Anatomy TMD evolution: solution ambiguity, ζ-prescription and all that

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Motivation

There was a significant progress in the theory of TMD distributions during last years.

The study of TMD distribution enters the new phase: precise extraction of TMD distributions.

- Global fits DY+SIDIS (currently only [Bacchetta, et al; 1703.10157])
- Usage of precise data (e.g. LHC)
- Inclusion of high-order perturbation correction (e.g. NNLO [Scimemi,AV; 1706.01473]).

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Questions of internal consistency and comparison of results became ultimately important

I will show that currently there is a fundamental problem within the "naive" TMD evolution approach. It is important/unimportant, but definitely potentially very dangerous. And should be fixed as early as possible.

Evidence of the problem (1)

Problem of comparison

Couple of mouths ago I tried to compare the unpolarized TMD PDF extracted in [Bacchetta, et al; 1703.10157] to one extracted in [Scimemi,AV; 1706.01473].

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Problem of comparison

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I was not able to do it

- I took the value of TMD PDF from [Bacchetta, et al; 1703.10157] substitute to `arTeMiDe`
- And obtain complete non-sense with $\chi^2/points \sim 40$.
- Huge disagreement for each experiment!
- Is it the problem of the code? do not think so
- Is it the problem of formulas misunderstanding? also, do not think so
- Is it the problem of formalism? yes, it is

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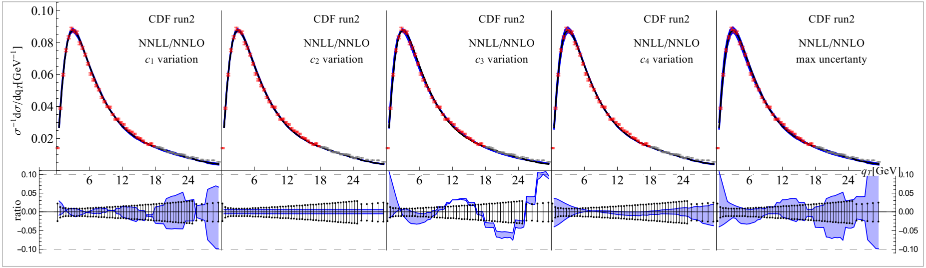
Evidence of the problem (2)

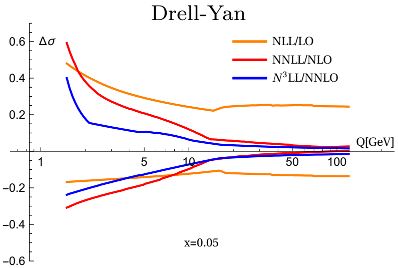


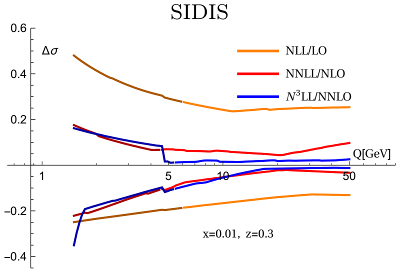
Anomalous behavior of variations

In [Scimemi,AV; 1706.01473] there was a study of a perturbative stability. With the help of variation of scales.

- The variations of constants c_1 and c_3 are the largest despite these are 3-loop series (compare to c_2 and c_4 which are 2-loop)
- The variation of c_1 and c_3 are numerically unstable (see artifacts)

Evidence of the problem (3)





Anomalous behavior of variations (2)

- The variations of constants does not decrease at large-Q.
- Opposite it start to increase at large-Q.

Evidence of the problem (4)

Strong dependence on μ

- It seems that TMD fits are seriously dependent on the values of μ (μ_b, μ^*, etc)
- Often the parameter μ is used as a subject of fit.
- Is it evidence of perturbative instability? Difficult to answer, since there is no dedicated study on it.

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In fact, these are consequences of a larger problem:

> not self-consistency of TMD evolution in the "traditional" form
> within perturbation theory.

Under "traditional" I refer to, say [Collins textbook]

$$\times \exp\left\{ \ln\frac{\sqrt{\zeta_A}}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_D(g(\mu');1) - \ln\frac{\sqrt{\zeta_A}}{\mu'}\gamma_K(g(\mu'))\right]\right\}. \tag{13.70}$$

This is probably the best formula for calculating and fitting TMD fragmentation functions;

Outline

Disclaimer:
part of talk is probably too elementary. But I have not found any dedicated discussion in the literature.
Thus, it is probably worth to spend some time and fix the terminology.

Outline

- TMD evolution in a nutshell
  - Equations, solutions, etc.
  - TMD evolution field and its structure
- Effects of truncation perturbation theory
  - Violation of integrability condition, and solution-dependence of TMD evolution
  - Methods to fix the ambiguity.
  - Improved γ approach.
- ζ-prescription
  - The advantages of ζ-prescription
  - Universal scale-independent TMD distribution.
- TMD cross-section and perturbative uncertainties.

TMD evolution: theory

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TMD evolution: theory

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta), \tag{1}$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}^f(\mu,b) F_{f\leftarrow h}(x,b;\mu,\zeta), \tag{2}$$

- γ_F – TMD anomalous dimension
- $\mathcal{D}$ – rapidity anomalous dimension ($= -\frac{\tilde{K}}{2}$[Collins' book], $= K$[Bacchetta, at al,1703.10157])
- Anomalous dimensions are *universal*, i.e. independent on hadron, polarization, PDF/FF(see proof [AV;1707.07606]).
- Anomalous dimension depend only on flavor (gluon/quark). Skip index f in the following.

Collinear overlap

There are collinearly divergent subgraphs (then gluon is parallel to Wilson line), which result to overlap of UV and rapidity divergent sectors. It gives interdependance of anomalous dimension on "opposite" scale

$$\zeta\frac{d}{d\zeta}\gamma_F(\mu,\zeta) = -\Gamma(\mu),$$

$$\mu\frac{d}{d\mu}\mathcal{D}(\mu,b) = \Gamma(\mu),$$

where Γ is the (light-like) cusp anomalous dimension.

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Thus the logarithmic part of AD's could be fixed

(exact) $$\gamma_F(\mu,\zeta) = \Gamma(\mu)\ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V(\mu)$$

(order-by-order) $$\mathcal{D}(\mu,b) = a_s(\mu)\frac{\Gamma_0}{2}\mathbf{L}_\mu + a_s^2\left(\frac{\Gamma_0\beta_0}{4}\mathbf{L}_\mu^2 + \frac{\Gamma_1}{2}\mathbf{L}_\mu + d^{(2,0)}\right) + ...$$

standard notation: $\mathbf{L}_X = \ln(C_0^{-2}b^2X^2)$, $C_0 = 2e^{-\gamma_E}$

TMD evolution is used for two practical purposes

- Compare different experiments
- Modeling TMD distribution

$$\frac{d\sigma}{dX} \sim \int d^2 b \, e^{i(bq_T)} H_{ff'}(Q,\mu) F_{f\leftarrow h}(x_1, b; \mu, \zeta_1) F_{f'\leftarrow h}(x_2, b; \mu, \zeta_2)$$

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Minimize $\ln(Q/\mu)$
$\mu = Q$

$\zeta_1\zeta_2 = Q^4$
or
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Minimize $\mathbf{L}_\mu$, $\mathbf{L}_{\sqrt{\zeta}}$
$\mu \sim \sqrt{\zeta} \sim b^{-1}$

$$F(x,b;\mu,\zeta) \sim C(x,b;\mu,\zeta) \otimes \mathrm{PDF}(x,\mu)$$

Typical model for TMD includes matching

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$$F(x,b;\mu_f,\zeta_f) = R[b,(\mu_f,\zeta_f)\to(\mu_i,\zeta_i)]F(x,b;\mu_i,\zeta_i)$$

Final scale ⟼ TMD evolution factor ⟼ Initial scale

Minimize $\mathbf{L}_\mu$, $\mathbf{L}_{\sqrt{\zeta}}$
$\mu \sim \sqrt{\zeta} \sim b^{-1}$

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The solution of TMD evolution equation (i.e. R) exists (in the strict mathematical sense) only if

$$\zeta \frac{d}{d\zeta} \frac{\gamma_F(\mu,\zeta)}{2} = -\mu^2 \frac{d}{d\mu^2} \mathcal{D}(\mu, b)$$

integrability condition

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Solution is

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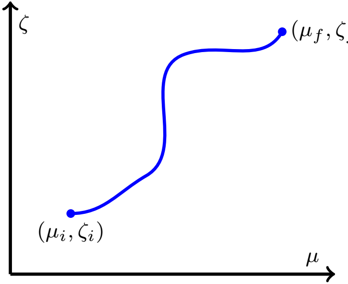
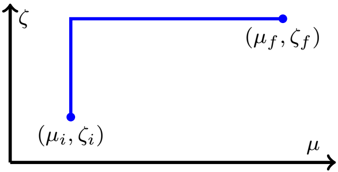


The solution is independent on the path of the integration due to *integrability condition*

Examples



Solution 1

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_f) - \mathcal{D}(\mu_i, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

given in [Collins' textbook]

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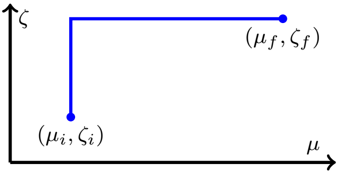

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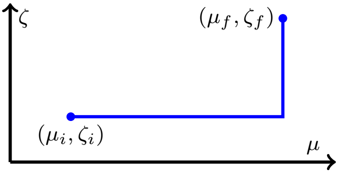

Solution 2

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu}\gamma_F(\mu,\zeta_i) - \mathcal{D}(\mu_f,b)\ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

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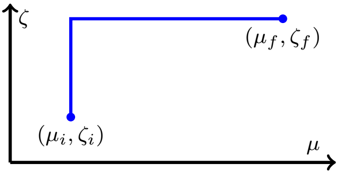

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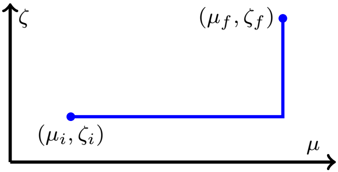

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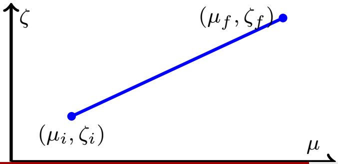

Solution 3

$$\ln R = \int_0^1 \Big(\gamma_F(\mu(t), \zeta(t)) \frac{\mu_f - \mu_i}{(\mu_f - \mu_i)t + \mu_i} - \mathcal{D}(\mu(t), b) \frac{\zeta_f - \zeta_i}{(\zeta_f - \zeta_i)t + \zeta_i}\Big) dt$$

TMD evolution is essentially 2D task.
Let me introduce convenient notation.

Evolution scales	Anomalous dimensions
$\boldsymbol{\nu} = \left(\ln\left(\frac{\mu^2}{1\ \text{GeV}^2}\right), \ln\left(\frac{\zeta}{1\ \text{GeV}^2}\right)\right).$	$\mathbf{E}(\boldsymbol{\nu}, b) = \left(\frac{\gamma_F(\boldsymbol{\nu})}{2}, -\mathcal{D}(\boldsymbol{\nu}, b)\right).$
2d vector	vector field

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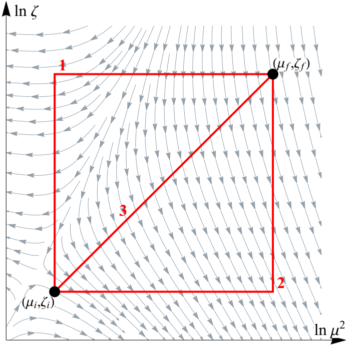

Evolution equation

$$\boldsymbol{\nabla} F(x, b; \boldsymbol{\nu}) = \mathbf{E}(\boldsymbol{\nu}, b) F(x, b; \boldsymbol{\nu})$$

Solution

$$\ln R[b, \boldsymbol{\nu}_f \to \boldsymbol{\nu}_i] = \int_P \mathbf{E} \cdot d\boldsymbol{\nu}$$

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Scalar potential

The integrability condition is the condition that evolution field $\mathbf{E}$ is *irrotational* (*conservative*)

$$\boldsymbol{\nabla} \times \mathbf{E} = 0$$

Thus, it is determined by a *scalar potential*

$$\mathbf{E}(\boldsymbol{\nu}, b) = \boldsymbol{\nabla} U(\boldsymbol{\nu}, b)$$

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Evolution is the difference between potentials

$$\ln R[b; \boldsymbol{\nu}_f \to \boldsymbol{\nu}_i] = U(\boldsymbol{\nu}_f, b) - U(\boldsymbol{\nu}_i, b).$$

Scalar potential can be easily found

$$U(\boldsymbol{\nu}, b) = \int^{\nu_1} \frac{\Gamma(s)s - \gamma_V(s)}{2} ds - \mathcal{D}(\boldsymbol{\nu}, b)\nu_2 + \text{const}(b),$$

Test of solution independence

$$(Q, Q^2) \to (\mu_b, \mu_b^2) \qquad \mu_b = \frac{C_0}{b} + 2\text{GeV}$$

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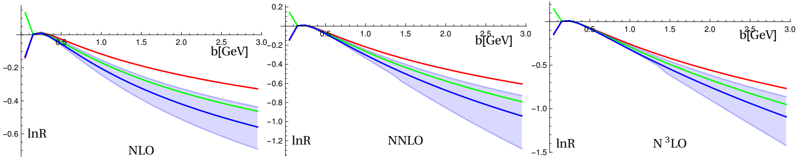
$Q = 10\text{GeV}$ (perturbation theory could work not very well)

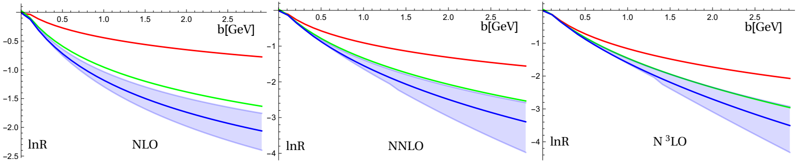


- Typical range of Fourier integration $b \in (0, 3)\text{GeV}^{-1}$
- The difference between $\ln R$ at $b = 1\text{GeV}^{-1}$ (1.74,1.39,1.23)
- The difference between R at $b = 1\text{GeV}^{-1}$ (1.09,1.08,1.06)
- Effect is almost negligible but non-zero(!)
- Improvement NLO→NNLO (~ 1.11) is (a bit) bigger then solution dependence
- Improvement NNLO→NNNLO (~ 1.04) is of the same order as solution dependence
- NP model for $\mathcal{D}$ could compensate the effect

Test of solution independence

$$(Q, Q^2) \to (\mu_b, \mu_b^2) \qquad \mu_b = \frac{C_0}{b} + 2\text{GeV}$$

$Q = M_Z$ (perturbation theory should work well)



- Typical range of Fourier integration $b \in (0, 1)\text{GeV}^{-1}$
- The difference between $\ln R$ at $b = 0.5\text{GeV}^{-1}$ (2.6,1.5,1.23)
- The difference between R at $b = 0.5\text{GeV}^{-1}$ (1.6,1.35,1.18)
- Effect is very sizable, $a_s \simeq 0.009$, b in perturbative region.
- Improvement NLO→NNLO (~ 1.22) is of the same order as solution dependence
- Improvement NNLO→NNNLO (~ 1.10) is smaller then solution dependence
- NP model for $\mathcal{D}$ could not compensate the effect, it is too large in PT region.

Effects of truncation of PT

Synopsis of the problem

- There is a solution dependence of TMD evolution
- It is almost negligible at smaller Q, but large at larger Q.
- It is not disappear (or disappear very slowly) with the increase of PT order.
- At 3-loop order it is the largest uncertainty that comes from perturbation theory

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The source of solution dependence is the violation of integrability condition.

In (truncated) perturbation theory

$$\zeta \frac{d}{d\zeta} \frac{\gamma_F(\mu,\zeta)}{2} \neq -\mu^2 \frac{d}{d\mu^2} \mathcal{D}(\mu, b) \qquad \Leftrightarrow \qquad \boldsymbol{\nabla} \times \mathbf{E} \neq 0 \tag{3}$$

The evolution flow is *non-conservative*, the scalar potential is undetermined

The TMD evolution equation has not a unique solution.

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Main effects

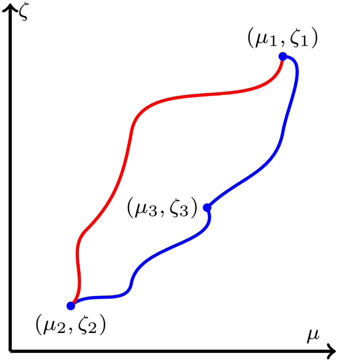
Although the discrepancy between evolution exponents is huge it is not the main problem. Much more serious problems are

- Violation of transitivity
- Violation of inverse rule

These are cornerstones of the evolution approach.

Transitivity

$$R[b;(\mu_1,\zeta_1)\to(\mu_2,\zeta_2)] = R[b;(\mu_1,\zeta_1)\to(\mu_3,\zeta_3)]R[b;(\mu_3,\zeta_3)\to(\mu_2,\zeta_2)]$$



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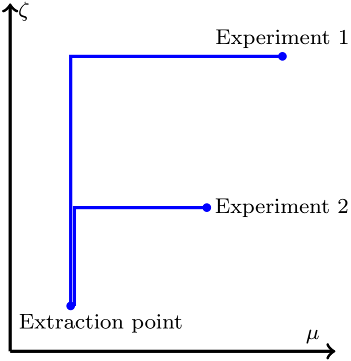

In the fitting procedure
different experiments (different Q)
define the same point (same b)

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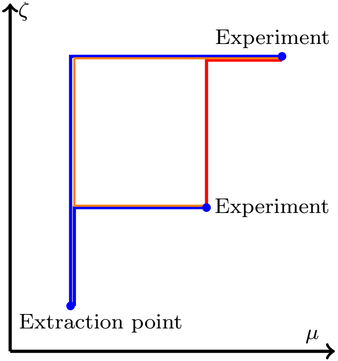

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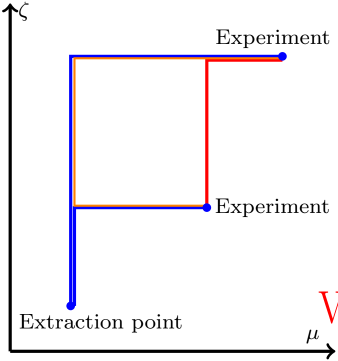

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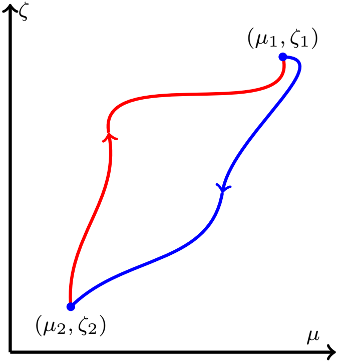
We have lost prediction power!

Inversion

$$R[b;\{\mu_1,\zeta_1\}\to\{\mu_2,\zeta_2\}] = R^{-1}[b;\{\mu_2,\zeta_2\}\to\{\mu_1,\zeta_1\}]$$



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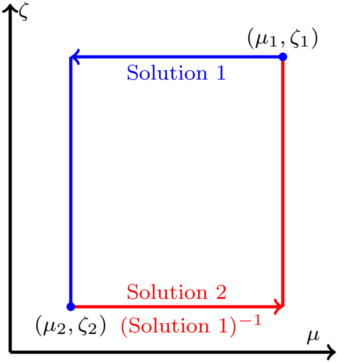

$$R[b;\{\mu_1,\zeta_1\}\xrightarrow{1}\{\mu_2,\zeta_2\}] = R^{-1}[b;\{\mu_2,\zeta_2\}\xrightarrow{2}\{\mu_1,\zeta_1\}]$$
$$\neq R^{-1}[b;\{\mu_2,\zeta_2\}\xrightarrow{1}\{\mu_1,\zeta_1\}]$$

That is the reason of why I was not able to compare fits

Reverse ingeneering in each case!

Why is it so large?

It is a unique situation

- The dependence on truncation of PT is normal situation
- The dependence on the solution is not normal situation

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Why is it so large?

It is a unique situation

- The dependence on truncation of PT is normal situation
- The dependence on the solution is not normal situation
- This effect is absolutely uncontrollable (could you guaranty that in your program you always evolve in the same direction?)
- It is even difficult to realize the possible consequences
- Numerical impact is large
- This effect is stable with respect to increase of PT

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The violation takes place in the *integrability* condition

$$\mu \frac{d\mathcal{D}(\mu)}{d\mu} \neq \Gamma(\mu)$$

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$$\mu\frac{d\mathcal{D}(\mu)}{d\mu} \neq \Gamma(\mu)$$

Simple example at 1-loop

$$\mathcal{D} = a_s(\mu)\frac{\Gamma_0}{2}\mathbf{L}_\mu$$

$$\begin{aligned}\mu\frac{d\mathcal{D}}{d\mu} &= a_s(\mu)\frac{\Gamma_0}{2}\left(\mu\frac{d}{d\mu}\mathbf{L}_\mu\right) + \left(\mu\frac{da_s(\mu)}{d\mu}\right)\frac{\Gamma_0}{2}\mathbf{L}_\mu \\ &= a_s(\mu)\Gamma_0 - \beta_0 a_s^2(\mu)\Gamma_0\mathbf{L}_\mu \neq a_s(\mu)\Gamma_0\end{aligned}$$

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$$= a_s(\mu)\Gamma_0 - \beta_0 a_s^2(\mu)\Gamma_0\mathbf{L}_\mu \neq a_s(\mu)\Gamma_0$$

$$\delta\Gamma(\mu,b) = \Gamma(\mu) - \mu\frac{d\mathcal{D}(\mu,b)}{d\mu}$$

At N'th order of perturbation theory $\delta\Gamma^{(N)} \sim a_s^{N+1}\mathbf{L}_\mu^N$

- Since $a_s \sim \ln^{-1}\mu$ there is always (at any finite N) value of b(fixed) then $\delta\Gamma \gg 1$
- The value of μ does not play a role
- In fact, this term is ALWAYS NLO, in the standard resummation counting $(a_s L \sim 1)$.
- The NP models for $\mathcal{D}$ only enforce the problem.

Helmeholz decomposition

$$\mathbf{E} = \tilde{\mathbf{E}} + \boldsymbol{\Theta}$$

$\tilde{\mathbf{E}}$	conservative (*irrotational*) component	$\text{curl}\tilde{\mathbf{E}} = 0$
$\boldsymbol{\Theta}$	*divergence-free* component	$\boldsymbol{\nabla} \cdot \boldsymbol{\Theta} = 0$

$$\tilde{\mathbf{E}} \cdot \boldsymbol{\Theta} = 0$$

$$\text{curl}\mathbf{E} = \text{curl}\boldsymbol{\Theta} = \frac{\delta\Gamma}{2}$$

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Ambiguous scalar potential

The *divergence-free* component is artifact of PT. But it prevents the definition of scalar potential

$$\boldsymbol{\nabla}\tilde{U} = \tilde{\mathbf{E}}, \qquad \mathbf{curl}V = \boldsymbol{\Theta}$$

$$\nabla^2\tilde{U} = \frac{d\gamma_F}{d\ln\mu}$$

Poisson equation solution is defined up to $\nabla^2 f = 0$.

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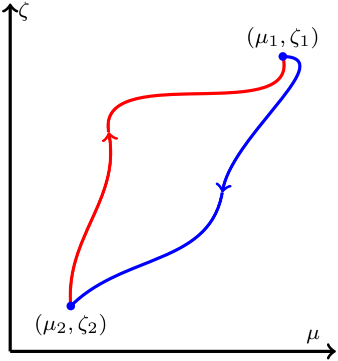
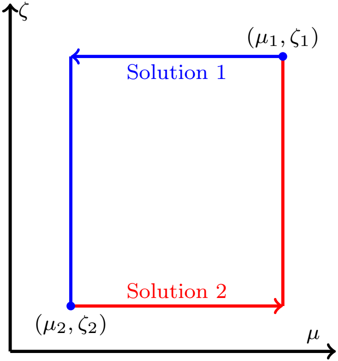
Non-conservative evolution



$$\oint_C \mathbf{E} \cdot d\nu = \int_\Omega d^2\nu \, \text{curl}\boldsymbol{\Theta} = \frac{1}{2} \int_\Omega d^2\nu \, \delta\Gamma(\nu, b)$$

Non-conservative evolution



$$\oint_C \mathbf{E}\cdot d\nu = \int_\Omega d^2\nu\,\mathrm{curl}\boldsymbol{\Theta} = \frac{1}{2}\int_\Omega d^2\nu\,\delta\Gamma(\nu,b)$$

$$\ln\frac{\text{solution 1}}{\text{solution 2}} = \ln\left(\frac{\zeta_f}{\zeta_i}\right)\int_{\mu_i}^{\mu_f}\frac{d\mu}{\mu}\delta\Gamma(\mu,b)$$

The "longer" evolution – the bigger error

That is why for $Z-$boson error is larger

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How to fix it?

There are many possibilities

- Lets use a single evolution line $\mu^2 = \zeta$, and the solution 3
  - \+ Restore self-consistency and inversion
  - \- - Everyone stick to a single line. No freedom for modeling.
  - \- Numerically more expensive
- Lets set $\mathbf{\Theta} = 0$, and use only $\tilde{\mathbf{E}}$
  - \+ + Ideal solution which does not restrict anything
  - \- The procedure is not unique, we need to set boundary conditions
- Lets repair the integrability condition by adding terms beyond PT
  - \+ + Very simple
  - \- The procedure is not unique (however, there is only single "good" solution)
  - Equivalent to some boundary condition (do not know which)

We modify anomalous dimensions such that integrability restored

$$\mu \frac{d\mathcal{D}(\mu, b)}{d\mu} = -\zeta \frac{d\gamma_F(\mu, \zeta)}{d\zeta}$$

It can be done from both sides of the equation.

Improved $\mathcal{D}$

Improved γ

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Improved $\mathcal{D}$

Facilitate

$$\mu \frac{d\mathcal{D}}{d\mu} = \Gamma$$

by

$$\mathcal{D}(\mu, b) = \int_{\mu_0}^{\mu} \frac{d\mu}{\mu} \Gamma(\mu) + \mathcal{D}(\mu_0, b)$$

- In the spirit of [Collins' text book].
- Already used in many studies

Improved γ

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- In the spirit of [Collins' text book].
- Already used in many studies
- However, it is not the best way

Improved γ

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Improved $\mathcal{D}$ solution

$$\ln R[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i);\mu_0] = \int_{\mu_i}^{\mu_f}\frac{d\mu}{\mu}\left(\Gamma(\mu)\ln\left(\frac{\mu^2}{\zeta_f}\right)-\gamma_V(\mu)\right) - \int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\Gamma(\mu)\ln\left(\frac{\zeta_f}{\zeta_i}\right)-\mathcal{D}(\mu_0,b)\ln\left(\frac{\zeta_f}{\zeta_i}\right).$$

- μ_0 is some scale where perturbation theory works. At larger b is to be modified (see e.g. b^*)
- In fact it is the composition of solution 1 and 2

Improved $\mathcal{D}$ solution

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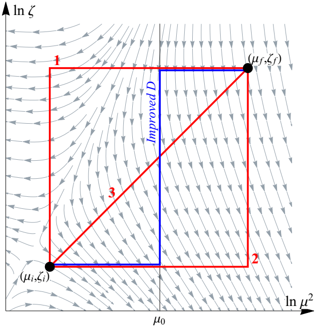

- Transitivity and inversion hold
- If different μ_0 are used, the problem returns
- If different non-perturbative models are used, the problem returns

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We modify anomalous dimensions such that integrability restored

$$\mu \frac{d\mathcal{D}(\mu, b)}{d\mu} = -\zeta \frac{d\gamma_F(\mu, \zeta)}{d\zeta}$$

It can be done from both sides of the equation.

Improved $\mathcal{D}$

Facilitate

$$\mu \frac{d\mathcal{D}}{d\mu} = \Gamma.$$

by

$$\mathcal{D}(\mu, b) = \int_{\mu_0}^{\mu} \frac{d\mu}{\mu} \Gamma(\mu) + \mathcal{D}(\mu_0, b)$$

- In the spirit of [Collins' text book].
- Already used in many studies
- However, it is not the best way

Improved γ

We set

$$\zeta \frac{d\gamma_F}{d\zeta} \equiv -\mu \frac{d\mathcal{D}}{d\mu} = \delta\Gamma - \Gamma$$

Or

$$\gamma_F(\mu, \zeta) \rightarrow \gamma_M(\mu, \zeta, b)$$

$$\gamma_M = (\Gamma - \delta\Gamma) \ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V$$

- Completely self consistent
- Very natural

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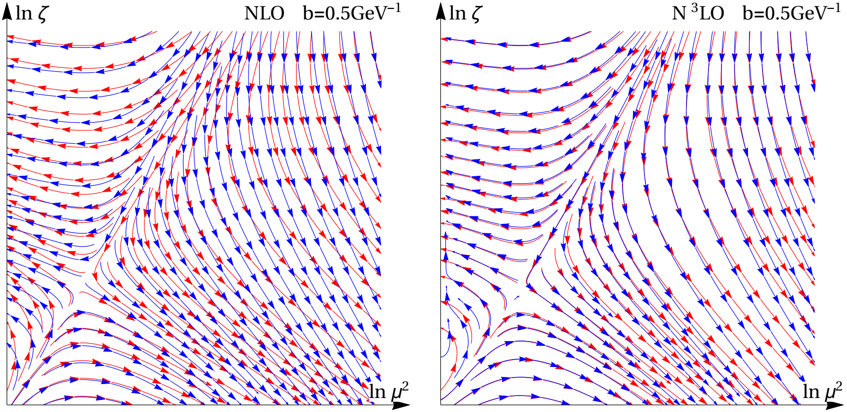
Improved γ-solution

$$\gamma_M = (\Gamma - \delta\Gamma) \ln\left(\frac{\mu^2}{\zeta}\right) - \gamma_V$$

$$\begin{aligned}\ln R[b; (\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] \quad = \quad & -\int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \left(2\mathcal{D}(\mu, b) + \gamma_V(\mu)\right) \\ & + \mathcal{D}(\mu_f, b) \ln\left(\frac{\mu_f^2}{\zeta_f}\right) - \mathcal{D}(\mu_i, b) \ln\left(\frac{\mu_i^2}{\zeta_i}\right).\end{aligned}$$

- Explicitly transitive, and inverse.
- Simple non-perturbative generalization ($\mathcal{D} \to \mathcal{D}_{NP}$)
- No extra scales. The evolution field is explicitly conservative.
- I suggest to use this formula for phenomenology, and avoid all previous complications
- Always publish $\mathcal{D}_{NP}$. Otherwise we cannot compare.

How strong is modification of the field?



Part 2:
ζ-prescription

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The final scales (μ_f, ζ_f) are fixed by process kinematics $\sim (Q, Q^2)$.
The initial scale are fixed only by model of TMD distribution.

Small-b matching

At small-b one can match TMD to collinear distribution by OPE

$$\text{TMD}(x, b; \mu_i, \zeta_i) = C(x, \mathbf{L}_\mu, \mathbf{L}_{\sqrt{\zeta}}, \mu) \otimes \text{PDF}(x, \mu)$$

- It is often used as an zero-level input to the model of TMD.
- It guaranties agreement with high energy experiments.
- It also requires the evolution from $(Q, Q^2) \to (\mu_i, \zeta_i)$, which are typically selected as

$$\mu_i^2 = \zeta_i \sim \frac{1}{b^2}$$

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$$\mu_i^2 = \zeta_i \sim \frac{1}{b^2}$$

It is an arbitrary choice without any profit.
Let us use 2D nature of TMD evolution to get rid of logarithms completely.

"Naive" ζ-prescription

$$\mathrm{TMD}(x,b;\mu_i,\zeta_i)=C(x,\mathbf{L}_\mu,\mathbf{L}_{\sqrt{\zeta}},\mu)\otimes \mathrm{PDF}(x,\mu)$$

The ζ appears only in the coefficient function. Let us use it to compensate $\mathbf{L}_\mu$.
1-loop example

$$C=\delta(\bar x)+a_sC_F\Bigg[-2\underbrace{\mathbf{L}_\mu p(x)}_{\substack{\text{never large}\\ \text{thanks to}\\ \text{charge}\\ \text{conservation}}}+2\bar x+\delta(\bar x)\Big(\overbrace{-\mathbf{L}_\mu\mathbf{L}_{\sqrt{\zeta}}+3\mathbf{L}_\mu}^{\text{usually large}}-\zeta_2\Big)\Bigg]$$

"Naive" ζ-prescription

$$\text{TMD}(x, b; \mu_i, \zeta_i) = C(x, \mathbf{L}_\mu, \mathbf{L}_{\sqrt{\zeta}}, \mu) \otimes \text{PDF}(x, \mu)$$

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We set $\zeta \to \zeta_\mu$: such that $\mathbf{L}_{\sqrt{\zeta}} = 3$.

$$\zeta_\mu = \frac{2\mu}{b} e^{-\gamma_E} \overbrace{e^{3/2 + a_s \ldots}}^{\text{PT-calculable}}$$

In this way we determine it in the PT. It has been used in [Scimemi,AV,1706.01473]
Let me call it "naive" implementation, because it is defined only in PT.

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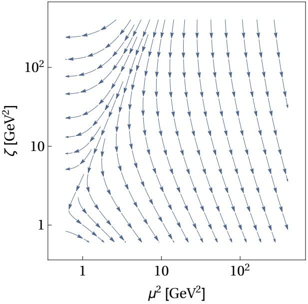
To understand ζ-prescription and to formulate it non-perturbatively
we should study
the structure of evolution field



The evolution field is determined by potential U

- In perturbation theory by potential $\tilde{U}$
- Evolution potential is smooth real-valued function without singularities in "physical" region.

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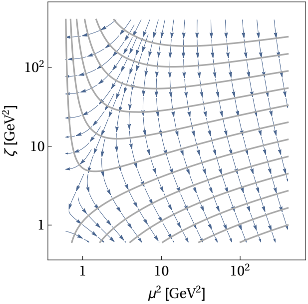

The evolution field is determined by potential U

- In perturbation theory by potential $\tilde{U}$
- Evolution potential is smooth real-valued function without singularities in "physical" region.
- There are lines of on which potential is same, or *equipotential* lines, or curves of *null-evolution*

The evolution from (μ_f, ζ_f) to (μ_i, ζ_i)

$$\ln R = U(\mu_f, \zeta_f) - U(\mu_i, \zeta_i)$$

It the same as the evolution to another point of *equipotential* line $\boldsymbol{\omega}$

$$\ln R = U(\mu_f, \zeta_f) - U(\boldsymbol{\omega}(\mu_i, \zeta_i))$$

- We are free to use the simplest path of evolution
- ~~Geodesic.~~ Just fixed-μ evolution

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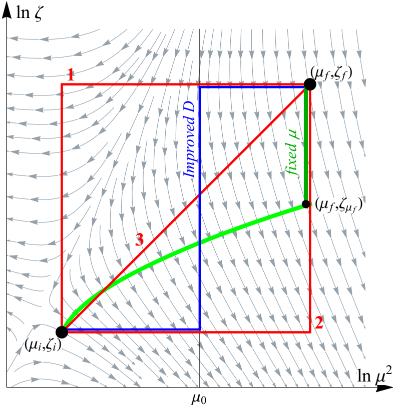

- We are free to use the simplest path of evolution
- ~~Geodesic.~~ Just fixed-μ evolution

$$\ln R[b; (\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = -\mathcal{D}(\mu_f, b) \ln\left(\frac{\zeta_f}{\zeta_{\mu_f}}\right)$$

- The **E** must be conservative
- (I found that) Psychologically it is difficult to accept, since there is "no resummation of large Sudakov logarithms", etc. But it is indeed just the same.
- Accepting that we should accept the following →

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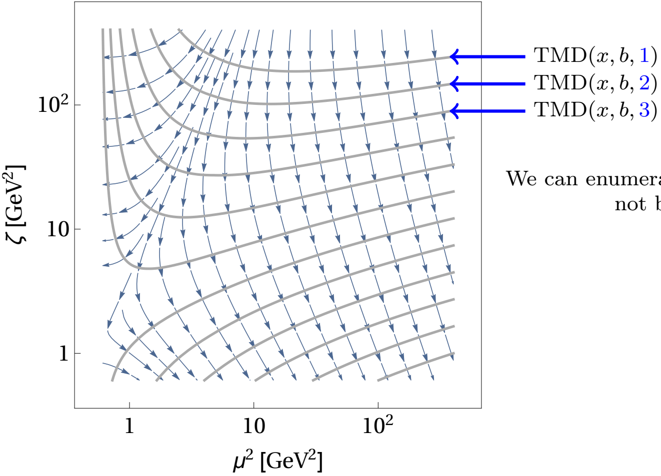
TMD distributions on the same equipotential line are equivalent.



We can enumerate them by a lines not by (μ, ζ)

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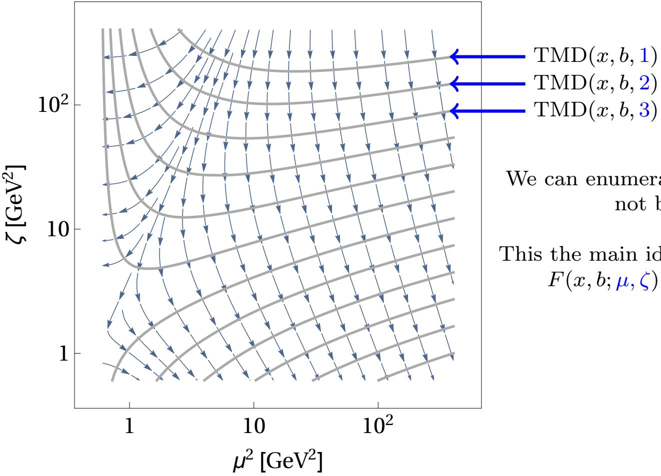

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This the main idea of ζ-prescription
$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$

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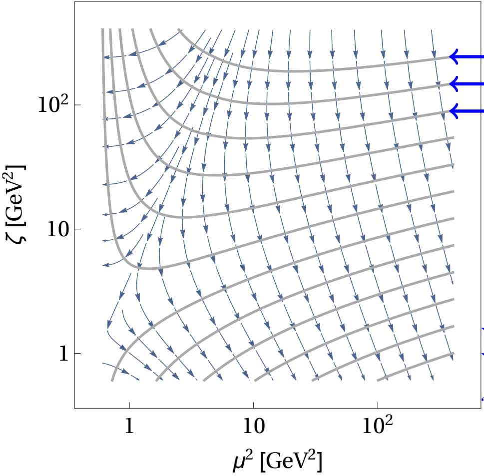


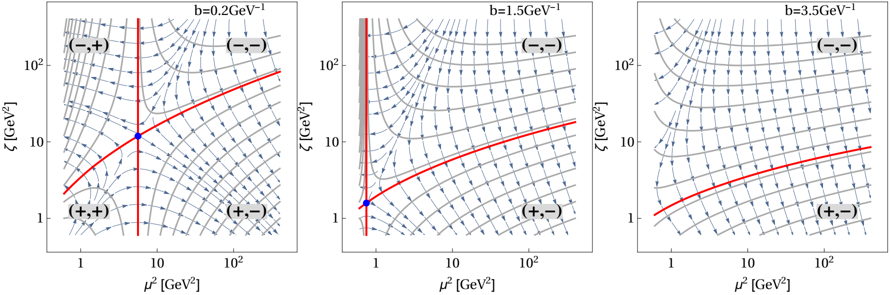
$\text{TMD}(x, b, 1)$
$\text{TMD}(x, b, 2)$
$\text{TMD}(x, b, 3)$

We can enumerate them by a lines not by (μ, ζ)

This the main idea of ζ-prescription
$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$

★ This is "non-perturbative" definition
★ In truncated PT it is equivalent to "naive" at some line.
?? Which line to select?

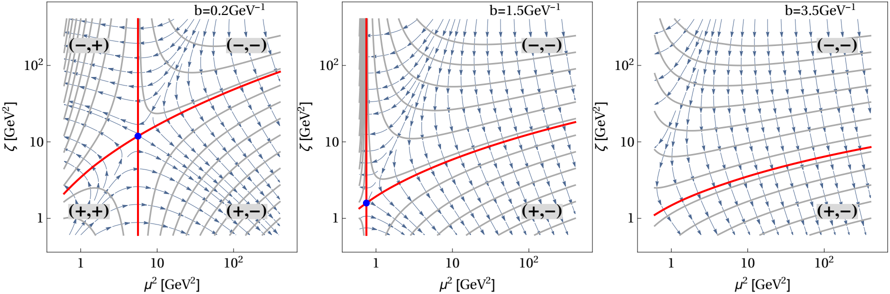
Singularities of evolution field



- Some non-interesting singularities at $\mu, \zeta \to \infty$
- Landau pole at $\mu = \Lambda$
- Saddle point (blue dot)

$$\mathcal{D}(\mu_{\text{saddle}}, b) = 0, \qquad \gamma_M(\mu_{\text{saddle}}, \zeta_{\text{saddle}}, b) = 0$$

Singularities of evolution field



- Due to presence of saddle point the set of uquipotential lines is split into subsets with restricted domains
- **Subset 1:** $\mu > \mu_{\text{saddle}}$
- **Subset 2:** $\mu < \mu_{\text{saddle}}$
- **Special line:** The one which passes though the saddle point (μ is unrestricted)
- Special lines dissect the evolution planes into quadratures of the "same evolution sign".

In ζ-prescription we set
$\zeta \to \zeta_\mu(\boldsymbol{\nu}_0)$

- TMDs are "enumerated" by $\boldsymbol{\nu}_0$ (the number of line)
- TMDs are scale independent

$$\mu \frac{d}{d\mu} F(x, b; \mu, \zeta_\mu) = 0.$$

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In ζ-prescription we set
$$\zeta \to \zeta_\mu(\boldsymbol{\nu}_0)$$

- TMDs are "enumerated" by $\boldsymbol{\nu}_0$ (the number of line)
- TMDs are scale independent

$$\mu \frac{d}{d\mu} F(x, b; \mu, \zeta_\mu) = 0.$$

- However, there is a redundant dependence on μ in the form of restriction

$$\text{at small b:} \quad F(x, b) = C(x, b, \mu_{\text{OPE}}) \otimes f(x, \mu_{\text{OPE}})$$

μ_{OPE} is restricted to the range of μ of equipotential line.

- Otherwise there is an evolutional "step" at $\mu_{\text{OPE}} = \mu_{\text{saddle}}$.
- It is not good, it makes difficult matching of low and high b in the perturbation theory (since saddle point migrates)

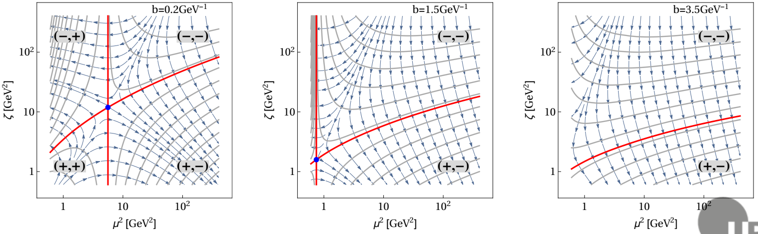
Universal scale-independent TMD

There is a unique line which passes though all μ's

The universal scale-independent TMD distribution

$$F(x,b)=F(x,b;\mu,\zeta_\mu)$$

where ζ_μ is the special line.



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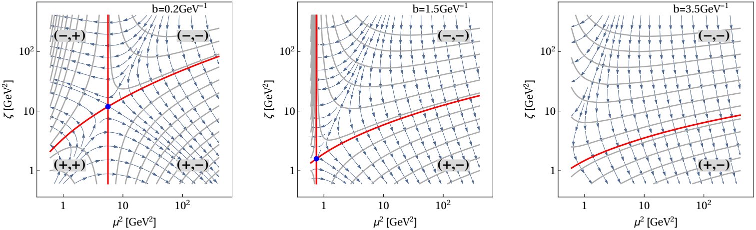
Universal scale-independent TMD

- The definition is non-perturbative
- The definition is the same for all TMD distributions (since the evolution is the same)
- **WARNING:** At large-b the saddle point can escape the observable region. Make sure that your $\mathcal{D}_{NP}$ keeps $\mu_{\text{saddle}} > \Lambda$
- Automatic in b^*.



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Part 3:
TMD cross-section and its uncertainties

To measure perturbative uncertainties, we typically vary scales μ.

- In exact PT, μ-dependence is absent, but at finite PT there is the perturbative mismatch between the evolution exponent and the fixed order coefficient function.
- In TMD case there is an additional source of scale-dependence, solution dependence

A TMD cross-section

$$\frac{d\sigma}{dX} = \sigma_0 \sum_f \int \frac{d^2b}{4\pi} e^{i(b\cdot q_T)} H_{ff'}(Q,\mu_f)$$
$$\times\{R^f[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i),\mu_0]\}^2 F_{f\leftarrow h}(x_1,b;\mu_i,\zeta_i)F_{f'\leftarrow h}(x_2,b;\mu_i,\zeta_i),$$

$$\mu_0 \to c_1\mu_0, \qquad \mu_f \to c_2\mu_f, \qquad \mu_i \to c_3\mu_i, \qquad \mu_{\text{OPE}} \to c_4\mu_{\text{OPE}}.$$

$$c_i \in (0.5, 2)$$

Some of these scales measure the solution dependence, some perturbative mismatch, some both.

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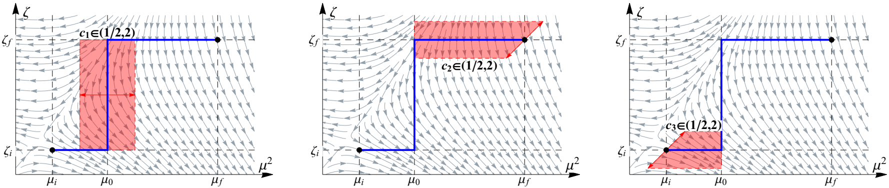
A TMD cross-section

$$\frac{d\sigma}{dX} = \sigma_0 \sum_f \int \frac{d^2b}{4\pi} e^{i(b\cdot q_T)} H_{ff'}(Q,\mu_f)$$
$$\times\{R^f[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i),\mu_0]\}^2 F_{f\leftarrow h}(x_1,b;\mu_i,\zeta_i)F_{f'\leftarrow h}(x_2,b;\mu_i,\zeta_i),$$



- c_1 measure only solution dependence
- c_2 measure mismatch between H and R + solution dependence
- c_3 measure mismatch between F and R + solution dependence
- c_4 measure mismatch between C and f

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Cross-section in the improved γ

In the improved γ there is no solution dependence

$$\frac{d\sigma}{dX} = \sigma_0 \sum_f \int \frac{d^2b}{4\pi} e^{i(b\cdot q_T)} H_{ff'}(Q,\mu_f) \{R^f[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i)]\}^2 F_{f\leftarrow h}(x_1,b;\mu_i,\zeta_i) F_{f'\leftarrow h}(x_2,b;\mu_i,\zeta_i),$$

where

$$R^f[b;(\mu_f,\zeta_f)\to(\mu_i,\zeta_i)] = \exp\Big\{-\int_{\mu_i}^{\mu_f}\frac{d\mu}{\mu}\Big(2\mathcal{D}^f_{\text{NP}}(\mu,b)+\gamma^f_V(\mu)\Big) + \mathcal{D}^f_{\text{NP}}(\mu_f,b)\ln\left(\frac{\mu_f^2}{\zeta_f}\right) - \mathcal{D}^f_{\text{NP}}(\mu_i,b)\ln\left(\frac{\mu_i^2}{\zeta_i}\right)\Big\}.$$

There are 3 scales and no solution dependence

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Cross-section in the ζ-prescription

$$\frac{d\sigma}{dX}=\sigma_0\sum_f\int\frac{d^2b}{4\pi}e^{i(b\cdot q_T)}H_{ff'}(Q,\mu_f)\{R^f[b;(\mu_f,\zeta_f)]\}^2F_{f\leftarrow h}(x_1,b)F_{f'\leftarrow h}(x_2,b),$$

where

$$R^f[b;(\mu_f,\zeta_f)]\quad=\quad\exp\Big\{-\int_{\mu_{\text{saddle}}}^{\mu_f}\frac{d\mu}{\mu}\Big(2\mathcal{D}^f_{\text{NP}}(\mu,b)+\gamma_V^f(\mu)\Big)+\mathcal{D}^f_{\text{NP}}(\mu_f,b)\ln\Big(\frac{\mu_f^2}{\zeta_f}\Big)\Big\}$$

WARNING: Special line boundary condition should be taken into account in the coefficient function (details in private)
However, we can exponentiate boundary conditions and get a simple **practical formula**

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Practical formula

$$\frac{d\sigma}{dX}=\sigma_0\sum_f\int\frac{d^2b}{4\pi}e^{i(b\cdot q_T)}H_{ff'}(Q,\mu_f)\{\tilde R^f[b;(\mu_f,\zeta_f)]\}^2\tilde F_{f\leftarrow h}(x_1,b)\tilde F_{f'\leftarrow h}(x_2,b),$$

with

$$\tilde R^f[b;(\mu_f,\zeta_f)] \quad=\quad \exp\Big\{-\mathcal{D}^f_{\text{NP}}(\mu_f,b)\Big[\ln\Big(\frac{\zeta_f b}{C_0\mu_f}\Big)+v^f(\mu,b)\Big]\Big\},$$

- v is given perturbative series, $v=\frac{3}{2}+a_s...$
- $\tilde F$ is TMD in the "naive" ζ-prescription
- There is no integrations in the "Sudakov exponent"
- There are no approximations (*ala* high energy expansion of integrals)
- The TMD at "final" point has curious form

$$F_{f\leftarrow h}(x,b;Q,Q^2)=(C_0Qb)^{-\mathcal{D}^f_{\text{NP}}(Q,b)}\,e^{-\mathcal{D}^f_{\text{NP}}(Q,b)v(Q)}\tilde F_{f\leftarrow h}(x,b).$$

There are only 2 scales and no solution dependence

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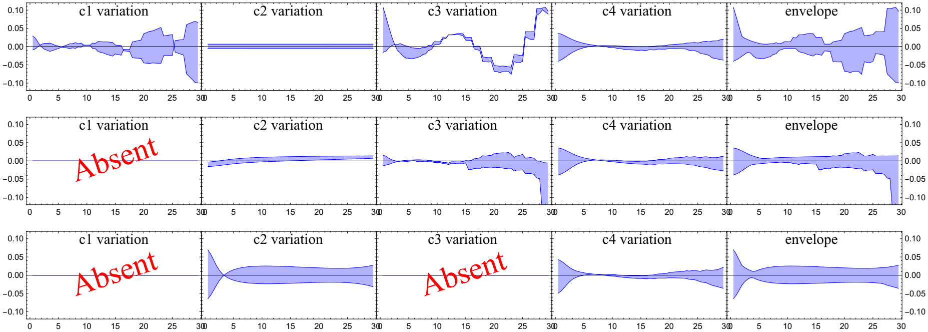
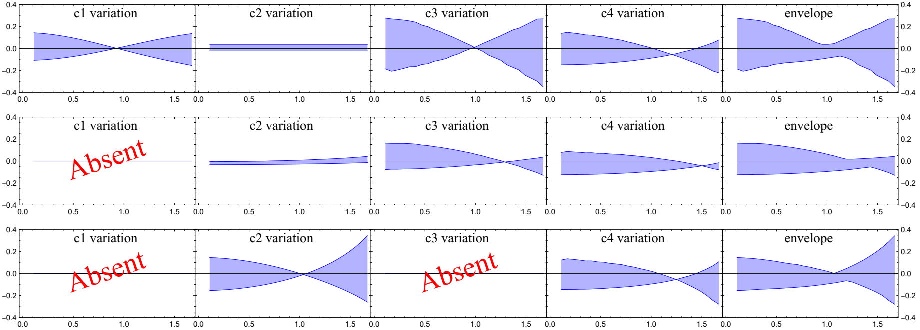
Uncertainties of TMD cross-section (1)

Z-boson production at CDF run 2



Uncertainties of TMD cross-section (2)

$$\text{E288 (200) } Q = 6 - 7 \text{ GeV}$$



Conclusion

Main message:
There is a serious problem within the TMD evolution (at finite order PT):
solution-dependence.
It brings inconsistencies into phenomenology and make impossible inter-comparison.

Message 1:
I suggest to use improved γ approach to avoid this problem

- Minimal modification of previous formulas
- Explicit path independence

Message 2:
It is even better to use ζ-prescription

- Non-perturbatively formulated
- Guarantied absence of (large) logarithms in coefficient function
- Universal for all quantum numbers
- Very simple practical formula (no integrations!)