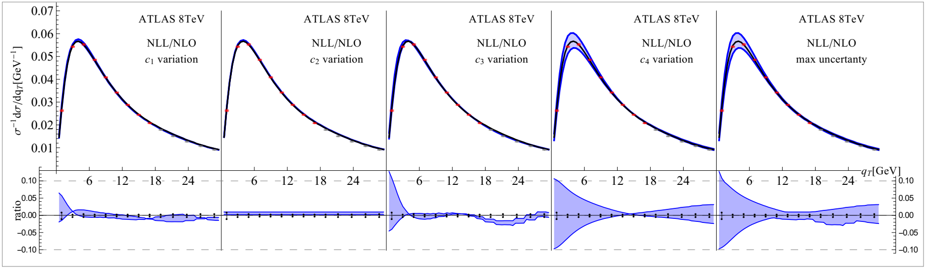

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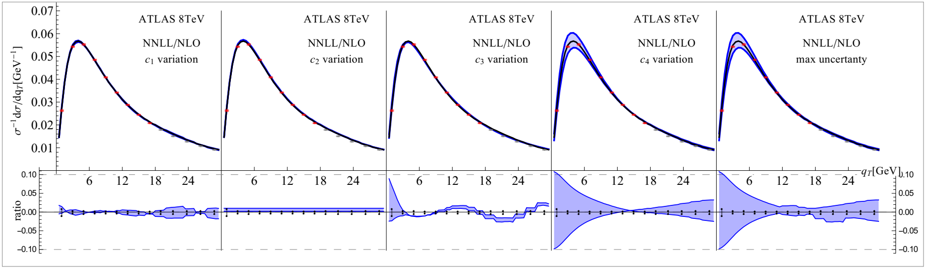
$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

High-energy example: ATLAS 8 TeV (best precision)



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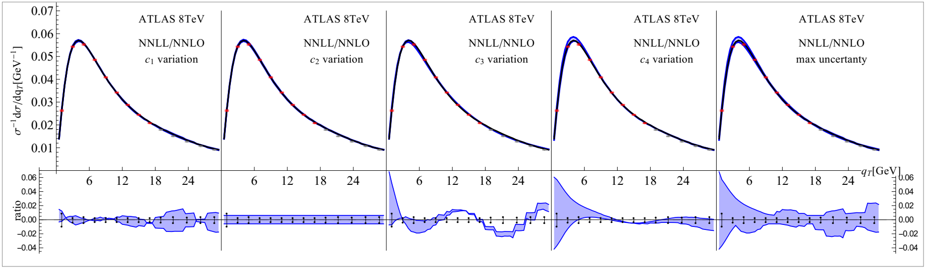
Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition	$c_2 \to$ uncertainty of hard matching	$c_3 \to$ uncertainty of small-b matching	Total uncertainty is the maximum of three
$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

High-energy example: ATLAS 8 TeV (best precision)



Perturbative uncertainties

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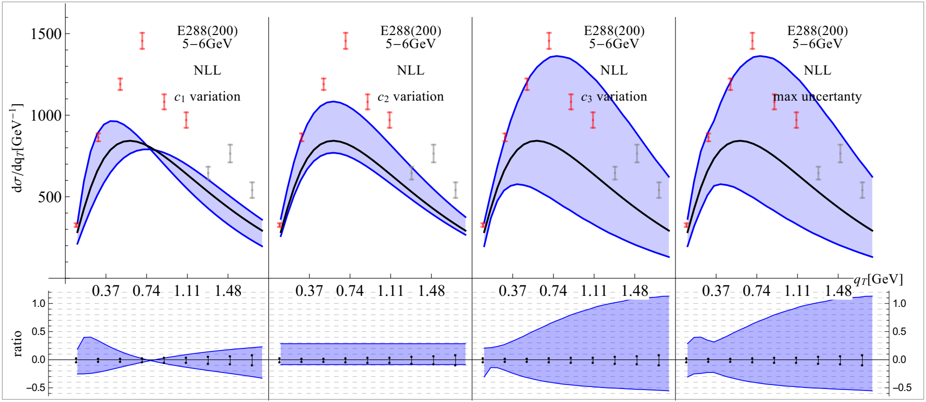
$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

Low-energy example: E288 $\sqrt{s} = 19.4$ GeV, $Q = 4 - 5$ GeV



Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition

$$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$$

$c_2 \to$ uncertainty of hard matching

$$H(c_2\mu)F(c_2\mu)F(c_2\mu)$$

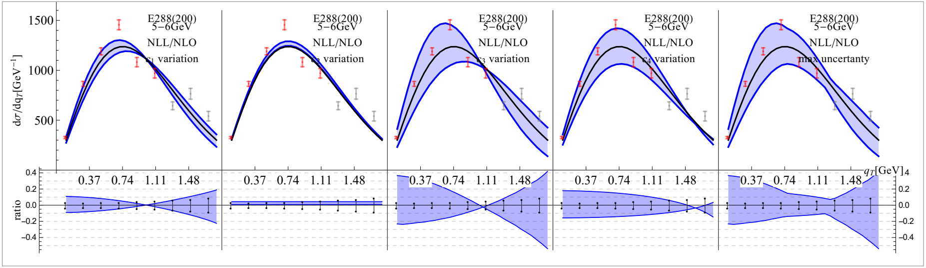
$c_3 \to$ uncertainty of small-b matching

$$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$$

Total uncertainty is the maximum of three

$$c_i \in (0.5, 2)$$

Low-energy example: E288 $\sqrt{s} = 19.4$ GeV, $Q = 4 - 5$ GeV



Perturbative uncertainties

$c_1 \to$ uncertainty of RAD definition	$c_2 \to$ uncertainty of hard matching	$c_3 \to$ uncertainty of small-b matching	Total uncertainty is the maximum of three
$\int_{c_1\mu_0}^{\mu} \Gamma + \mathcal{D}^{\text{pert}}(c_1\mu_0)$	$H(c_2\mu)F(c_2\mu)F(c_2\mu)$	$C(c_3\mu_{\text{low}}) \otimes f(c_3\mu_{\text{low}})$	$c_i \in (0.5, 2)$

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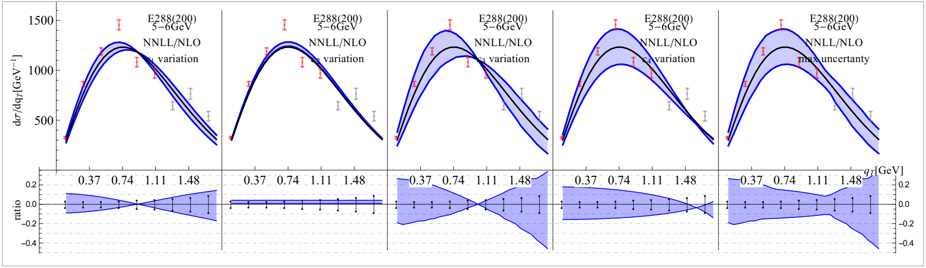

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Perturbative uncertainties

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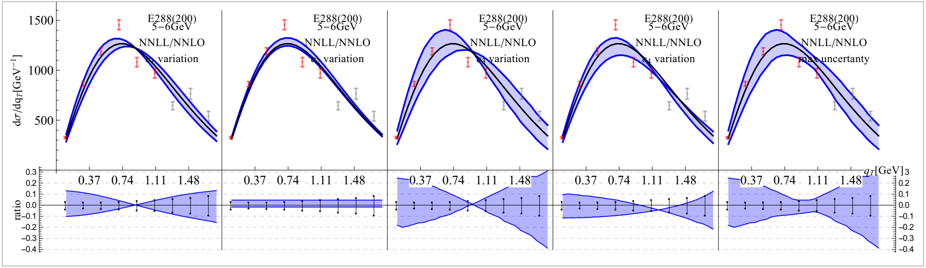

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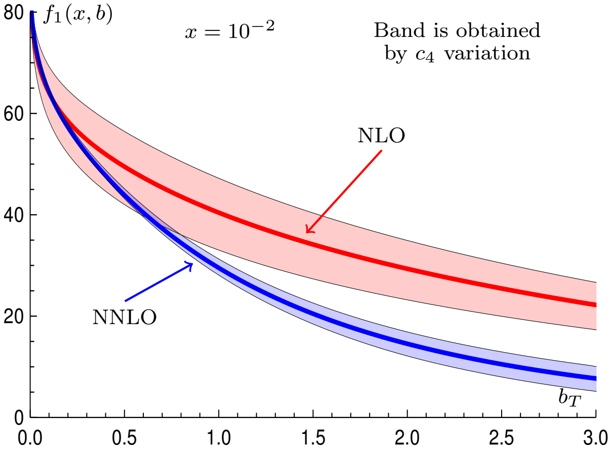
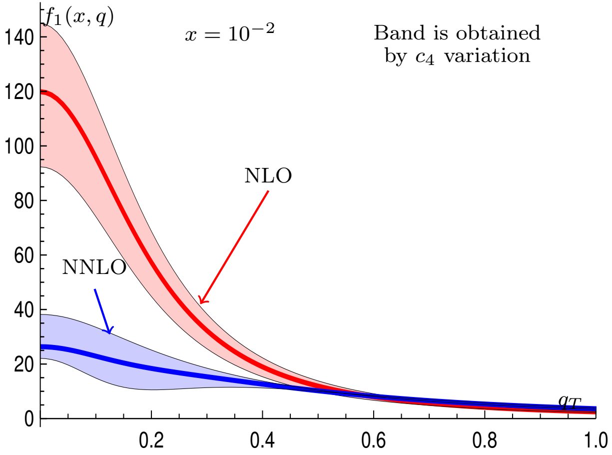
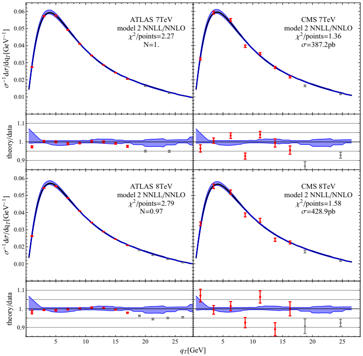
Uncertainties in TMD



Uncertainties in TMD





General

- This is **the first extraction** of uTMDPDF at NNLL/NNLO
- `arTeMiDe` ver.1.1 released DY, Z-boson, uTMDPDFs upto NNLL/NNLO teorica.fis.ucm.es/artemide

Particular

- ζ-prescription is a very nice tool.
- TMD factorization works at $q_T \lesssim 0.2Q$
- NLL/LO is completely unstable. At least NLO is needed (better go NNLO).
- g_K reduces with increase of perturbative order

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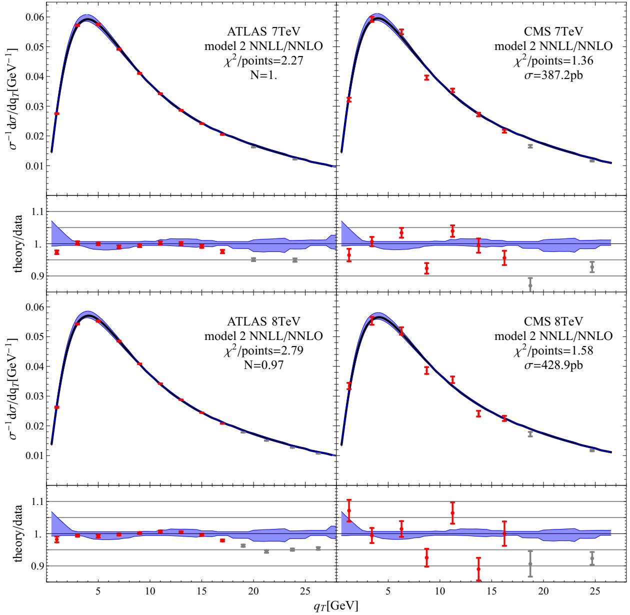
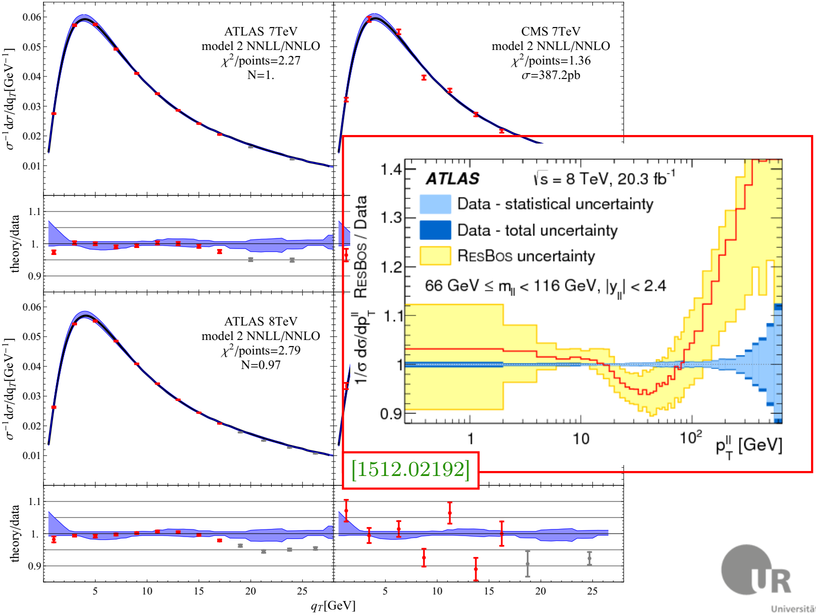
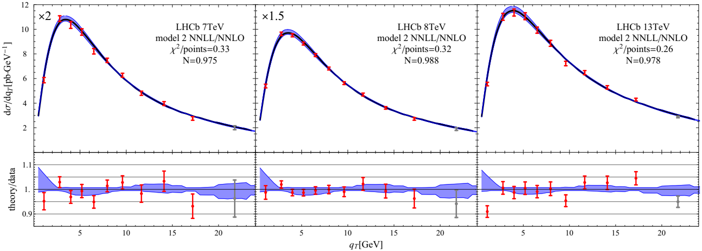
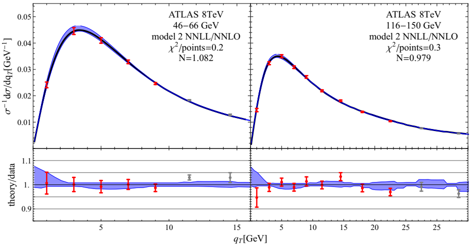
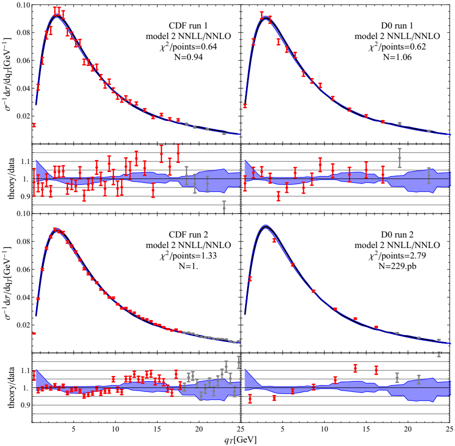
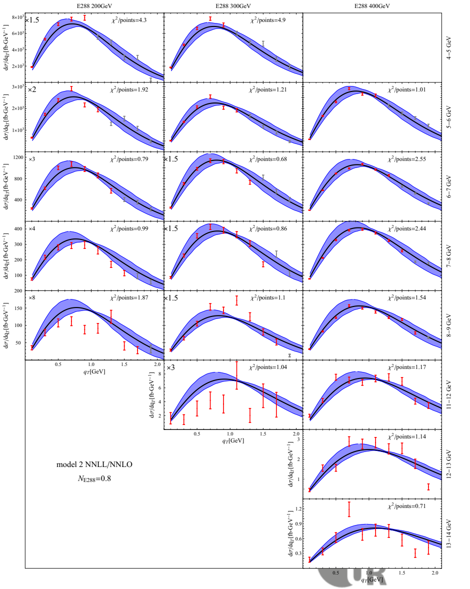

ATLAS 7TeV
model 2 NNLL/NNLO
χ^2/points=2.27
N=1.
CMS 7TeV
model 2 NNLL/NNLO
χ^2/points=1.36
σ=387.2pb
ATLAS 8TeV
model 2 NNLL/NNLO
χ^2/points=2.79
N=0.97
CMS 8TeV
model 2 NNLL/NNLO
χ^2/points=1.58
σ=428.9pb
$\sigma^{-1}d\sigma/dq_T[\text{GeV}^{-1}]$
theory/data
q_T[GeV]


ATLAS 7TeV
model 2 NNLL/NNLO
χ^2/points=2.27
N=1.
CMS 7TeV
model 2 NNLL/NNLO
χ^2/points=1.36
σ=387.2pb
ATLAS 8TeV
model 2 NNLL/NNLO
χ^2/points=2.79
N=0.97
$\sigma^{-1}d\sigma/dq_T$[GeV^{-1}]
theory/data
q_T[GeV]
ATLAS
$\sqrt{s}$ = 8 TeV, 20.3 fb^{-1}
Data - statistical uncertainty
Data - total uncertainty
ResBos uncertainty
66 GeV ≤ m_{ll} < 116 GeV, $|y_{ll}|$ < 2.4
$1/\sigma\, d\sigma/dp_T^{ll}$ ResBos / Data
p_T^{ll} [GeV]
[1512.02192]


×2
LHCb 7TeV
model 2 NNLL/NNLO
χ²/points=0.33
N=0.975
×1.5
LHCb 8TeV
model 2 NNLL/NNLO
χ²/points=0.32
N=0.988
LHCb 13TeV
model 2 NNLL/NNLO
χ²/points=0.26
N=0.978
dσ/dq_T[pb·GeV^−1]
theory/data
q_T[GeV]


ATLAS 8TeV
46−66 GeV
model 2 NNLL/NNLO
χ²/points=0.2
N=1.082
ATLAS 8TeV
116−150 GeV
model 2 NNLL/NNLO
χ²/points=0.3
N=0.979
σ^−1dσ/dq_T[GeV^−1]
theory/data
q_T[GeV]





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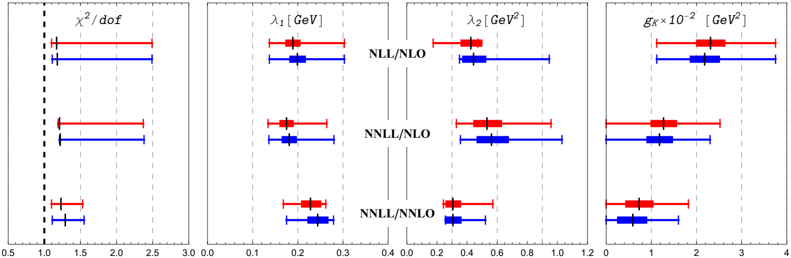

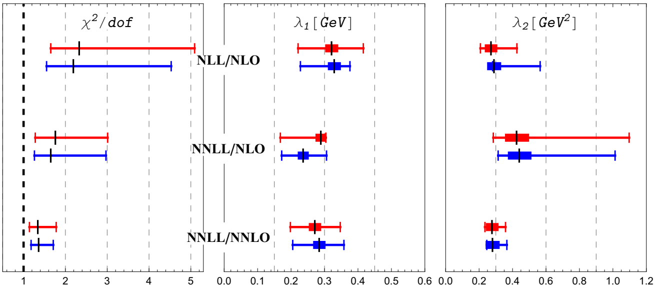

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Order	$\frac{\chi^2}{d.o.f.}$	λ_1	λ_2	$g_K \quad \times 10^{-2}$
Model 1				
NLL/NLO	$1.17\ ^{+1.32}_{-0.07}$	$0.189^{+0.009}_{-0.009}{}^{+0.114}_{-0.052}$	$0.425^{+0.054}_{-0.045}{}^{+0.047}_{-0.250}$	$2.31^{+0.25}_{-0.24}{}^{+1.44}_{-1.19}$
NNLL/NLO	$1.21\ ^{+1.16}_{-0.02}$	$0.175^{+0.008}_{-0.008}{}^{+0.089}_{-0.041}$	$0.532^{+0.076}_{-0.067}{}^{+0.426}_{-0.203}$	$1.27^{+0.22}_{-0.21}{}^{+1.19}_{-1.27}$
NNLL/NNLO	$1.23\ ^{+0.30}_{-0.13}$	$0.228^{+0.016}_{-0.013}{}^{+0.034}_{-0.060}$	$0.306^{+0.031}_{-0.026}{}^{+0.265}_{-0.063}$	$0.73^{+0.24}_{-0.23}{}^{+1.09}_{-0.73}$
Model 2				
NLL/NLO	$1.18\ ^{+1.31}_{-0.07}$	$0.199^{+0.011}_{-0.010}{}^{+0.104}_{-0.062}$	$0.443^{+0.061}_{-0.052}{}^{+0.503}_{-0.093}$	$2.18^{+0.26}_{-0.25}{}^{+1.57}_{-1.06}$
NNLL/NLO	$1.22\ ^{+1.16}_{-0.01}$	$0.181^{+0.009}_{-0.009}{}^{+0.099}_{-0.045}$	$0.562^{+0.092}_{-0.075}{}^{+0.468}_{-0.206}$	$1.18^{+0.22}_{-0.21}{}^{+1.12}_{-1.18}$
NNLL/NNLO	$1.29\ ^{+0.26}_{-0.18}$	$0.244^{+0.016}_{-0.015}{}^{+0.035}_{-0.069}$	$0.306^{+0.034}_{-0.029}{}^{+0.216}_{-0.050}$	$0.59^{+0.24}_{-0.27}{}^{+1.01}_{-0.59}$

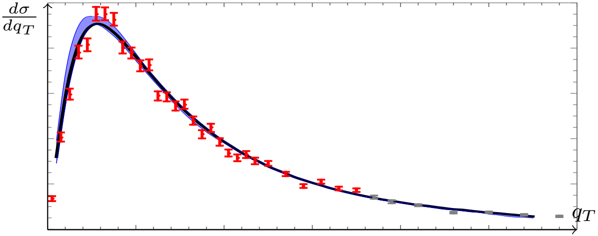
Order	$\frac{\chi^2}{d.o.f.}$	λ_1	λ_2
Model 1			
NLL/NLO	$2.33\ ^{+2.76}_{-0.68}$	$0.321^{+0.008\ +0.095}_{-0.007\ -0.100}$	$0.271^{+0.014\ +0.155}_{-0.013\ -0.063}$
NNLL/NLO	$1.76\ ^{+1.25}_{-0.48}$	$0.289^{+0.004\ +0.007}_{-0.004\ -0.121}$	$0.424^{+0.051\ +0.673}_{-0.045\ -0.139}$
NNLL/NNLO	$1.34\ ^{+0.44}_{-0.20}$	$0.271^{+0.007\ +0.076}_{-0.006\ -0.073}$	$0.277^{+0.015\ +0.081}_{-0.012\ -0.042}$
Model 2			
NLL/NLO	$2.19\ ^{+2.34}_{-0.64}$	$0.329^{+0.008\ +0.047}_{-0.008\ -0.101}$	$0.289^{+0.019\ +0.276}_{-0.017\ -0.008}$
NNLL/NLO	$1.65\ ^{+1.32}_{-0.39}$	$0.236^{+0.005\ +0.070}_{-0.004\ -0.064}$	$0.440^{+0.049\ +0.573}_{-0.044\ -0.126}$
NNLL/NNLO	$1.36\ ^{+0.35}_{-0.18}$	$0.284^{+0.007\ +0.074}_{-0.006\ -0.079}$	$0.280^{+0.019\ +0.086}_{-0.017\ -0.034}$

Limits of application of TMD factorizaiton $\leftrightarrow$ size of Y-term

$$\frac{d\sigma}{dQdy\,d^2q_T} = H(Q,\mu)\int\frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta)+Y$$

TMD factorization derived at small q_T

the leading correction $\sim q_T^2/Q^2$



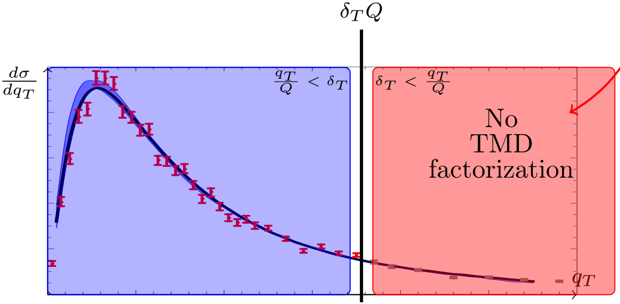
Limits of application of TMD factorizaiton ↔ size of Y-term

$$\frac{d\sigma}{dQdy\,d^2q_T} = H(Q,\mu)\int \frac{d^2b}{(2\pi)^2}\,e^{-ibq_T}\,F(x_A,b;\mu,\zeta)F(x_B,b;\mu,\zeta) + Y$$

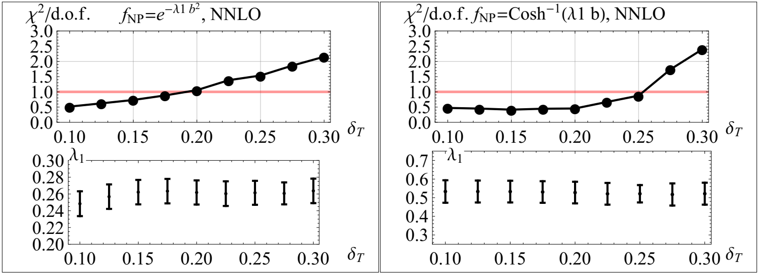
We include all points with $q_T < \delta_T Q$

To find the value of δ_T, we check **the stability of the fit**

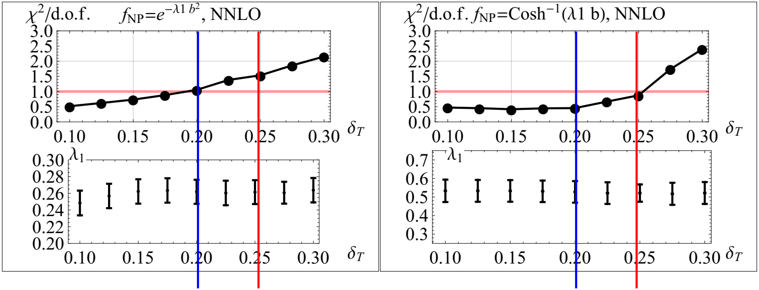
- Make fits with increasing δ_T (0.1→0.3)(165→399 points)
- The value of $\chi^2/d.o.f.$ **blows up** for δ_T **outside allowed region**



Scans of δ_T (E288 not included)



Scans of δ_T (E288 not included)



- $\delta_T < 0.2$ save region,
- $\delta_T < 0.25$ un-save region,
- $\delta_T > 0.25$ TMD factorization does not work.

To be on the save side we used $\delta_T = 0.2$
There are 309 data points

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