# TMD evolution as a double-scale evolution

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in collaboration with Ignazio Scimemi
based on [1803.11089]

QCD evolution
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Santa Fe, NM

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TMD evolution
is
a double-scale evolution

$$F(x, b; \mu, \zeta)$$

TMD evolution must be taken into account with a great care

### More freedom

- Relations between anomalous dimensions
- $\zeta$-prescription

### More ambiguity

- Many scales to "tune"
- Violation of transitivity in "naive" formulation (see [1803.11089])

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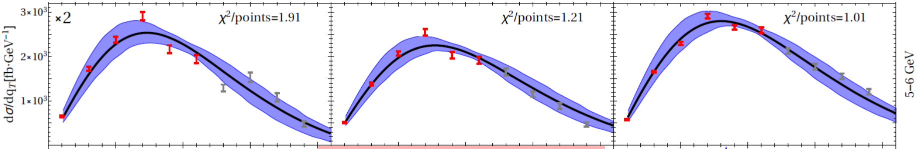
# NNLO fit and extraction of (unpol.) TMDPDF
[Scimemi,AV;1706.01473]

- The largest number of data point (DY)
- The largest energy separation
- Consideration of various orders (NLO,NNLL,NNLO)
- Studies of theory error-bands

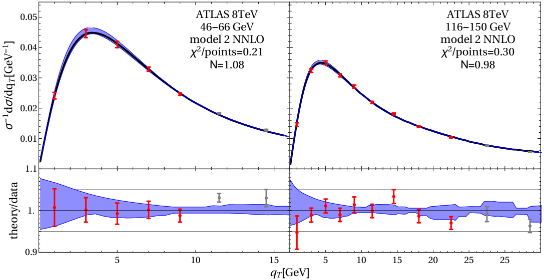
**Included data (at $q_T < 0.2Q$)**

| | reaction | $\sqrt{s}$ | $Q$ | comment | points |
|---|---|---|---|---|---|
| E288 | $p + Cu \to \gamma^* \to \mu\mu$ | 19.4 GeV | 4-9 GeV | norm=0.8 | 35 |
| E288 | $p + Cu \to \gamma^* \to \mu\mu$ | 23.8 GeV | 4-9 GeV | norm=0.8 | 45 |
| E288 | $p + Cu \to \gamma^* \to \mu\mu$ | 27.4 GeV | 4-9 & 11-14 GeV | norm=0.8 | 66 |
| CDF+D0 | $p + \bar{p} \to Z \to ee$ | 1.8 TeV | 66-116 GeV | | 44 |
| CDF+D0 | $p + \bar{p} \to Z \to ee$ | 1.96 TeV | 66-116 GeV | | 43 |
| ATLAS | $p + p \to Z \to \mu\mu$ | 7 & 8 TeV | 66-116 GeV | tiny errors! | 18 |
| CMS | $p + p \to Z \to \mu\mu$ | 7 & 8 TeV | 60-120 GeV | | 14 |
| LHCb | $p + p \to Z \to \mu\mu$ | 7 & 8 & 13 TeV | 60-120 GeV | | 30 |
| ATLAS | $p + p \to Z/\gamma^* \to \mu\mu$ | 8 TeV | 46-66 GeV | | 5 |
| ATLAS | $p + p \to Z/\gamma^* \to \mu\mu$ | 8 TeV | 116-150 GeV | | 9 |
| | | | | Total | **309** |




Drell-Yan at $Q = 5 - 6$GeV

Drell-Yan at $Q = 116 - 150$GeV




TMD evolution is a key element

$$\frac{\chi^2_{\text{global}}}{d.o.f.} \simeq 1.25$$

Here:

- 3-loop evolution
- 2-loop coefficient function
- 2-loop matching
- **$\zeta$-prescription**

plots from [1706.01473]

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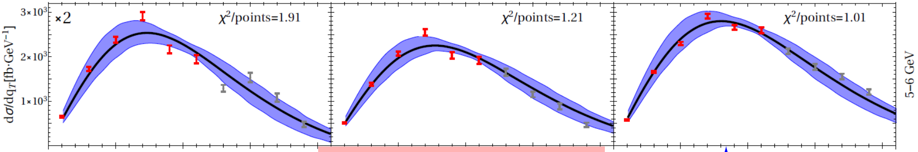
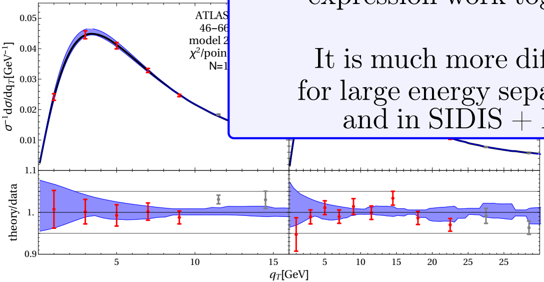



Drell-Yan at $Q = 5 - 6$GeV

The main difficulty is to make parts of factorized expression work together

It is much more difficult for large energy separation and in SIDIS + DY

Drell-Ya

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plots from [1706.01473]

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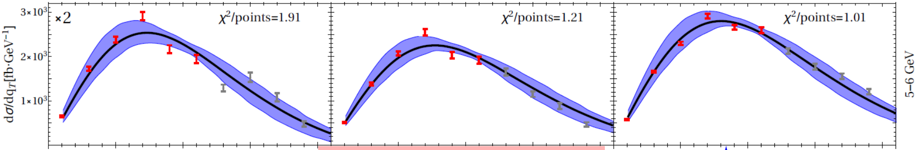


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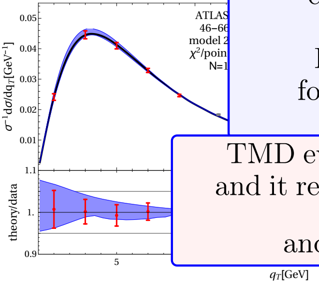


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$\frac{\chi^2_{\text{global}}}{d.o.f.} \simeq 1.25$

3-loop evolution

2-loop coefficient unction

matching

scription

1706.01473]

TMD evolution is the central element, and it requares critical "reexamination"

and it is the topic of this talk

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## TMD evolution is used for two practical purposes

- Compare different experiments
- Modeling TMD distribution

$$\frac{d\sigma}{dX} \sim \int d^2b\, e^{i(bq_T)} H_{ff'}(Q,\mu) F_{f\leftarrow h}(x_1, b; \mu, \zeta_1) F_{f'\leftarrow h}(x_2, b; \mu, \zeta_2)$$

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Minimize $\ln(Q/\mu)$

$\mu = Q$

$\zeta_1\zeta_2 = Q^4$

or

$\zeta_1 = \zeta_2 = Q^2$

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Minimize $\mathbf{L}_\mu$, $\mathbf{L}_{\sqrt{\zeta}}$
$\mu \sim \sqrt{\zeta} \sim b^{-1}$

$$F(x,b;\mu,\zeta) \sim C(x,b;\mu,\zeta) \otimes \text{PDF}(x,\mu)$$

Typical model for TMD includes matching

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$$F(x,b;\mu_f,\zeta_f) = R[b,(\mu_f,\zeta_f) \to (\mu_i,\zeta_i)] F(x,b;\mu_i,\zeta_i)$$

Final scale ⟼ TMD evolution factor ⟼ Initial scale

Minimize $\mathbf{L}_\mu$, $\mathbf{L}_{\sqrt{\zeta}}$
$\mu \sim \sqrt{\zeta} \sim b^{-1}$

$$F(x,b;\mu,\zeta) \sim C(x,b;\mu,\zeta) \otimes \mathrm{PDF}(x,\mu)$$

Typical model for TMD includes matching

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# TMD evolution equations

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta), \quad (1)$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}^f(\mu,b) F_{f\leftarrow h}(x,b;\mu,\zeta), \quad (2)$$

Solution: $F(x,\mathbf{b};\mu_f,\zeta_f) = R[\mathbf{b};(\mu_f,\zeta_f)\to(\mu_i,\zeta_i)]F(x,\mathbf{b};\mu_i,\zeta_i)$

- $\gamma_F$ – TMD anomalous dimension
- $\mathcal{D}$ – rapidity anomalous dimension ($= -\frac{\tilde{K}}{2}$[Collins' book], $= K$[Bacchetta, at al,1703.10157])
- Anomalous dimensions are *universal*, i.e. depend only on flavor (gluon/quark).

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# TMD evolution is two-dimensional

$$\begin{pmatrix} \mu^2 \dfrac{d}{d\mu^2} \\ \zeta \dfrac{d}{d\zeta} \end{pmatrix} F = \begin{pmatrix} \dfrac{\gamma_F}{2} \\ -\mathcal{D} \end{pmatrix} F$$

⟷

$$\vec{\boldsymbol{\nabla}} F = \vec{\mathbf{E}} F$$

$\vec{\mathbf{E}}$ is 2D evolution field in $\vec{\boldsymbol{\nu}} = (\ln \mu^2, \ln \zeta)$ coordinates

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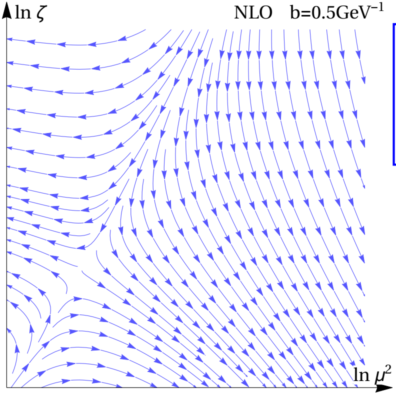


Solution

$$R[(\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left(\int_P d\vec{\boldsymbol{\nu}} \cdot \vec{\mathbf{E}}\right)$$

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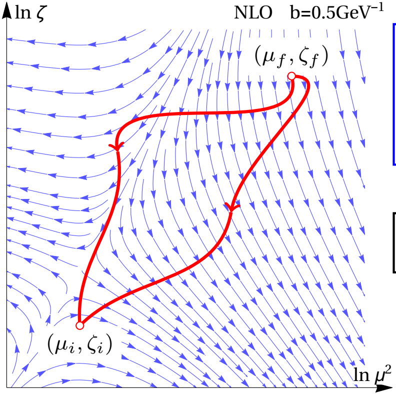
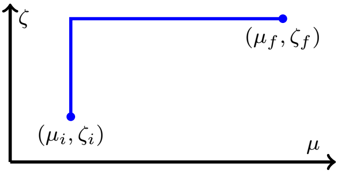



Solution

$$R[(\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left(\int_P d\vec{\boldsymbol{\nu}} \cdot \vec{\mathbf{E}}\right)$$

**The integration path is unimportant!**

# Examples



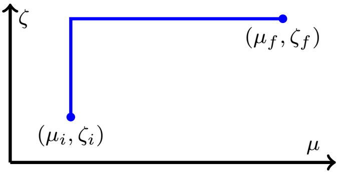

Solution 1

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_f) - \mathcal{D}(\mu_i, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

[Collins' textbook],[Aybat,Rogers,1101.5057],...

99% popular

## Examples

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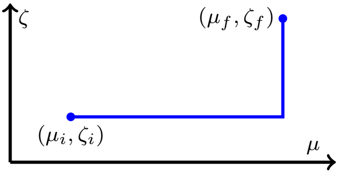
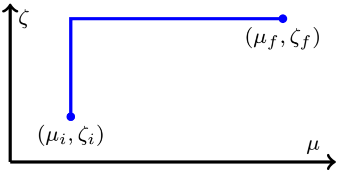



Solution 2

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu}\gamma_F(\mu,\zeta_i) - \mathcal{D}(\mu_f,b)\ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

# Examples

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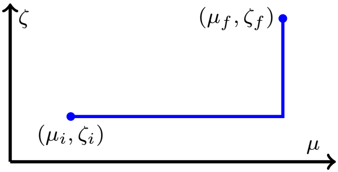


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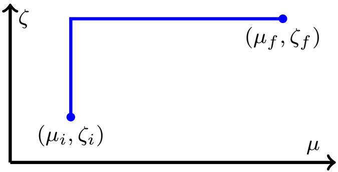


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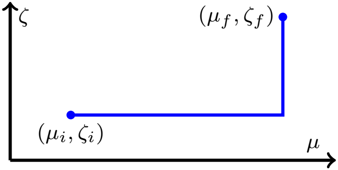


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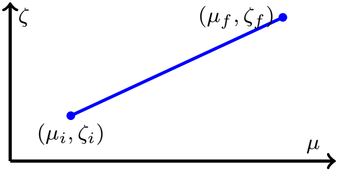


Solution 3

$$\ln R = \int_0^1 \Big(\gamma_F(\mu(t),\zeta(t))\frac{\mu_f-\mu_i}{(\mu_f-\mu_i)t+\mu_i} - \mathcal{D}(\mu(t),b)\frac{\zeta_f-\zeta_i}{(\zeta_f-\zeta_i)t+\zeta_i}\Big)dt$$

# Unique solution

Solution exist only if *integrability condition* holds

$$\zeta \frac{d\gamma_F}{d\zeta} = -\mu^2 \frac{d\mathcal{D}}{d\mu^2}$$

⟷

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0$$

$\vec{\mathbf{E}}$ is conservative field

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*Integrability condition* is trivially satisfied due to collinear overlap of divergences

$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) = -\Gamma(\mu), \qquad \mu^2 \frac{d}{d\mu^2}\mathcal{D}(\mu,b) = \Gamma(\mu)$$

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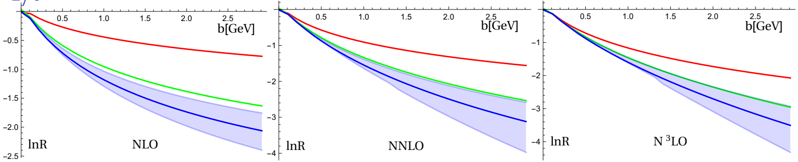
$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) = -\Gamma(\mu), \qquad \mu^2 \frac{d}{d\mu^2}\mathcal{D}(\mu,b) \neq \Gamma(\mu)$$

In fixed order PT integrability condition is violated.
The restoration procedure is ambigous
(large impact at large-$b$)
See extended dicussion in [Scimemi,AV;1803.11089]

$M_Z \to 1/b^*$



# Evolution potential

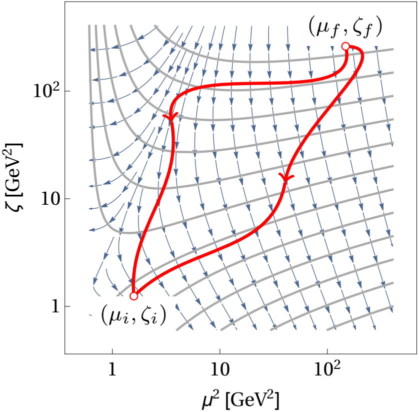
Solution exist only if *integrability condition* holds

$$\zeta \frac{d\gamma_F}{d\zeta} = -\mu^2 \frac{d\mathcal{D}}{d\mu^2}$$

$\longleftrightarrow$

$$\vec{\nabla} \times \vec{\mathbf{E}} = 0$$

$\vec{\mathbf{E}}$ is conservative field




Conservative field is determined by a potential

$$\vec{\mathbf{E}} = \vec{\nabla} U$$

Evolution is a difference between potentials

$$R[(\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left(U_f - U_i\right)$$

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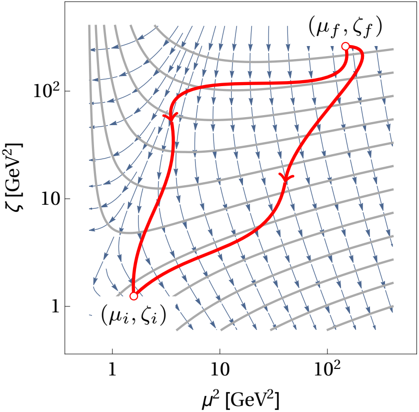


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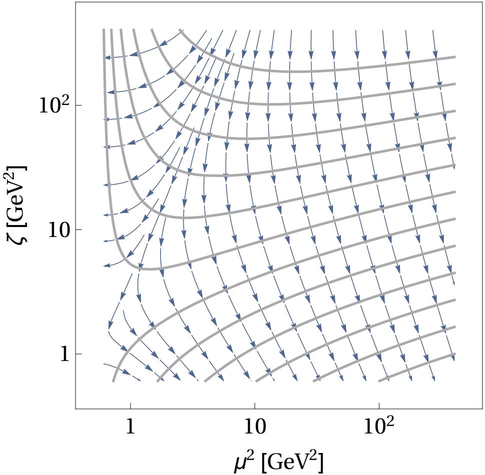
**This absolutely standard picture contains an important message.**

The TMD distribution is not defined by a scale $(\mu, \zeta)$
It is defined by an equipotential line.




The scaling is defined by
~~a difference between scales~~
a difference between potentials

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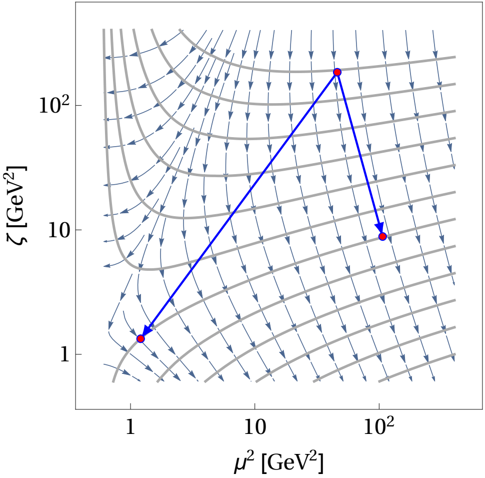
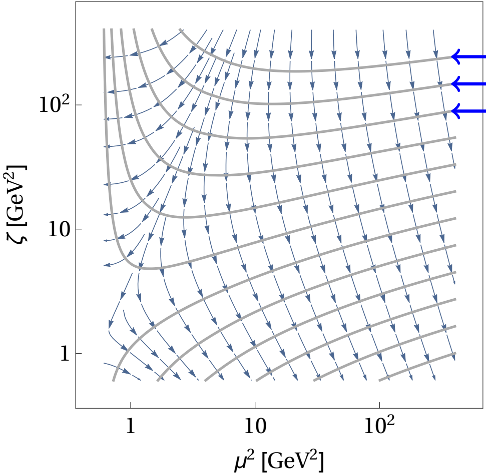



The scaling is defined by
~~a difference between scales~~
a difference between potentials

Evolution factor to both points
is the same
although the scales are
different by $10^2\text{GeV}^2$

**TMD distributions on the same equipotential line are equivalent.**




TMD$(x, b, 1)$
TMD$(x, b, 2)$
TMD$(x, b, 3)$

We can enumerate them by a lines not by $(\mu, \zeta)$

$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

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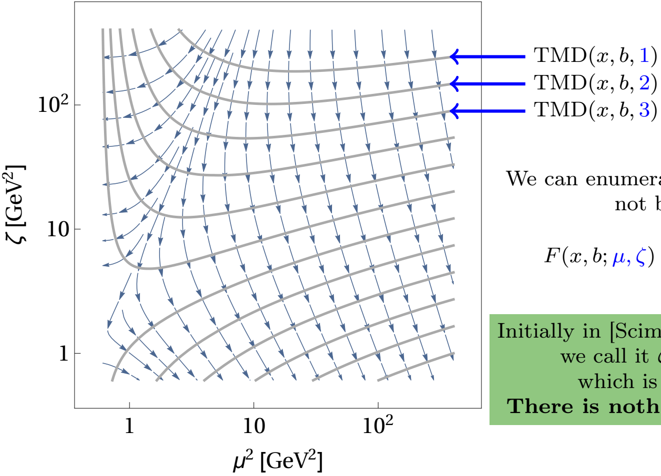


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$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

Initially in [Scimemi,AV,1706.01473] we call it $\zeta$-prescription which is misleading.
**There is nothing to prescribe.**

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## In $\zeta$-prescription we set $\zeta \to \zeta_\mu(\boldsymbol{\nu})$

- TMDs are "enumerated" by $\boldsymbol{\nu}$ (the number of line)
- TMDs are "naive" scale-independent

$$\mu \frac{d}{d\mu} F(x, b; \mu, \zeta_\mu) = 0 \qquad \Rightarrow \text{No double-logs in the matching.}$$

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TMD distribution depends only on the "number" of equipotential line

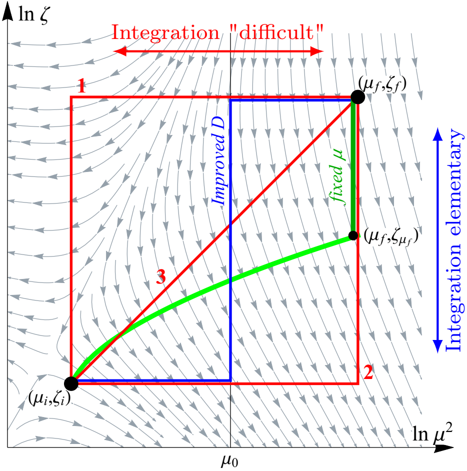
$$F(x, \mathbf{b}; \mu, \zeta) \to F(x, \mathbf{b}; \nu)$$

$$\frac{dF(x, \mathbf{b}; \nu)}{d\nu} = \frac{dU(\mathbf{b}; \nu)}{d\nu} F(x, \mathbf{b}; \nu)$$

$$\Updownarrow$$

$$F(x, \mathbf{b}; \nu) = e^{U(\mathbf{b};\nu) - U(\mathbf{b};\nu_0)} F(x, \mathbf{b}; \nu_0)$$

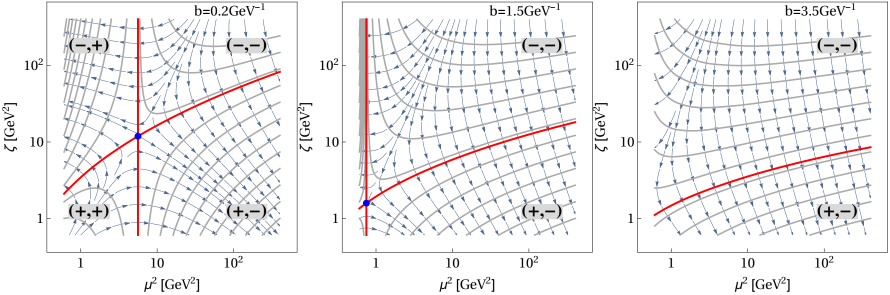
## Best way to measure the difference between potentials




$$R = \left(\frac{\zeta_f}{\zeta_{\mu_f}}\right)^{-\mathcal{D}(\mu_f, b)}$$

- Numerically simple (and fast)
- $\mu_f = Q$ thus $a_s$ is small
- Does just the same job as the Sudakov exponent

# Which line is the best?




- Some non-interesting singularities at $\mu, \zeta \to \infty$
- Landau pole at $\mu = \Lambda$
- Saddle point (blue dot)

$$\mathcal{D}(\mu_{\text{saddle}}, b) = 0, \qquad \gamma_M(\mu_{\text{saddle}}, \zeta_{\text{saddle}}, b) = 0$$

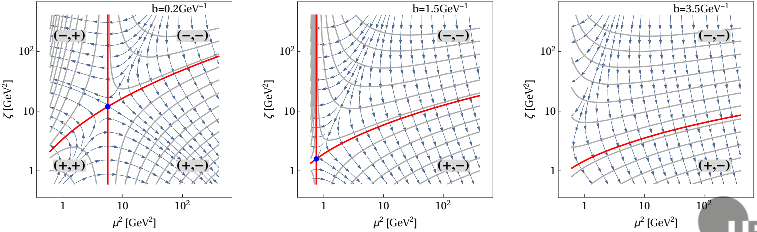
# Universal scale-independent TMD

There is a unique line which passes though all $\mu$'s

**The optimal TMD distribution**

$$F(x,b) = F(x,b;\mu,\zeta_\mu)$$

where $\zeta_\mu$ is the special line.



# TMD cross-section

$$\frac{d\sigma}{dX} = \sigma_0 \sum_f \int \frac{d^2 b}{4\pi} e^{i(b\cdot q_T)} H_{ff'}(Q) \{\tilde{R}^f[b;Q]\}^2 \tilde{F}_{f\leftarrow h}(x_1,b)\tilde{F}_{f'\leftarrow h}(x_2,b),$$

with $\zeta_f = \mu_f^2 = Q^2$

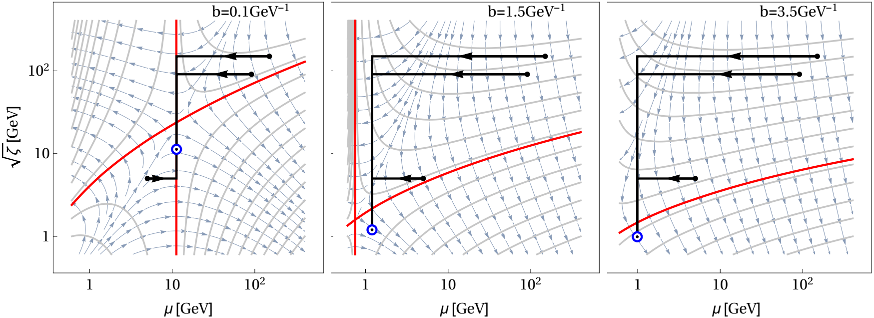
$$\tilde{R}^f[b;Q] \quad = \quad (Qb)^{-\mathcal{D}^f_{\text{NP}}(Q,b)} \exp\{-\mathcal{D}^f_{\text{NP}}(Q,b) v^f(Q,b)\}$$

- $v$ is given perturbative series, $v = \frac{3}{2} + a_s \ldots$
- $\tilde{F}$ is TMD in the "naive" $\zeta$-prescription

- There are no approximations (*ala* high energy expansion of integrals).
- There are only $(\mu_f, \zeta_f)$ scales and no solution dependence.
- **Clear separation of TMD evolution from the model for TMD distribution.**

# Evolution with $b$-dependent scale (CSS-like)

$$(Q, Q^2) \to (\mu_b, \mu_b^2)$$



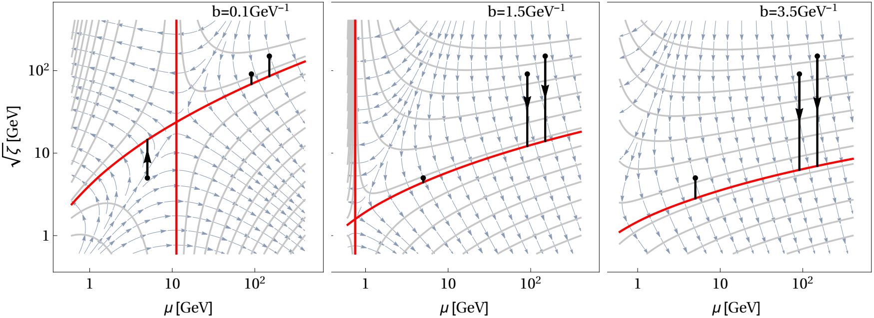

Here $\mu_b = \dfrac{C_0}{b^*}$ with $b_{\max} = 1.2\text{GeV}^{-1}$

Analogy in DIS

Scale depends on paremeter $\leftrightarrow$ $d\sigma = C(Q)R[Q \to \text{ch}(x)]f(x, \text{ch}(x))$

PDF $f(x, \text{ch}(x))$ has no interpretation, no sense, and **depends on the order of evolution in use.**

## Optimal version
$(Q, Q^2) \to (Q, \zeta_Q)$




Analogy in DIS

Scale (potential) is fixed $\leftrightarrow$ $d\sigma = C(Q)R[Q \to 2\text{GeV}]f(x, 2\text{GeV})$

PDF $f(x, 2\text{GeV})$ is just a model and

**is dependent on the order of evolution in use.**

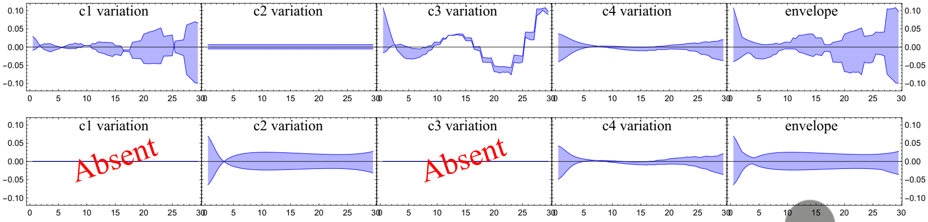
# Uncertainties of TMD cross-section

## Less number of scales = less theory error

- The variation of $c_1$ is the variation of the evolution path (only).
- The variation of $c_3$ is the variation of the evolution path (almost).

We found significant reduction of theory error band.

## Z-boson production at CDF run 2

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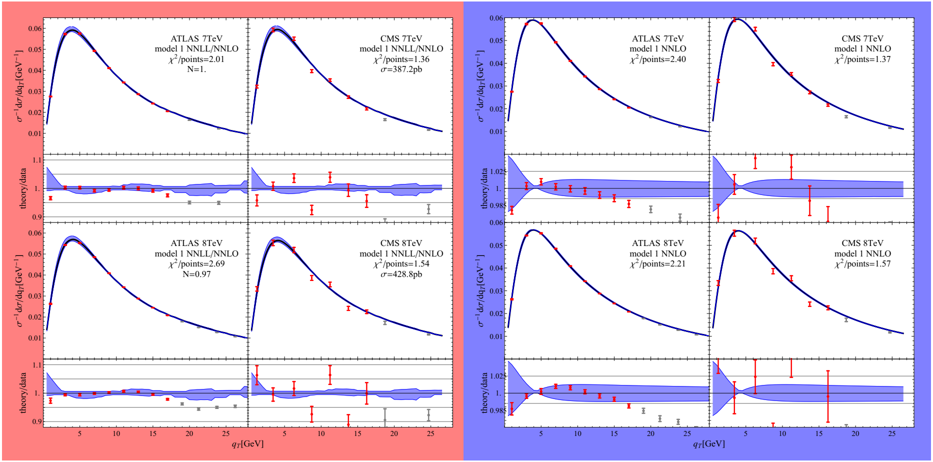
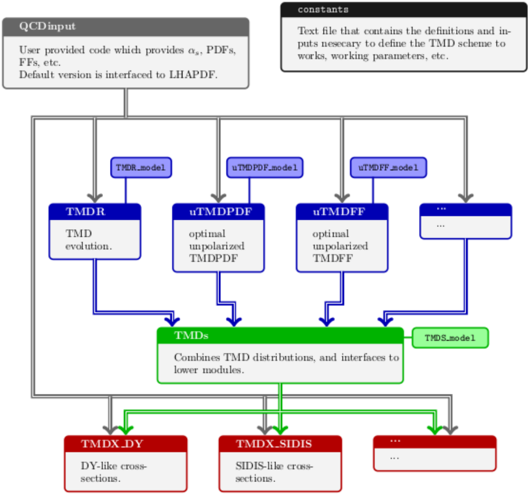
CSS-like evolution ($\zeta$-prescription) | Optimal evolution




Update of the NNLO DY fit,
$\chi^2$-values practically the same (a bit better), parameters within (previous) error-bars
significant reduction of theory uncertainties.

# arTeMiDe v1.3




- Variety of evolutions
- LO, NLO, NNLO
- No restriction for NP models
- Fast code
- DY cross-sections
- SIDIS cross-sections (not tuned yet)
- Theory uncertainty bands

https://teorica.fis.ucm.es/artemide/

# Conclusion

Main message:
TMD evolution is a double scale evolution.
Therefore, it should be considered with care, and then it grants many simplifications.

Message 1:
TMD distributions on a same equipotential line are equivalent. Enumerate them with lines!

- Universal for all quantum numbers
- Very simple practical formula (no integrations!)
- Guarantied absence of (large) logarithms in the matching coefficient
- TMD model is independent on evolution order.
  **E.g You can use NNLO unpolarized and LO Sivers together, without theory tensions**

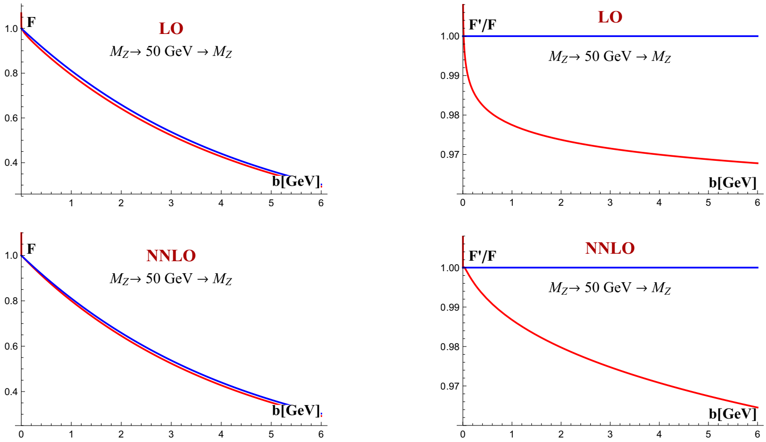
Message 2 (unpresented):
In truncated PT there is the solution-dependence of evolution (see [1803.11089])

- It could be strong.
- There is no unique way to fix it.

Double-scale evolution is not unique for TMD case. It also appears in $k_T$-resummation, joint resummation, DPDs, etc.

## Problem 1: Violation of transitivity

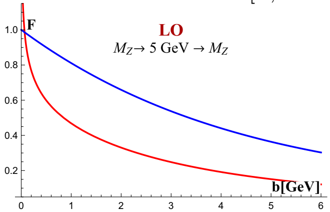
$$R[\mathbf{b}; X \to Y]\ R[\mathbf{b}; Y \to X] = 1$$

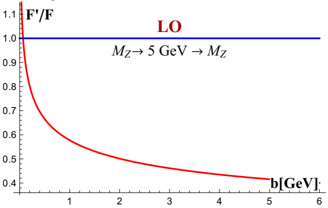


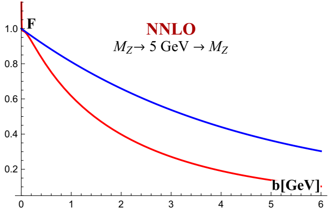

There is a violation of transitivity $\sim 2$ % which seems better at NNLO

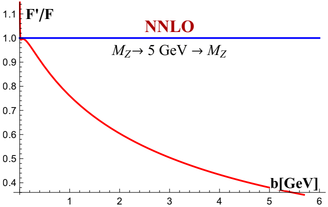
## Problem 1: Violation of transitivity

$$R[\mathbf{b}; X \to Y]\ R[\mathbf{b}; Y \to X] = 1$$



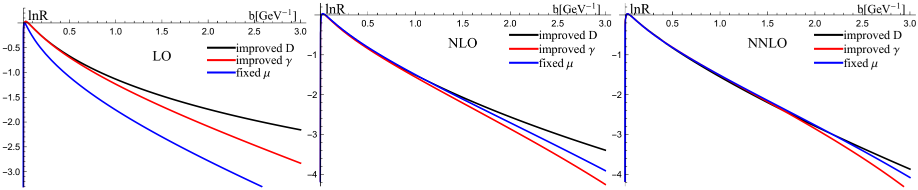




There is a VERY strong violation of transitivity, which seems worse at NNLO

The difference between solutions is $\underbrace{\sim a^{N+1}\mathbf{L}_\mu}_{\text{main b}}$ or $\underbrace{\sim a^{N+1}\mathbf{L}_\mu\mathbf{L}_{\mu_0}^N}_{\text{large b}}$



# How strong is modification of the field?