Transverse momentum dependent (TMD) factorization in perturbation theory

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TMD factorization is a complicated composition
of perturbative and non-perturbative functions.

$$\frac{d\sigma}{dX} \simeq \int db e^{ibq_T} H(Q) \quad \{R(Q \to (\mu_i, \zeta_i))\}^2 \quad F_1(x, b, \mu_i, \zeta_i) F_2(x, b, \mu_i, \zeta_i)$$

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Hard coef.
Perturbative

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Evolution factor
γ_F $\mathcal{D}(b)$

UV anomalous dimension
Perturbative

Rap. anomalous dimension
$b \ll \Lambda^{-1}$ **Perturbative**
$b \gtrsim \Lambda^{-1}$ **Non-Perturbative**

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Hard coef. $H(Q)$: **Perturbative**

Evolution factor $\{R(Q \to (\mu_i,\zeta_i))\}^2$: γ_F, $\mathcal{D}(b)$

TMD distributions F_1, F_2: **Non-Perturbative**

UV anomalous dimension γ_F: **Perturbative**

Rap. anomalous dimension $\mathcal{D}(b)$:
- $b \ll \Lambda^{-1}$ **Perturbative**
- $b \gtrsim \Lambda^{-1}$ **Non-Perturbative**

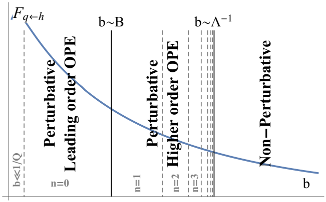
at $b \ll \Lambda^{-1}$ **OPE to collinear**

$$F = C \otimes f \cdot f_{NP}$$

C is **Perturbative**

f is **Non-Perturbative**(known)

f_{NP} is **Non-Perturbative**



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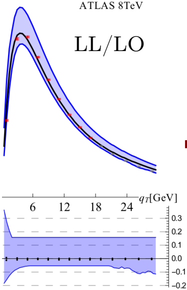
Balance

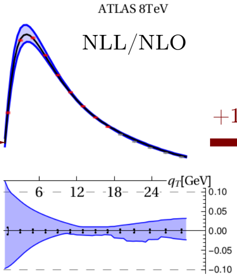
	Perturbative	**Non-Perturbative**	
H	Hard coeff.	collinear distributions	$f, d, ..$
γ	UV anomalous dimension	TMD behaviour	f_{NP}
$\mathcal{D}$	rap. anomalous dimension	rap. anomalous dimension	$\mathcal{D}_{NP}$
C	mathing coefficients		

Perturbative input is essential for presice phenomenology.
All perturbative inputs are important.

High energy



+1 loop



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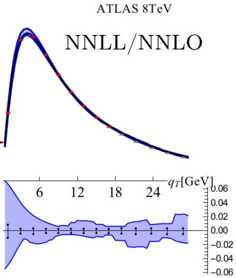

18%

3-10%

2-6%

plots from [I.Scimemi,AV; 1706.01473]

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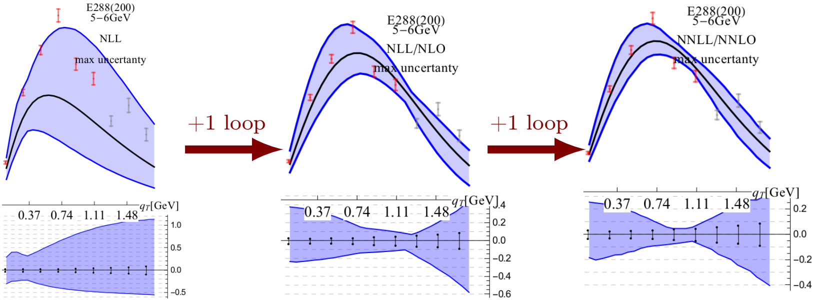
Low energy



50-100% **10-40%** **5-20%**

plots from [I.Scimemi,AV; 1706.01473]

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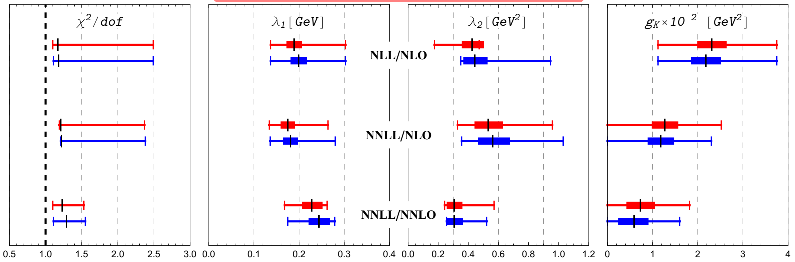
Perturbative input is essential for presice phenomenology.
All perturbative inputs are important.

Effect on the TMD extraction

LL/LO is not shown (errors too large)



plots from [I.Scimemi,AV; 1706.01473]

Conlusion

Perturbation input is important at both low and high energies.
The higher order – the better.

There was significant progress in the perturbative calculus

- Schemes of perturbative calculation were consistently formulated
- 3-loop TMD evolution [Li,Zhu,1604.01404][AV,1610.05791]
- All TMD distributions (tw-2) were recalculated in the same scheme [Scimemi,AV,1702.06558][Buffing,Diehl,Kasemets,1708.03528].
- (Rapidity divergences part) of TMD factorization was proven [AV,1707.07606].
- More to come soon.

	known	in this talk
H (universal)	3-loop [2010]	
γ_F (universal)	3-loop [2010]	
$\mathcal{D}$ (universal)	3-loop [2016]	✓
f_1(unpol. PDF)	2-loop [2015]	
d_1(unpol.FF)	2-loop [2016]	
g_1(helicity)	1-loop [2013]	
h_1(transvercity)	1-loop [2013]	✓
$h_{1T}^{\perp}$(pretzelocity)	1-loop (=0) [2017]	✓
$f_{1T}^{\perp}$(Sivers)	?tree [2014]	✓
$d_{1T}^{\perp}$(Collins)	unknown	✓
$h_1^{\perp}$(Boer-Mulders)	unknown	✓
g_{1T}, h_{1L} (worm-gear T,L)	unknown	✓

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Not everything requares (full) calculation.
Universal quantities could be taken from outside.

Vector form factor (of parton)

Directly

$$H(Q,\mu) = |C_V(Q,\mu)|^2$$

Hard coefficient

$$\gamma_F(\mu,\zeta) = \Gamma_{\text{cusp}}(\mu)\ln(\mu^2/\zeta) - \gamma_V(\mu)$$

TMD anomalous dimension

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$$H(Q,\mu) = |C_V(Q,\mu)|^2 \qquad \gamma_F(\mu,\zeta) = \Gamma_{\text{cusp}}(\mu)\ln(\mu^2/\zeta) - \gamma_V(\mu)$$

Hard coefficient — TMD anomalous dimension

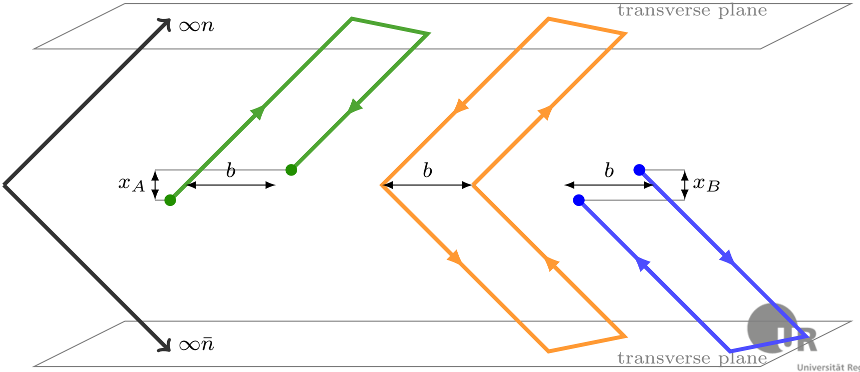
Soft/rapidity anomalous dimension correspondance

Rapidity anomalous dimension		Soft anomalous dimension
$\mathcal{D}(\mu,b)$ $(= -\tilde{K}/2)$	⟷	$\gamma_{\text{soft}}(\mu, v_{ij})$
TMD physics		jet production

Byproduct of rapidity divergences renormalization theorem
that completes the TMD factorization

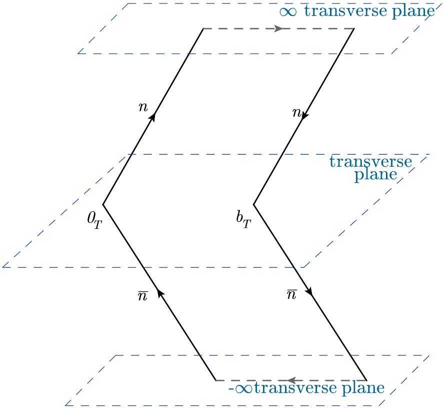
TMD factorization is two successive factorizations
The first: collinear factorization [Collins' book], [SCET]
The second: rapidity divergences factorization [AV,1707.07606]

$$\frac{d\sigma}{dX} \simeq H(Q) \int \frac{d^2b}{(2\pi)^2} e^{i(bk)_T} f(x_A, b) S(b) f(x_B, b)$$



Soft factor is vacuum matrix element of a Wilson loop.

$$S(b) = \langle 0|[0, \pm\infty n][\pm\infty n + \mathbf{b}, \mathbf{b}][\mathbf{b}, \pm\infty\bar{n} + \mathbf{b}][\pm\infty\bar{n}, 0]|0\rangle$$



Various kinds of divergences

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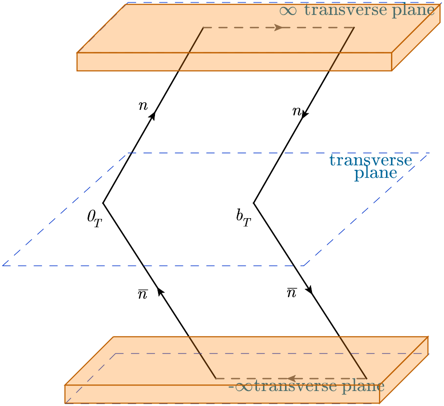


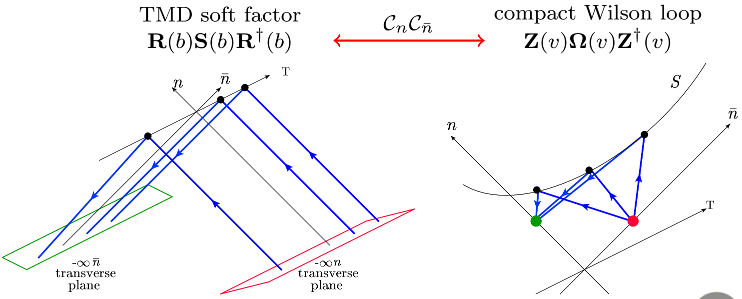
Various kinds of divergences

- **Ultraviolet** (renormalized)
- **Collinear** (cancel in sum)
- **Mass divergences** (cancel in sum)
- **Rapidity divergences** (???)

There is a geometrical transformation that turns rapidity divergence to UV

$$\mathcal{C}_{\bar n}:\{x^+,x^-,x_\perp\}\to\{\frac{-1}{2a}\frac{1}{\lambda+2ax^+},x^-+\frac{ax_\perp^2}{\lambda+2ax^+},\frac{x_\perp}{\lambda+2ax^+}\}$$

- The UV renormalization imposes rapidity divergence renormalization

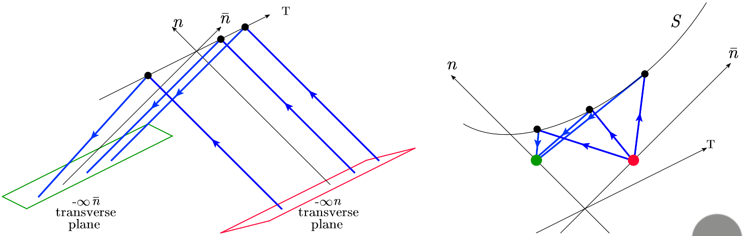


Rapidity divergence renormalization theorem

Rapidity divergences for TMD soft factor can be renormalized for each direction separately (also holds for MPS).

- Immediate in conformal field theory
- Proof by iteration in QCD (1-loop in conformal)

TMD soft factor $\mathbf{R}(b)\mathbf{S}(b)\mathbf{R}^\dagger(b)$ $\xleftrightarrow{\mathcal{C}_n\mathcal{C}_{\bar{n}}}$ compact Wilson loop $\mathbf{Z}(v)\mathbf{\Omega}(v)\mathbf{Z}^\dagger(v)$



Soft/Rapidity anomalous dimension correspondence

Exact relation

$$\gamma_{\text{soft}}(v) = 2\mathcal{D}(\mathbf{b}, \epsilon^*)$$

How to use it?

- Physical value is $\mathbf{D}(\{\mathbf{b}\}, 0)$
- $\epsilon^* = 0 - a_s\beta_0 - a_s^2\beta_1 - a_s^3\beta_2 - \ldots.$
- We can compare order by order in PT

$$\mathbf{D}_1(\{b\}) = \frac{1}{2}\boldsymbol{\gamma}_1(\{v\}),$$

$$\mathbf{D}_2(\{b\}) = \frac{1}{2}\boldsymbol{\gamma}_2(\{v\}) + \beta_0\mathbf{D}_1'(\{b\}),$$

$$\mathbf{D}_3(\{b\}) = \frac{1}{2}\boldsymbol{\gamma}_3(\{v\}) + \beta_0\mathbf{D}_2'(\{b\}) + \beta_1\mathbf{D}_1'(\{b\}) - \frac{\beta_0^2}{2}\mathbf{D}_1''(\{b\}),$$

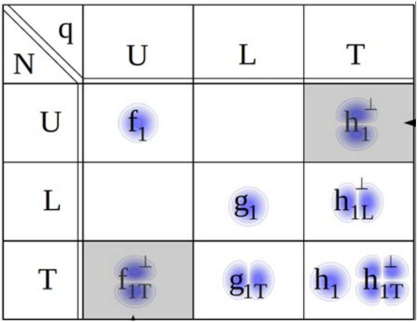
Always gain one order!

This simple exercise gives 3-loop rapidity anomalous dimension.

$$\mathcal{D}^{(3)}_{L=0} = -\frac{C_A^2}{2}\left(\frac{12328}{27}\zeta_3 - \frac{88}{3}\zeta_2\zeta_3 - 192\zeta_5 - \frac{297029}{729} + \frac{6392}{81}\zeta_2 + \frac{154}{3}\zeta_4\right)$$
$$-\frac{C_A N_f}{2}\left(-\frac{904}{27}\zeta_3 + \frac{62626}{729} - \frac{824}{81}\zeta_2 + \frac{20}{3}\zeta_4\right) -$$
$$\frac{C_F N_f}{2}\left(-\frac{304}{9}\zeta_3 + \frac{1711}{27} - 16\zeta_4\right) - \frac{N_f^2}{2}\left(-\frac{32}{9}\zeta_3 - \frac{1856}{729}\right)$$

- Coincides with the one calculated directly [Li,Zhu,1604.01404]

Unfortunately, there is something to calculate



OPE at small values of b

$$F(x,b) = C(x,b;\mu_{\text{OPE}}) \otimes f(x,\mu_{\text{OPE}}) + \dots$$

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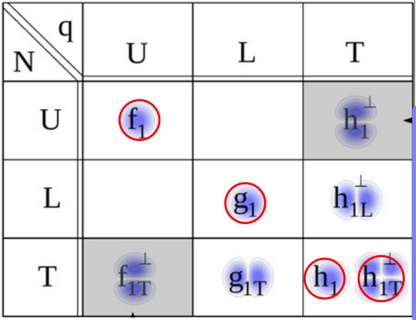

OPE at small values of b

$$F(x,b) = C(x,b;\mu_{\text{OPE}}) \otimes f(x,\mu_{\text{OPE}}) + \ldots$$

○ Leading twist-2

- f_1 →Unpol.PDF (2-loop) [1403.6451,1604.07869]
- d_1 →Unpol.FF (2-loop) [1509.06392,1604.07869]
- g_1 →helicity PDF (1-loop) [1303.2129,1702.06558,1708.03528]
- h_1 →transvercity PDF (1-loop) [1303.2129,1702.06558,1708.03528]
- $h_{1T}^\perp$ →transvercity PDF(1-loop) **(=0)** [1702.06558]

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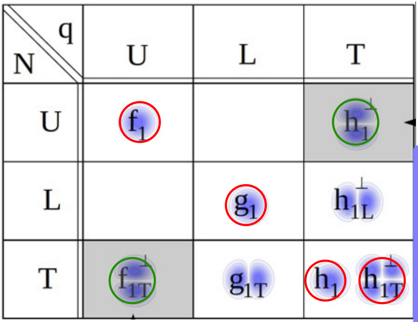
Unfortunately, there is something to calculate



N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

OPE at small values of b

$$F(x,b) = C(x,b;\mu_{\rm OPE}) \otimes f(x,\mu_{\rm OPE}) + \dots$$

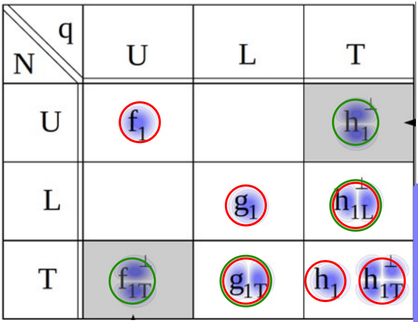
○ Leading twist-2

○ Leading twist-3

- $f_{1T}^{\perp}$ →Qui-Sterman
  all calculations made for cross-section
- $h_1^{\perp}$ → **??**

These function must change sign under DY↔SIDIS

Unfortunately, there is something to calculate



OPE at small values of b

$$F(x,b) = C(x,b;\mu_{\text{OPE}}) \otimes f(x,\mu_{\text{OPE}}) + ...$$

- Leading twist-2 (red circle)
- Leading twist-3 (green circle)
- Leading twist-2 and 3 (red and green circle)

- $h_{1L}^{\perp}$ →??+WW(helicity PDF)
- g_{1T} → ??+WW(transvercity PDF)

WW part can be found in [Kanazawa,et al;1512.07233]

New 2-loop results

$$\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma_5]}(x,\mathbf{b}) = s_T^\alpha h_1(x,\mathbf{b}) + 2\left(\frac{g_T^{\alpha\mu}}{2} + \frac{b^\alpha b^\mu}{\mathbf{b}^2}\right) s_{T\mu} h_{1T}^\perp(x,\mathbf{b}) + b_\mu(...)^{\alpha\mu}$$

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transvercity TMDPDF
⇓
transvercity PDF
$h_1(x,\mathbf{b}) = C \otimes \delta f(x) + \mathbf{b}^2...$
tree+1loop

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$h_1(x,\mathbf{b}) = C \otimes \delta f(x) + \mathbf{b}^2 ...$
tree+1loop

linear in $\mathbf{b}$
⇓
twist-3

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$h_1(x,\mathbf{b})$: transvercity TMDPDF
⇓
transvercity PDF
$h_1(x,\mathbf{b}) = C \otimes \delta f(x) + \mathbf{b}^2...$
tree+1loop

$h_{1T}^{\perp}(x,\mathbf{b})$: pretzelocity TMDPDF
⇓
transvercity PDF
$h_{1T}^{\perp}(x,\mathbf{b}) = C_T^{\perp} \otimes \delta f(x) + \mathbf{b}^2...$

$b_{\mu}(...)^{\alpha\mu}$: linear in $\mathbf{b}$
⇓
twist-3

$$C_T^{\perp}(x,\mathbf{b}) = -4a_s^2\left[4\left(C_F^2 - \frac{C_F C_A}{2}\right)\left(\bar{x}\ln\bar{x} + x\ln x - \frac{3}{2}\bar{x}\right) - C_F^2\bar{x}(x - \ldots\right]$$

Preliminary

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New 2-loop results

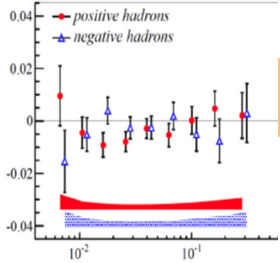
$$\Phi_{q\leftarrow h}^{[i\sigma^{\alpha+}\gamma_5]}(x,\mathbf{b}) = s_T^\alpha h_1(x,\mathbf{b}) + 2\left(\frac{g_T^{\alpha\mu}}{2} + \frac{b^\alpha b^\mu}{\mathbf{b}^2}\right) s_{T\mu} h_{1T}^\perp(x,\mathbf{b}) + b_\mu(...)^{\alpha\mu}$$

transvercity TMDPDF
$\Downarrow$
transvercity PDF
$h_1(x,\mathbf{b}) = C \otimes \delta f(x) + \mathbf{b}^2 ...$
tree+1loop+**2loop**

pretzelocity TMDPDF
$\Downarrow$
transvercity PDF
$h_{1T}^\perp(x,\mathbf{b}) = C_T^\perp \otimes \delta f(x) + \mathbf{b}^2$

linear in $\mathbf{b}$
$\Downarrow$
twist-3

$$C_T^\perp(x,\mathbf{b}) = -4a_s^2\left[4\left(C_F^2 - \frac{C_F C_A}{2}\right)\left(\bar{x}\ln\bar{x} + x\ln x - \right.\right.$$



The first model independent result for pretzelocity
Natural explanation of smallness of $\sin(3\varphi - \varphi_s)$ assymetry

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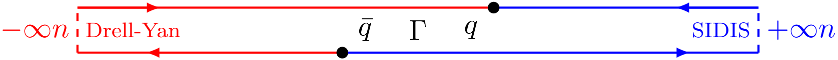
Leading matching of all polarized distributions at twist-3

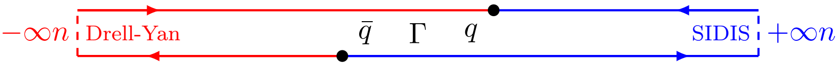
$$U^{\Gamma}_{\text{DY}}(z,\mathbf{b}) = \bar q(zn+\mathbf{b})[zn+\mathbf{b},-\infty n+\mathbf{b}]\Gamma[-\infty n-\mathbf{b},-zn-\mathbf{b}]q(-zn-\mathbf{b}),$$



$$U^{\Gamma}_{\text{SIDIS}}(z,\mathbf{b}) = \bar q(zn+\mathbf{b})[zn+\mathbf{b},+\infty n+\mathbf{b}]\Gamma[+\infty n-\mathbf{b},-zn-\mathbf{b}]q(-zn-\mathbf{b}),$$

Leading matching of all polarized distributions at twist-3

$$U^{\Gamma}_{\mathrm{DY}}(z,\mathbf{b}) = \bar q(zn+\mathbf{b})[zn+\mathbf{b},-\infty n+\mathbf{b}]\Gamma[-\infty n-\mathbf{b},-zn-\mathbf{b}]q(-zn-\mathbf{b}),$$



$$U^{\Gamma}_{\mathrm{SIDIS}}(z,\mathbf{b}) = \bar q(zn+\mathbf{b})[zn+\mathbf{b},+\infty n+\mathbf{b}]\Gamma[+\infty n-\mathbf{b},-zn-\mathbf{b}]q(-zn-\mathbf{b}),$$

$$U^{\Gamma}_{\mathrm{DY(SIDIS)}}(z,\mathbf{b}) = O^{\Gamma}(z)+\mathbf{b}^{\mu}\Big\{\lim_{x\to z}\frac{\partial}{\partial x^{\mu}}O^{\Gamma}(x)-i\int_{-1}^{1}dv\,vz\,T^{\Gamma}_{\mu}(z,vz,-z)$$
$$+(-)i\left(\int_{-\infty}^{z}+\int_{-\infty}^{-z}\right)d\tau T^{\Gamma}_{\mu}(z,\tau,-z)\Big\}+\mathcal{O}(\mathbf{b}^2),$$

$$O^{\Gamma}(z)=\bar q(z)\Gamma[z,-z]q(z), \qquad T^{\Gamma}_{\mu}(z,\tau,-z)=\bar q(z)[z,\tau]\Gamma F_{\mu+}(\tau)[\tau,-z]q(-z)$$

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Leading matching of all polarized distributions at twist-3

Collins	$f_{1T}^{\perp}(x,\mathbf{b}) = +(-)\pi\, T(x,x) + O(\mathbf{b}^2)$
Boer-Mulders	$h_1^{\perp}(x,\mathbf{b}) = -(+)\pi\, \delta T_\epsilon(x,x) + O(\mathbf{b}^2)$
Worm-gear T	$g_{1T}^{\perp}(x,\mathbf{b}) = 2\int \frac{dx_2}{x_2^2}\int_{x-x_2}^{x} du \Delta T(u,u+x_2) - \frac{1}{2}\int du \frac{u}{\lvert u\rvert}\left(g_1(u) - g_T(u)\right) + O(\mathbf{b}^2)$
Worm-gear L	$h_{1L}^{\perp}(x,\mathbf{b}) = -2\int \frac{dx_2}{x_2^2}\int_{x-x_2}^{x} du \delta T_g(u,u+x_2) + \frac{1}{2}\int du \frac{u}{\lvert u\rvert}\left(h_1(u) - h_L(u)\right) + O(\mathbf{b}^2)$

Qiu-Sterman functions

$$\langle \mathcal{T}_\mu^{\gamma^+}\rangle \sim \tilde{s}_{\mu T} T \qquad \langle \mathcal{T}_\mu^{\gamma^+\gamma^5}\rangle \sim s_{\mu T}\Delta T \qquad \langle \mathcal{T}_\mu^{\sigma^{\alpha+}\gamma^5}\rangle \sim \epsilon_T^{\mu\alpha}\delta T_\epsilon + g_T^{\mu\alpha}\delta T_g$$

Functions g_T and h_L are compositions of twist-2/3 PDFs.

Conclusion

- Factorization of rapidity divergences is proven (for SIDIS assuming universality)
- TMD evolution is complete 3-loop
- Small-b matching is known for all TMDs (see table)

	known	soon to come
H (universal)	3-loop [2010]	
γ_F (universal)	3-loop [2010]	
$\mathcal{D}$ (universal)	3-loop [2016]	
f_1(unpol. PDF)	2-loop [2015]	
d_1(unpol.FF)	2-loop [2016]	
g_1(helicity)	1-loop [2013]	
h_1(transvercity)	1-loop [2013]	**2-loop**
$h_{1T}^{\perp}$(pretzelocity)	1-loop (=0) [2017]	**2-loop**
$f_{1T}^{\perp}$(Sivers)	?tree [2014]	**tree**
$d_{1T}^{\perp}$(Collins)	unknown	**tree**
$h_1^{\perp}$(Boer-Mulders)	unknown	**tree**
g_{1T}, h_{1L} (worm-gear T,L)	unknown	**tree**

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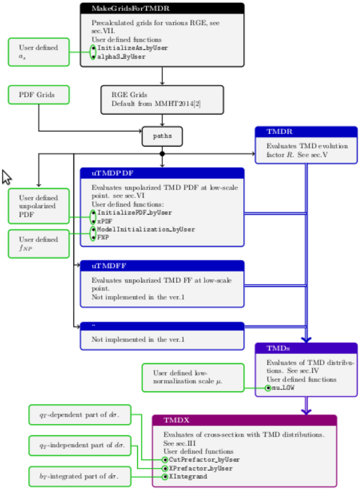

Tool for phenomenological studies of TMDs with high-end theory input

- FORTRAN 90 code
- Module structure
- Convolutions, evolution (LO,NLO,NNLO)
- Fourier to q_T-space, integrations over phase space
- Scale-variation (ζ-prescription)
- User defined PDFs, scales, f_{NP}
- Efficient code ($\sim 10^9$ TMDs $\sim$ 6. min at NNLO)

Currently ver 1.1 (soon 1.2 with TMD FFs)

Available at: https://teorica.fis.ucm.es/artemide

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