Transverse momentum dependent (TMD) factorization: status and progress

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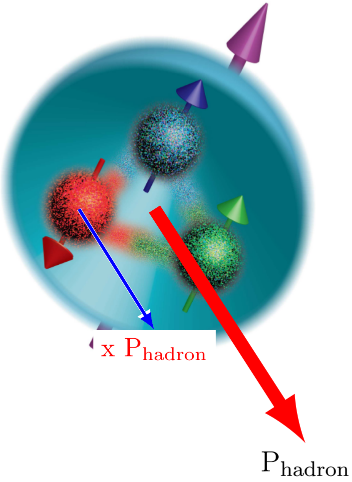

Original parton model

- Hadrons are build from partons (quarks and gluons)
- In the fast moving hadron all partons move collinearly and carry the fraction of momentum x_i ($\sum_i x_i = 1$)
- The probability distribution of partons is given by Parton Distribution Functions (PDFs)

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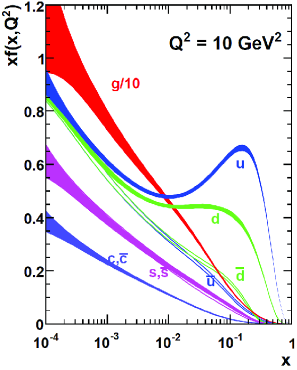

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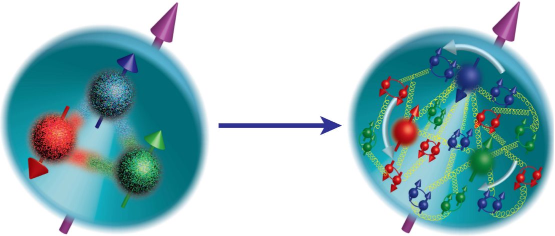
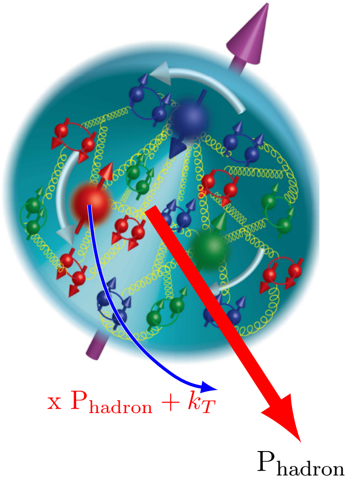
$$f_{q \leftarrow h}(x)$$

Partons distribution function are central objects of high-energy QCD

Nowadays, we known that the structure of hadron is much more involved then parton distribution functions.

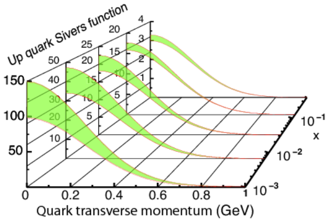


My talk is about one of the generalizations of parton model, that take into account transverse momentum of partons



Transverse momentum dependent (TMD)

- Partons (quarks and gluons) moves almost collinear to the hadrom momentum
- Partons (quarks and gluons) also have transverse momentum
- The probability distribution of partons is given by Transverse Momentum dependent Parton Distribution Functions (TMD PDFs)



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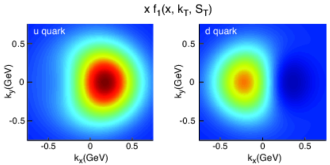

TMD distributions are naturally defined in b_T-space

$$F_{q\leftarrow h}(x, k_T) \quad \Leftrightarrow \quad F_{q\leftarrow h}(x, b_T)$$

There is no special interpretation in b_T space

Transverse momentum was always presented in the parton model
as small (power) corrections.
Why and where TMD corrections are large?

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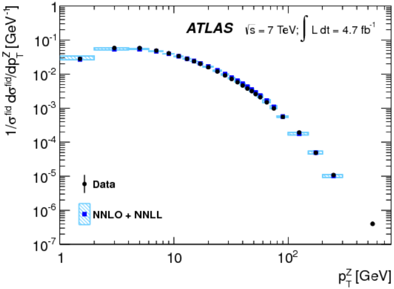
① ↓

Transverse momentum spectrum
of particle production.

In the region where
produced transverse momentum
is compatible to
inrinsic transverse momentum

Example: pT-spectum of Z-boson



Different parts of pT-spectrum
are dominated
by different physics

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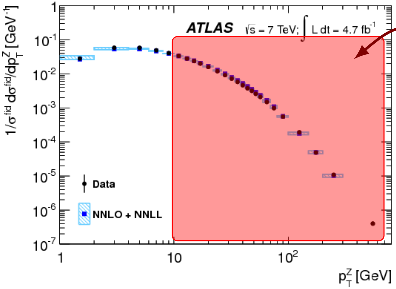


Pertubative (large) transverse momentum

Collinear factorization

Monte-Carlo event generators

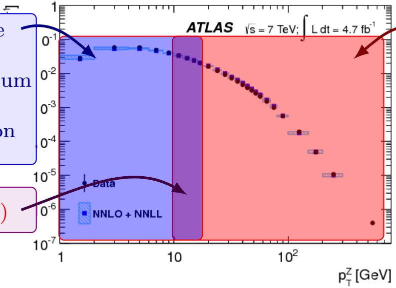
Different parts of pT-spectrum are dominated by different physics

Example: pT-spectum of Z-boson

Non-Pertubative (small) transverse momentum

TMD factorization

Overlap region (?)



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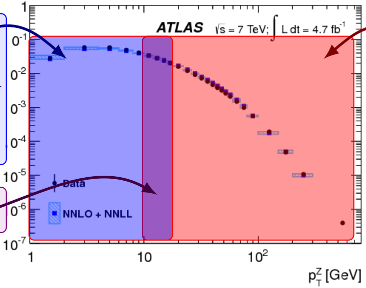

Pertubative
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Transverse momentum dependent factorization
and
collinear factorization
independent and complimentary

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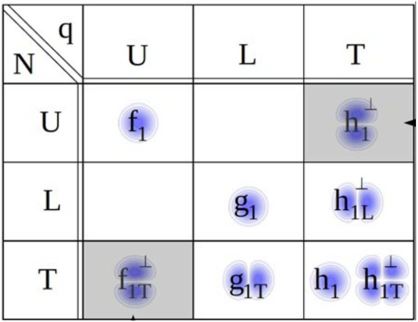
② Spin and angular momentum structure of hadron.

Intrinsic transverse momentum
is
natural sorce of the angular momentum

Transverse momentum distributions of leading order



N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Transverse momentum distributions of leading order

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○ Have analog in "naive" parton model

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- ○○ Have not analog
- ○ Change orientation of spin
- ○ Produce spin

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- ○ Have analog in "naive" parton model
- ○○ Have not analog
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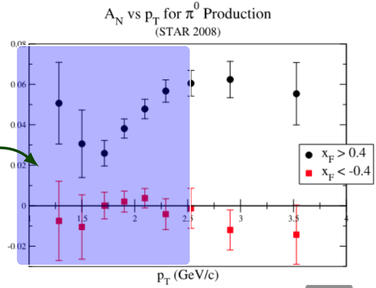
- ○ Produce spin

○ Effect is significant



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③ ④ ..

Jet sub-structure, multi-parton scattering, ...

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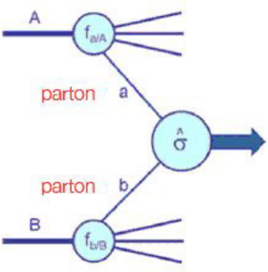
Factorization theorems

In perturbative QCD cross-sections are expressed as products (integral convolutions) of

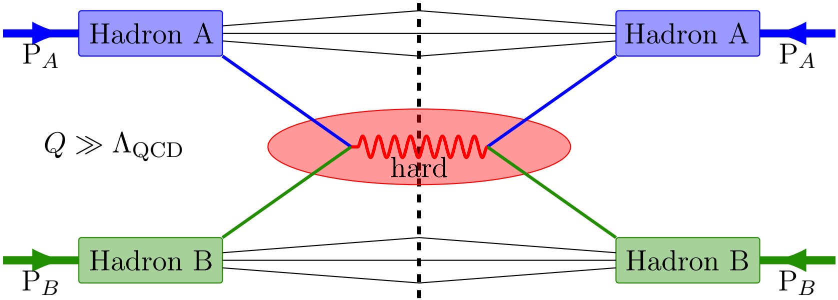
- Process dependent, but calculable coefficient functions
- Process independent, but non-calculable distribution functions

$$\frac{d\sigma}{dX} \sim f_A(x_A) \otimes \hat{\sigma}(x_A, x_B) \otimes f_B(x_B) + \text{power corrections}$$

- The statement of the theorems is the statement on singularities of Feynman diagrams
- The (artificial) split of singularities, introduces (artificial) scales, and evolution
- TMD factorization deals with larger set of singularities



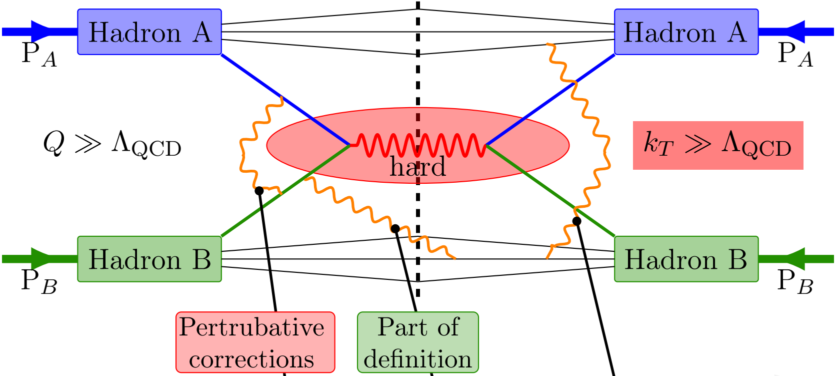
Factorization of Drell-Yan process



$$\frac{d\sigma}{dX} \sim f_A(x_A) \otimes \hat{\sigma}(x_A, x_B) \otimes f_B(x_B)$$

Tree order

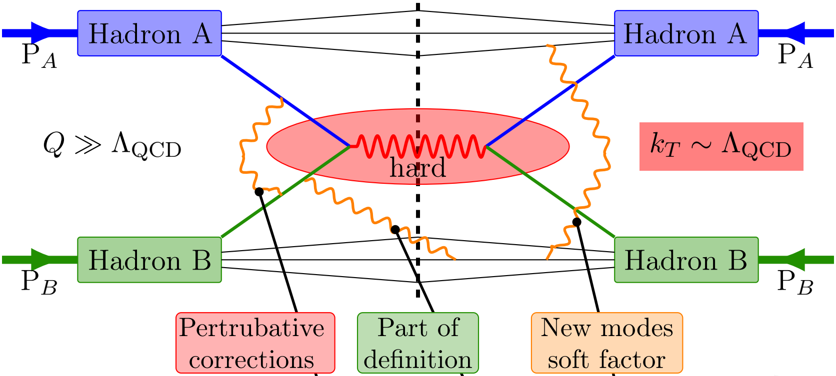
Factorization of Drell-Yan process



$$\frac{d\sigma}{dX} \sim f_A(x_A) \otimes \hat{\sigma}(x_A, x_B) \otimes f_B(x_B) + \text{power corrections}$$

Collinear factorization

Factorization of Drell-Yan process



$$\frac{d\sigma}{dX} \sim f_A(x_A, k_T) \otimes \hat{\sigma}(x_A, x_B) \otimes f_B(x_B, k_T) S(k_T) + \text{power cor.}$$

TMD factorization

Simpler in soft-collinear effective theory (SCET)

Summation of soft gluon exchanges ⇒ Wilson lines

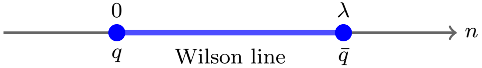
$$[x, y] = P \exp\left(ig \int_x^y dz^\mu A_\mu(z)\right)$$

Parallel transporter of gluon field.

- Central objects of soft physics
- Preserve gauge invariance
- Present in all elements of factorization theorems

Example: parton distribution function

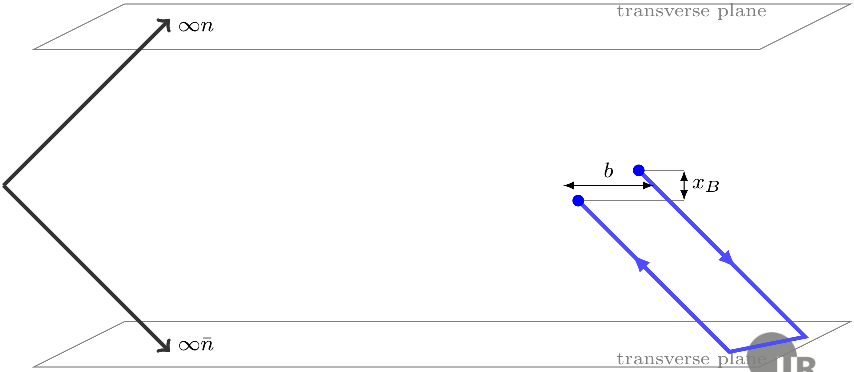
$$f(x) = \int \frac{d\lambda}{2\pi} e^{ixp\lambda} \langle \text{hadron} | \bar{q}(\lambda n)[\lambda n, 0] q(0) | \text{hadron} \rangle$$



$n^2 = 0$
on light-cone

TMD factorization is full of (light-like) Wilson lines

$$\frac{d\sigma}{dX} \simeq H(Q) \int \frac{d^2b}{(2\pi)^2} e^{i(bk)_T}\ f(x_A, b)\ S(b)\ f(x_B, b)$$



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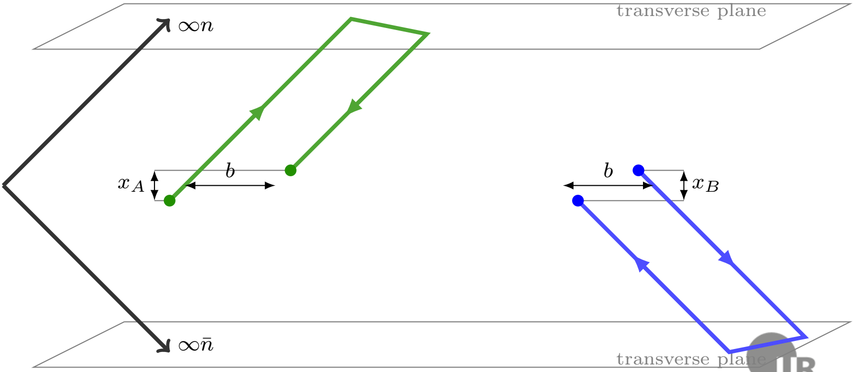

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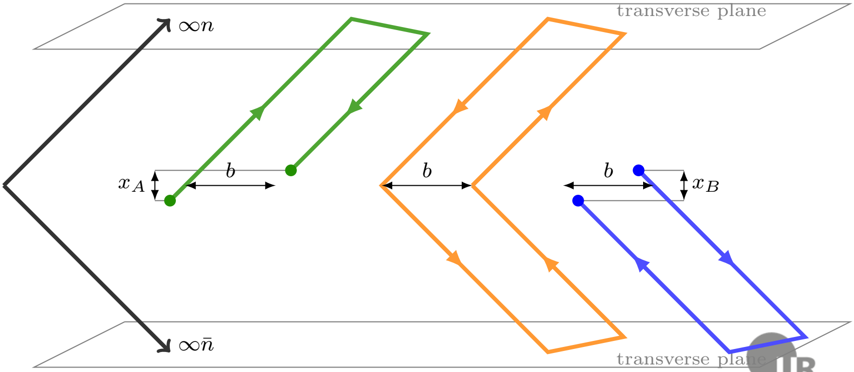
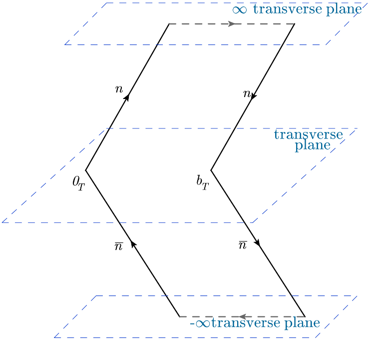


Soft factor is vacuum matrix element of a Wilson loop.

$$S(b) = \langle 0|[0, \pm\infty n][\pm\infty n + \mathbf{b}, \mathbf{b}][\mathbf{b}, \pm\infty\bar{n} + \mathbf{b}][\pm\infty\bar{n}, 0]|0\rangle$$



- Geometrically non-compact
- Light-cone directions
- Process dependent (± depends on the process)

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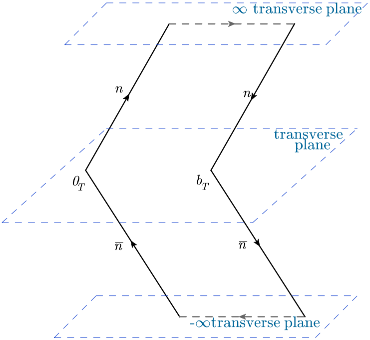


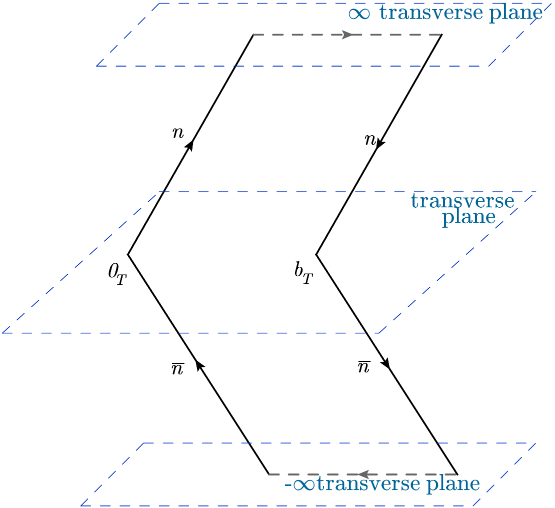
- Geometrically non-compact
- Light-cone directions
- Process dependent ($\pm$ depends on the process)
- **Singular**

$$H(Q)\int d^2b e^{i(bk)_T} f(x_A, b) S(b) f(x_B, b)$$

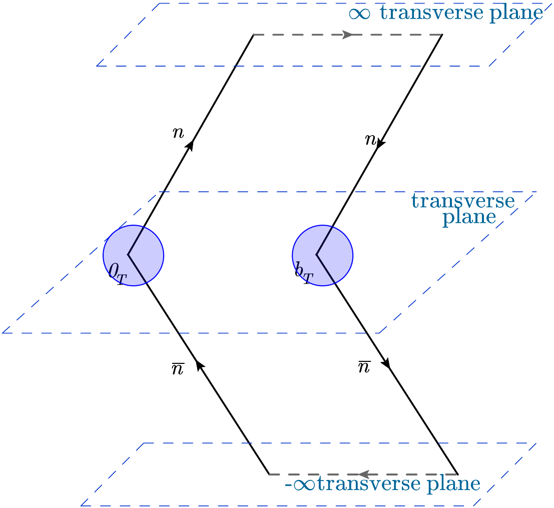
regular

singular **singular** **singular**

The problem is in the soft factor.
It mixes divergences of different kinds.
Factorization is incomplete

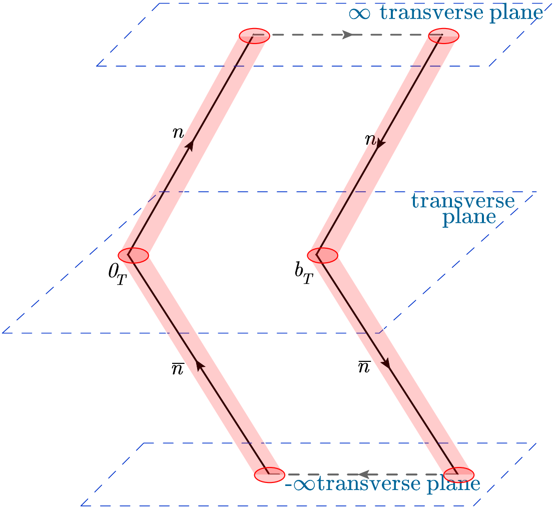


Various kinds of divergences



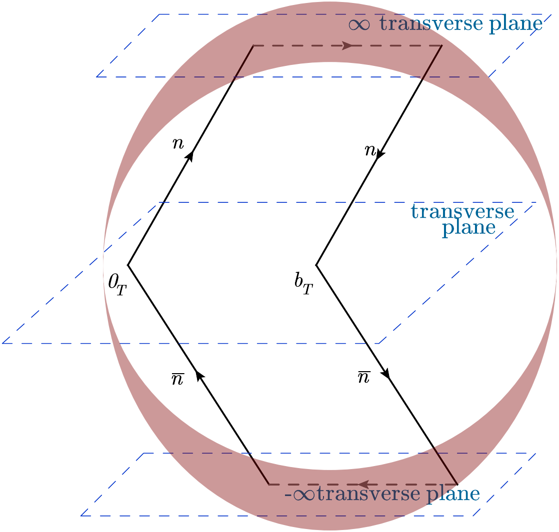
Various kinds of divergences

Ultraviolet (renormalized)



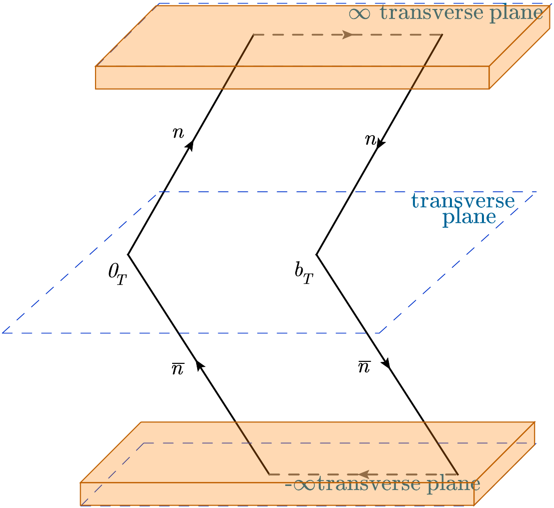
Various kinds of divergences

- **Ultraviolet** (renormalized)
- **Collinear** (cancel in sum)



Various kinds of divergences

- **Ultraviolet** (renormalized)
- **Collinear** (cancel in sum)
- **Mass divergences** (cancel in sum)



Various kinds of divergences

- **Ultraviolet** (renormalized)
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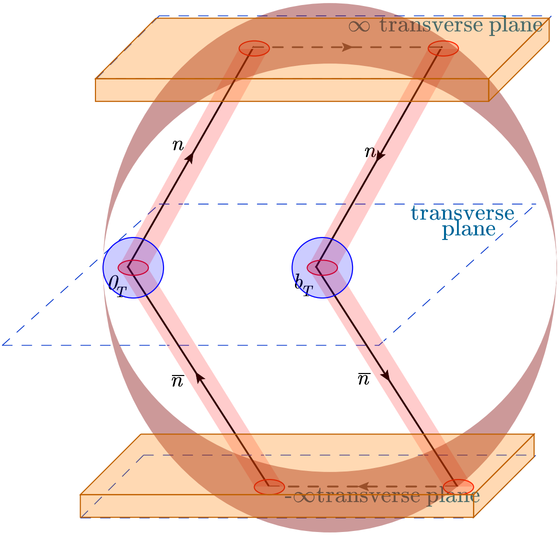
- **Mass divergences** (cancel in sum)
- **Rapidity divergences** (???)



Various kinds of divergences

- **Ultraviolet** (renormalized)
- **Collinear** (cancel in sum)
- **Mass divergences** (cancel in sum)
- **Rapidity divergences** (???)

Only recently [1707.07606] it has been **proven** that rapidity divergences factorize

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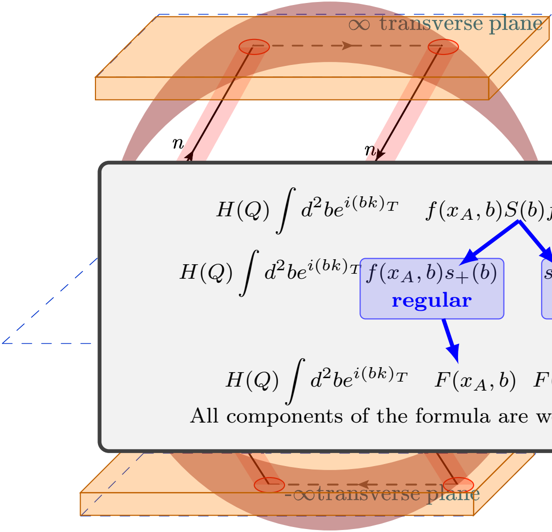

$$H(Q)\int d^2b e^{i(bk)_T} \quad f(x_A,b)S(b)f(x_B,b)$$

$$H(Q)\int d^2b e^{i(bk)_T} \quad f(x_A,b)s_+(b) \quad s_-(b)f(x_B,b)$$

regular **regular**

$$H(Q)\int d^2b e^{i(bk)_T} \quad F(x_A,b) \quad F(x_B,b)$$

All components of the formula are well-defined

Various kinds of divergences

- **Ultraviolet** (renormalized)
- n)
- **gences** n)
- **ivergences**

ntly
06]
it has been **proven**
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rapidity divergences
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In fact, TMD factorization is **two factorizations**
collinear + factorization of soft factor
Each factorization brings its own (factorization) scale
TMD distribution depends on **two scales**

$$F(x, b; \mu, \zeta)$$

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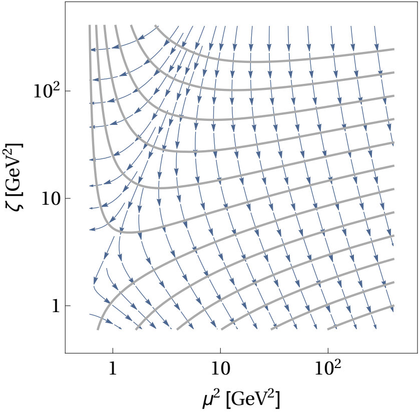

TMD evolution
is 2D evolution

$$\mu^2 \frac{dF(x, b; \mu, \zeta)}{d\mu^2} = \gamma_F(\mu, \zeta) F(x, b; \mu, \zeta)$$

$$\zeta \frac{dF(x, b; \mu, \zeta)}{d\zeta} = -\mathcal{D}(\mu, b) F(x, b; \mu, \zeta)$$

- Anomalous dimensions γ_F and $\mathcal{D}$ are known up to three-loops.
- Anomalous dimension are universal for all TMD distributions

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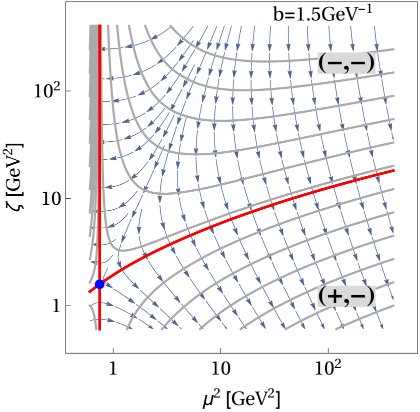
$$F(x, b; \mu, \zeta)$$

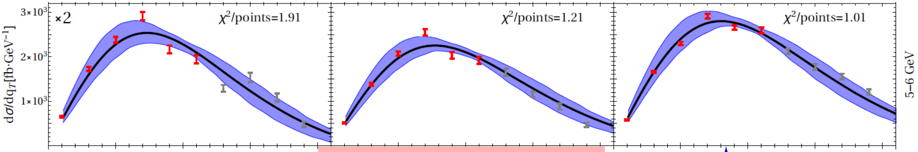


Evolutionless TMD distribution

TMD distribution on the null-evolution curve is scale-less

$$\mu^2 \frac{dF(x, b; \mu, \zeta_\mu)}{d\mu^2} = 0$$

- TMD distribution is universal
- No questions on the scale determinations
- Technically simpler then PDFs



Drell-Yan at $Q = 5 - 6$GeV

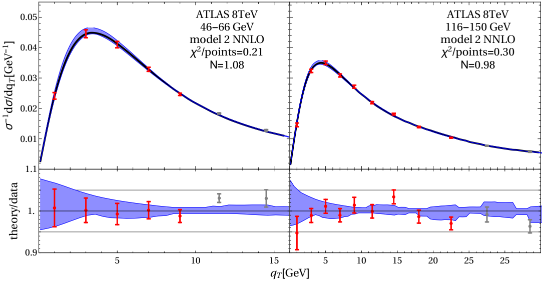
TMD evolution works!

Here:

- 3-loop evolution
- 2-loop coefficient function
- 2-loop matching

plots from [1706.01473]

Drell-Yan at $Q = 116 - 150$GeV



In TMD factorization the perturbative interpenetrates the non-perturbative

Perturbative

- Hard coefficient functions
- Anomalous dimensions
- Small-b matching (for each distribution)

Non-Perturbative

- 8 TMD parton distribution functionss (spin 1/2)
- 2 TMD fragmentation functions (spin 0)
- Non-perturbative part of evolution kernel

	known	expected progress
H (universal)	3-loop [2010]	
γ_F (universal)	3-loop [2010]	
$\mathcal{D}$ (universal)	3-loop [2016]	
f_1(unpol. PDF)	2-loop [2015]	
d_1(unpol.FF)	2-loop [2016]	
g_1(helicity)	1-loop [2017]	
h_1(transvercity)	1-loop [2017]	2-loop
$h_{1T}^{\perp}$(pretzelocity)	1-loop (=0) [2017]	2-loop
$f_{1T}^{\perp}$(Sivers)	tree [2014]	1-loop
$d_{1T}^{\perp}$(Collins)	unknown	tree
$h_1^{\perp}$(Boer-Mulders)	unknown	tree+1-loop
g_{1T}, h_{1L} (worm-gear T,L)	unknown	tree

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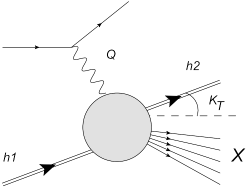
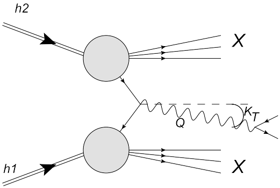
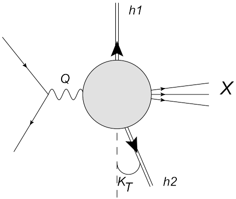






There are 3 main processes to extract TMD distributions

Semi-inclusive deep inelastic scattering
PDF×FF

Drell-Yan
PDF×PDF

$e^+e^- \to 2$ hadrons
FF×FF

Main efforts are dedicated to unpolarized distributions

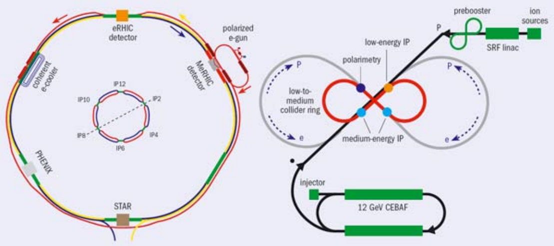
table by A.Bacchetta EICUG meeting

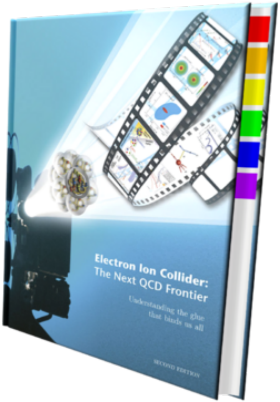
	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	NLL/NLO	✘	✘	✔	✔	98
Pavia 2013 arXiv:1309.3507	No evo	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	No evo	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL/NLO	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	NLL/LO	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2016 arXiv:1703.10157	NLL/LO	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLL/ NNLO	✘	✘	✔	✔	309

There are no systematic extractions of polarized TMD distributions

However, there are many independent fits

Electron-Ion collider

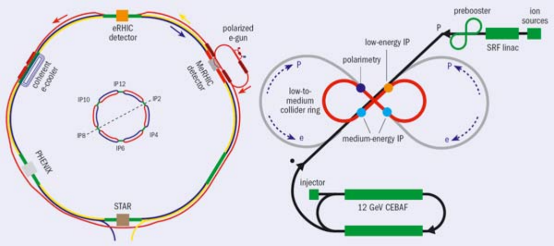


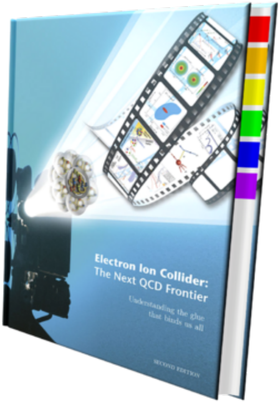


Dedicated QCD machine

- Proton/ion, polarized beam
- Large coverage of Q^2 (moderate values)
- Large dedicated program for nucleon tomography

Electron-Ion collider





Dedicated QCD machine

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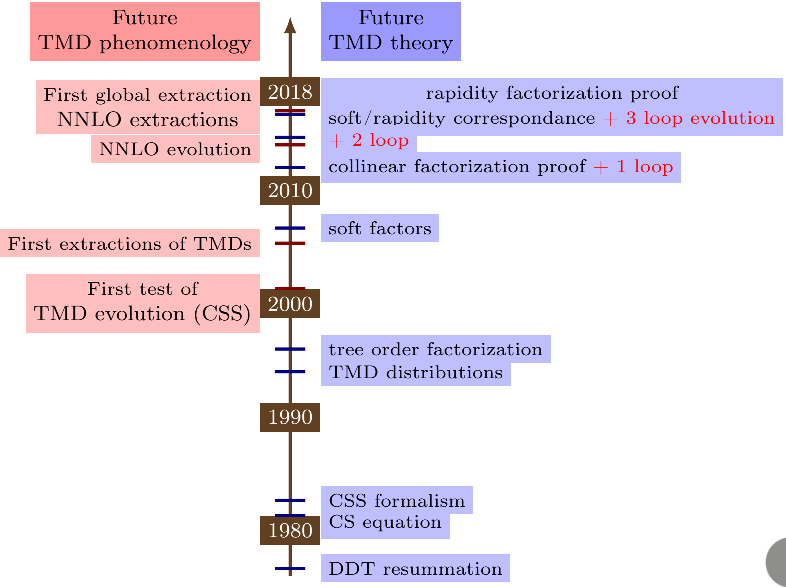
Critical decision 0
autumn 2018



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Future
TMD phenomenology

Future
TMD theory

rapidity factorization proof
/rapidity correspondance + 3 loop evolution

ctorization proof + 1 loop

actorization
butions

Theory driven global phenomenology

Global extractions FF+PDF (unpol.)
with maximum perturbative input

Consistent extractions of polarized (mathing + evolution)

Studies low-to-high p_T transition

CSS formalism
CS equation

1980

DDT resummation

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Future TMD phenomenology

Future TMD theory

Theory driven global phenomenology

Global extractions FF+PDF (unpol.) with maximum perturbative input

Consistent extractions of polarized (mathing + evolution)

Studies low-to-high p_T transition

Presice TMD theory

Matching of polarized TMDs (tw-3,tw-4)

Power corrections

Wider TMD theory

TMD based event generators

Elements of TMD factorization in jet physics, MPS, small-x

Correspondances between processes

1980