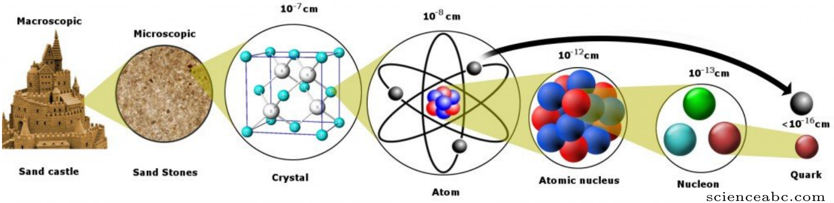
Nucleon Tomography

Alexey Vladimirov
(Universität Regensburg)



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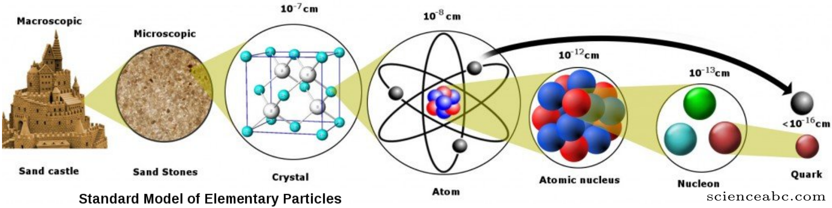
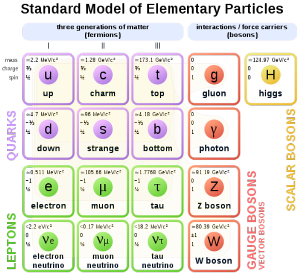
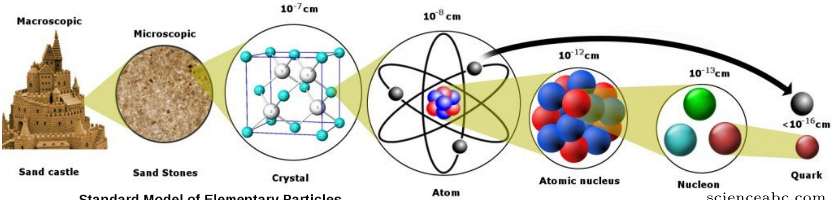




We know
how the world is built
at $\sim 10^{-16}$cm



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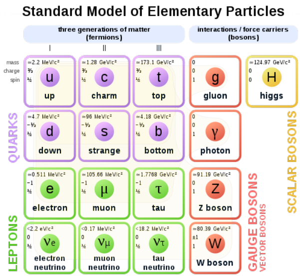
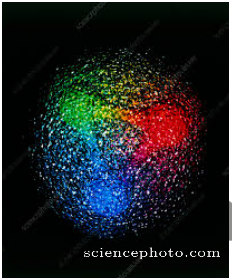


We know
how the world is built
at $\sim 10^{-16}$cm

better
than
at $\sim 10^{-14}$cm

? ? ??? ? ?



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Success of parton model

- The higher energy $\Rightarrow$ smaller distances

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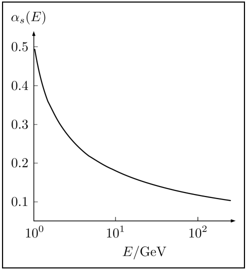
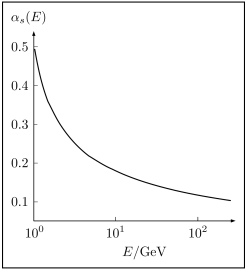
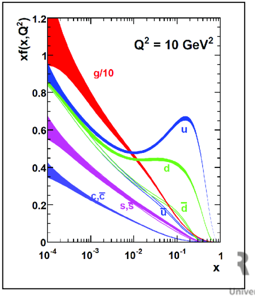
- The higher energy $\Rightarrow$ asymptotic freedom $\Rightarrow$ free fields



Success of parton model

- The higher energy ⇒ smaller distances
- The higher energy ⇒ asymptotic freedom ⇒ free fields
- High energetic hadron ⇒ all motions are collinear



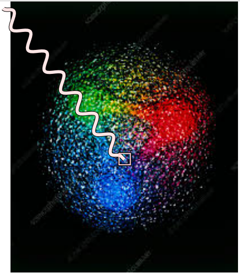


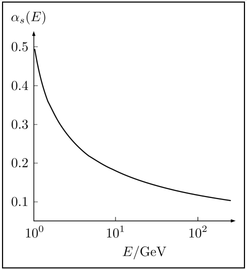
Success of parton model

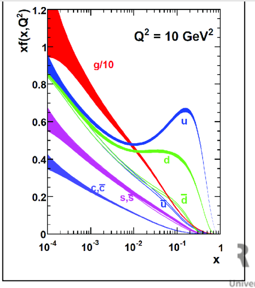
- The higher energy ⇒ smaller distances
- The higher energy ⇒ asymptotic freedom ⇒ free fields
- High energetic hadron ⇒ all motions are collinear

(Almost)* Parton model
Weakly interacting fields collinearly move in a single direction with some distribution of momentum (parton distribution function)

* In fact, it is not a parton model but the result of **collinear factorization theorem**. However, in a way it is used at high energy, they are practically indistinguishable.







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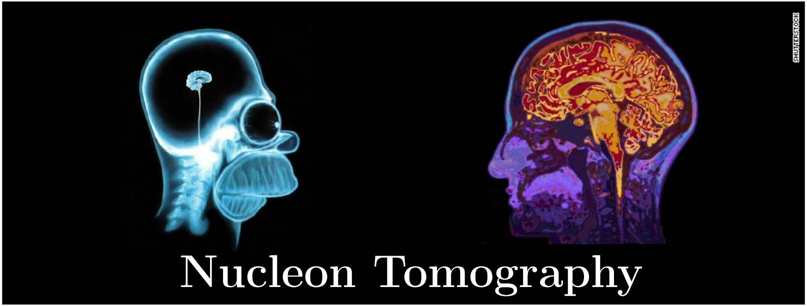


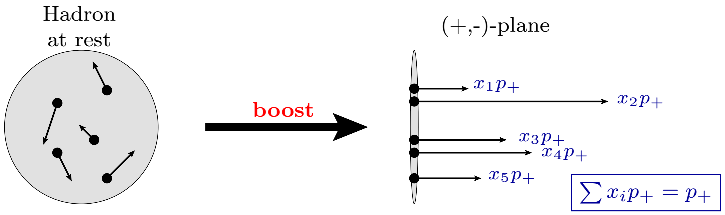
Nucleon tomography: direction(s) of study which aim is to restore the "tomographic" picture of hadron.

Outline

In this talk, I will **shortly** present two main topics

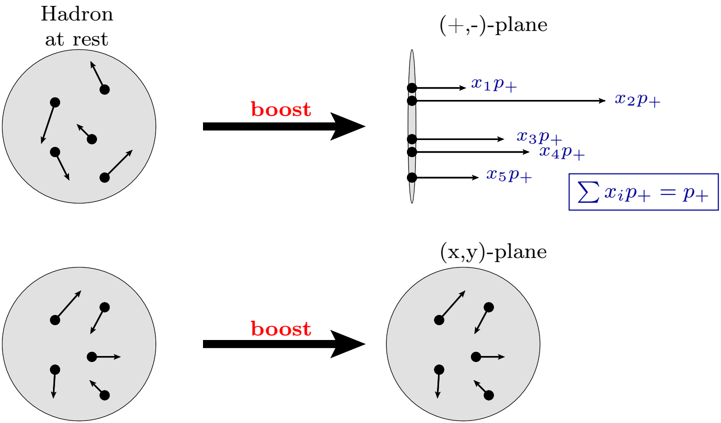
- Generalized Parton distributions (GPD)
- Transverse Momentum distributions (TMD)

Transverse degrees of freedom



- Q is generic hard scale
- Λ is generic soft scale ($\sim \Lambda_{\text{QCD}} \simeq 0.25\text{GeV} \sim (1.3\text{fm})^{-1}$)
- In the factorization frame $p_+ \sim Q$
- p_- components are small $p_- \sim m^2/p^+ \sim \Lambda^2/Q$

Transverse degrees of freedom

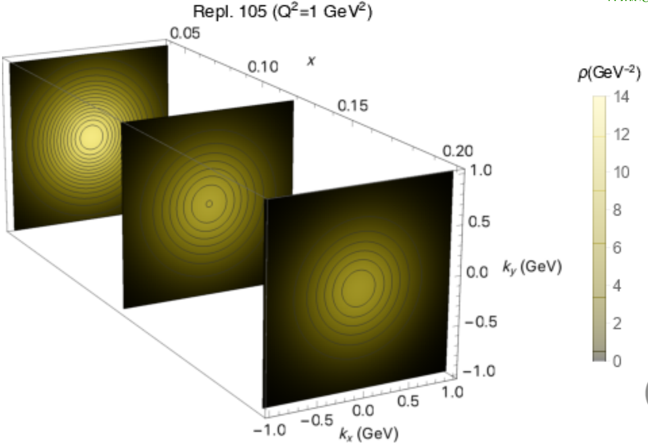


- All transverse momenta are of hardronic scale $\sim \Lambda$
- Hadron size is of the same scale $\sim \Lambda^{-1}$

Tomographic picture of a hadron

Reconstruction of transverse position/motion of a parton
at a given value of collinear momentum.

[Bacchetta,et al,2017]



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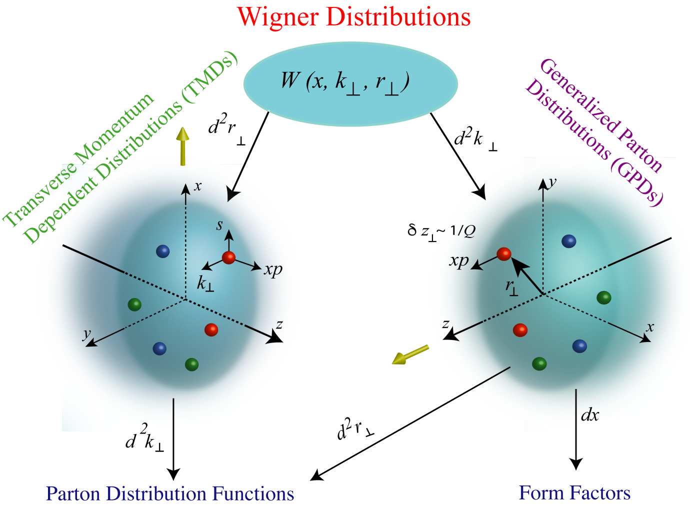


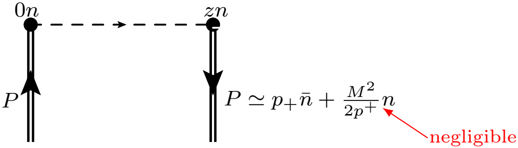
Wigner distribution

5D image of hadron – ultimate goal of nucleon tomography

Operators & matrix elements

Operator for Parton distribution function

$$\langle p|\bar{q}(zn)[zn,0]\gamma^+ q(0)|p\rangle = p^+ \int dx e^{ixzp^+} f_1(x)$$



Wilson line

$$[a,b] = P\exp\left(ig\int_b^a dx^\mu A_\mu(x)\right)$$

- Gauge link : $[a,b] \xrightarrow{\text{gauge trans.}} U(a)[a,b]U^\dagger(b)$
- Transitivity: $[a,b][b,c] = [a,c]$

Operators & matrix elements

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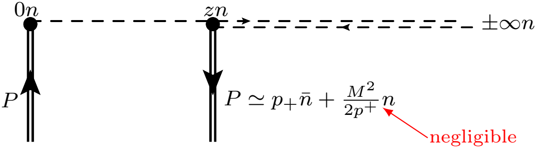

$$\langle p|\bar{T}\{\bar{q}(zn)[zn,-\infty n]\}\gamma^+ T\{[-\infty n,0]q(0)\}|p\rangle = p^+ \int dx e^{ixzp^+} f_1(x)$$

see e.g.[Jaffe, Nucl.Phys.B229]

Wilson line

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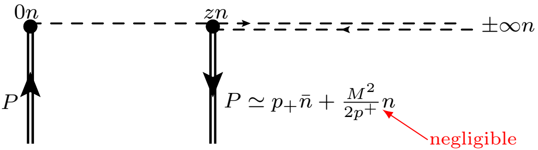


$$\langle p|\bar{T}\{\bar{q}(zn)[zn,-\infty n]\}\gamma^+ T\{[-\infty n,0]q(0)\}|p\rangle = p^+\int dx e^{ixzp^+} f_1(x)$$

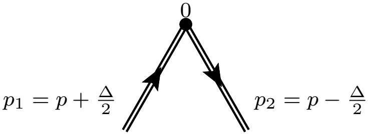
see e.g.[Jaffe, Nucl.Phys.B229]

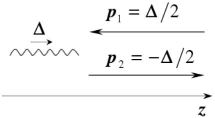
It can be shown (in the light-cone quantization) that

$$f_1(x)|_{x>0} = \frac{1}{2x}\sum_{\lambda=\uparrow\downarrow}\int \frac{d^2k_T}{(2\pi)^3}\frac{\langle p|b^\dagger_\lambda(xp^+,\mathbf{k}_T)b_\lambda(xp^+,\mathbf{k}_T)|p\rangle}{\langle p|p\rangle}$$

Form factor

$$\langle p-\frac{\Delta}{2}|\bar q(0)\gamma^\mu q(0)|p+\frac{\Delta}{2}\rangle = \bar u(p_2)\{\gamma^\mu F_1(\Delta^2)+\frac{i\sigma^{\Delta\mu}}{2M}F_2(\Delta^2)\}u(p_1)$$





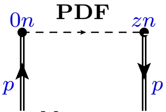
Interpretation

- In interpretation exists only in the Breit frame

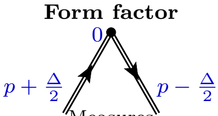
$$F(\mathbf{r}) = \int d^2\Delta e^{i(\mathbf{r}\boldsymbol{\Delta})}F(\boldsymbol{\Delta})$$

- Nucleon is "2D" pancake (like in light-front !)

Generalized parton distributions

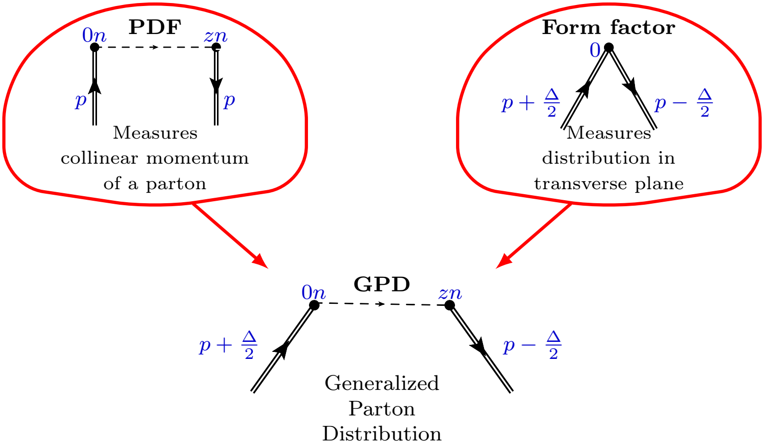


Measures collinear momentum of a parton

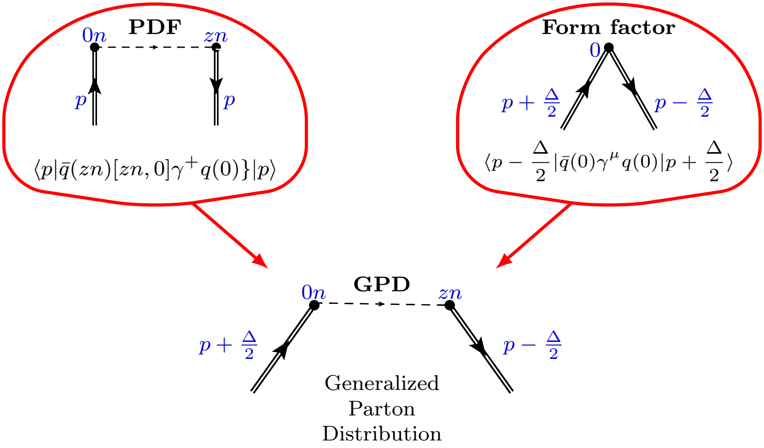


Measures distribution in transverse plane

Generalized parton distributions



Generalized parton distributions

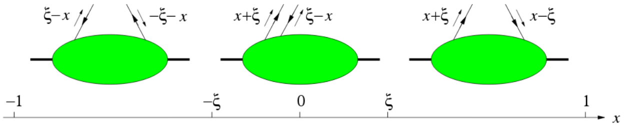


$$\langle p-\frac{\Delta}{2}|\bar{q}\left(\frac{zn}{2}\right)\left[\frac{zn}{2},\frac{-zn}{2}\right]\gamma^+ q\left(\frac{-zn}{2}\right)|p+\frac{\Delta}{2}\rangle$$

GPD: main properties

$$\langle p-\frac{\Delta}{2}|\bar q\left(\frac{zn}{2}\right)\left[\frac{zn}{2},\frac{-zn}{2}\right]\gamma^+ q\left(\frac{-zn}{2}\right)|p+\frac{\Delta}{2}\rangle$$
$$=p^+\int xe^{ixzp^+}\left[H^q(x,\xi,\Delta^2)\bar u(p_2)\gamma^+u(p)+E^q(x,\xi,\Delta^2)\bar u(p_2)\frac{i\sigma^{+\Delta}}{2M}u(p)\right]$$

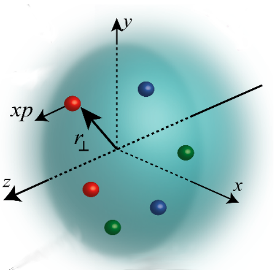
- $\xi=2\Delta^+/p^+$ "skewedness" parameter.
- $|x|>|\xi|$ similar to PDF
- $|x|<|\xi|$ similar to Distribution amplitude

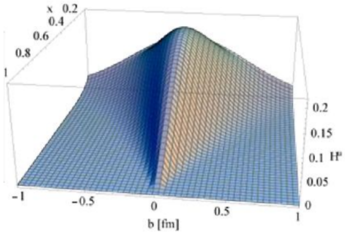


The impact parameter distribution appears at
$\xi \to 0$

$$H(x,0,\mathbf{b}) = \int d^2\mathbf{\Delta} e^{i(\mathbf{b}\mathbf{\Delta})} H(x,0,\mathbf{\Delta})$$

The knowledge on GPDs is rudimentary (only models)
Reason: Very complicated phenomenology





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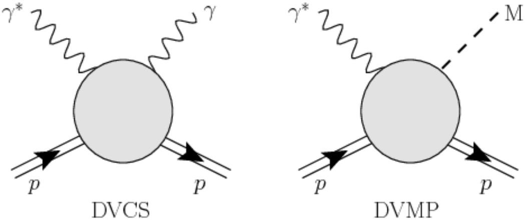
How to measure GPD?

To measure off-forward hadron state
one must consider
exclusive reaction at the level of amplitude

Main reactions for GPDs are Deeply-Virtual Compton scattering (DVCS) or Deeply-Virtual meson production (DVMP)

$$\gamma^* p \to p\gamma, \qquad \gamma^* p \to p\pi$$



- Measurements from JLab, COMPASS, HERA, HERMES

Main complication

$$\text{cross-section} = |\text{amplitude}|^2$$

- Amplitude is a complex number
- Amplitude is a composite of several Compton Form-Factors (CFF)

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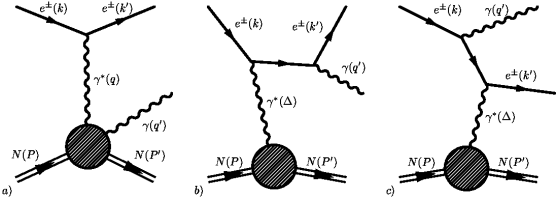
Structure of DVCS

- Accompanying process (Bether-Heitler scattering)



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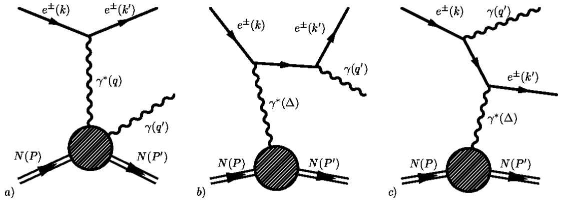
Extraction of GPDs

$$\text{cross-section} = |A_{\rm DVCS} + A_{\rm BH}|^2 \sim {\rm Re}A^2_{\rm DVCS} + 2{\rm Re}A_{\rm DVCS}{\rm Re}A_{\rm BH} + ...$$

- $A_{\rm BH} \sim A_{\rm Comp.}F(\Delta)$ is "known"
- $A_{\rm DVCS}$ is wanted
- Plenty of terms with different angular modulations

$$\frac{d\sigma_{\rm VCS}}{d\phi\, dt\, dQ^2\, dx_B} = \frac{1}{Q^2}\sum_{\lambda'\lambda}\Big[\ldots|M_{\lambda'+,\lambda+}|^2 + \ldots\cos\phi\,{\rm Re}\,(M^*_{\lambda'+,\lambda+}\,M_{\lambda'+,\lambda 0})$$
$$+\ldots P_\ell\,\sin\phi\,{\rm Im}\,(M^*_{\lambda'+,\lambda+}\,M_{\lambda'+,\lambda 0}) + O(Q^{-2}) + O(\alpha_s)\Big]$$



Extraction of GPDs

Measurement: Measure unpolarized and polarized DVCS in 4D binning (dependence on Q, Δ, x and φ)

$$\frac{d\sigma}{d\phi dt dQ^2 dx}$$

First step: Measuring all(4) angular modulations of unpolarized and transversely polarized DVCS one can resolve the system and extract a minimal set (8) of Compton form factors (CFF).

$$\mathrm{Re}\mathcal{H}, \mathrm{Re}\mathcal{E}, \mathrm{Re}\tilde{\mathcal{H}}, \mathrm{Re}\tilde{\mathcal{E}}$$
$$\mathrm{Im}\mathcal{H}, \mathrm{Im}\mathcal{E}, \mathrm{Im}\tilde{\mathcal{H}}, \mathrm{Im}\tilde{\mathcal{E}}$$

Second step: each CFF could be presented in terms of GPDs (factorization theorem)

$$H, E, \tilde{H}, \tilde{E}$$

$$H_T, E_T, \tilde{H}_T, \tilde{E}_T$$

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$$H, E, \tilde{H}, \tilde{E}$$

$$H_T, E_T, \tilde{H}_T, \tilde{E}_T$$

With the recent data from COMPASS, CLAS and Hall A (2015-..) this program became realistic

[H.Moutarde,P.Sznajder,J.Wagner,1905.02089]

- Different **kinds** of observables (2624 data point)
- Trained neural network for Compton Form Factor (PARTONS)

No.	Collab.	Year	Ref.	Observable		Kinematic dependence	No. of points used / all
1	HERMES	2001	[39]	A^{+}_{LU}		ϕ	10 / 10
2		2006	[40]	$A^{\cos i\phi}_{C}$	$i=1$	t	4 / 4
3		2008	[41]	$A^{\cos i\phi}_{C}$	$i=0,1$	x_{Bj}	18 / 24
				$A^{\sin(\phi-\phi_S)\cos i\phi}_{UT,\mathrm{DVCS}}$	$i=0$		
				$A^{\sin(\phi-\phi_S)\cos i\phi}_{UT,\mathrm{I}}$	$i=0,1$		
				$A^{\cos(\phi-\phi_S)\sin i\phi}_{UT,\mathrm{I}}$	$i=1$		
4		2009	[42]	$A^{\sin i\phi}_{LU,\mathrm{I}}$	$i=1,2$	x_{Bj}	35 / 42
				$A^{\sin i\phi}_{LU,\mathrm{DVCS}}$	$i=1$		
				$A^{\cos i\phi}_{C}$	$i=0,1,2,3$		
5		2010	[43]	$A^{+,\sin i\phi}_{UL}$	$i=1,2,3$	x_{Bj}	18 / 24
				$A^{+,\cos i\phi}_{LL}$	$i=0,1,2$		
6		2011	[44]	$A^{\cos(\phi-\phi_S)\cos i\phi}_{LT,\mathrm{DVCS}}$	$i=0,1$	x_{Bj}	24 / 32
				$A^{\sin(\phi-\phi_S)\sin i\phi}_{LT,\mathrm{DVCS}}$	$i=1$		
				$A^{\cos(\phi-\phi_S)\cos i\phi}_{LT,\mathrm{I}}$	$i=0,1,2$		
				$A^{\sin(\phi-\phi_S)\sin i\phi}_{LT,\mathrm{I}}$	$i=1,2$		
7		2012	[45]	$A^{\sin i\phi}_{LU,\mathrm{I}}$	$i=1,2$	x_{Bj}	35 / 42
				$A^{\sin i\phi}_{LU,\mathrm{DVCS}}$	$i=1$		
				$A^{\cos i\phi}_{C}$	$i=0,1,2,3$		
8	CLAS	2001	[46]	$A^{-,\sin i\phi}_{LU}$	$i=1,2$	—	0 / 2
9		2006	[47]	$A^{-,\sin i\phi}_{UL}$	$i=1,2$	—	2 / 2
10		2008	[48]	A^{-}_{LU}		ϕ	283 / 737
11		2009	[49]	A^{-}_{LU}		ϕ	22 / 33
12		2015	[50]	A^{-}_{LU}, A^{-}_{UL}, A^{-}_{LL}		ϕ	311 / 497
13		2015	[51]	$d^4\sigma^{-}_{UU}$		ϕ	1333 / 1933
14	Hall A	2015	[33]	$\Delta d^4\sigma^{-}_{LU}$		ϕ	228 / 228
15		2017	[34]	$\Delta d^4\sigma^{-}_{LU}$		ϕ	276 / 358
16	COMPASS	2018	[35]	$d^3\sigma^{\pm}_{UU}$		t	2 / 4
17	ZEUS	2009	[36]	$d^3\sigma^{+}_{UU}$		t	4 / 4
18	H1	2005	[37]	$d^3\sigma^{+}_{UU}$		t	7 / 8
19		2009	[38]	$d^3\sigma^{\pm}_{UU}$		t	12 / 12
						SUM:	2624 / 3996

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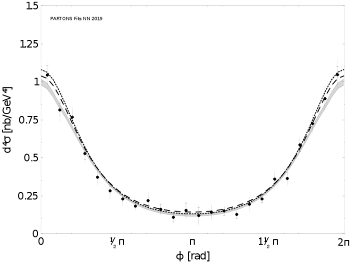
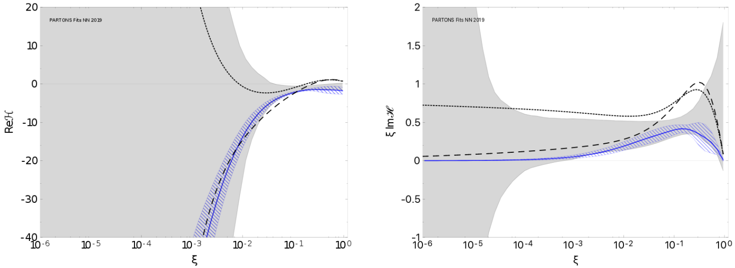
[H.Moutarde,P.Sznajder,J.Wagner,1905.02089]

- Different **kinds** of observables (2624 data point)
- Trained neural network for Compton Form Factor (PARTONS)



[H.Moutarde,P.Sznajder,J.Wagner,1905.02089]

- Different **kinds** of observables (2624 data point)
- Trained neural network for Compton Form Factor (PARTONS)
- So far, no analysis of GPDs



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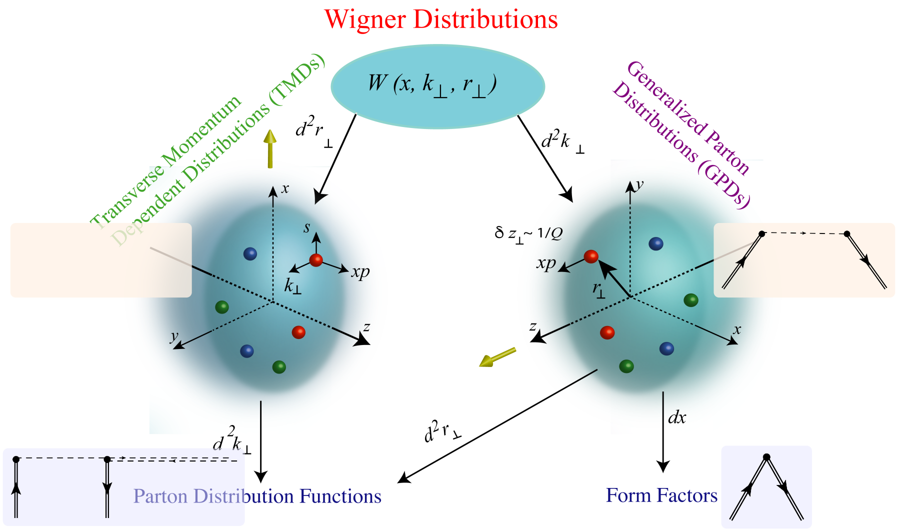
GPD state

- Beautiful and well-developed theory
- Recently plenty of data appear (even more to appear from JLab12)
- Very difficult phenomenology (not too many groups)
- First accurate extractions for FCC
- We close to accurate "model-independent" extraction of GPDs

Further reading

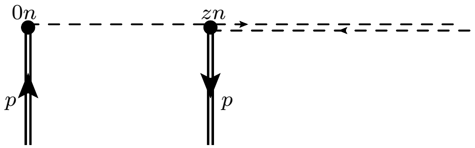
- M.Diehl, *"Generalized Parton Distributions"* [arXiv:0307382]
- A.V. Belitsky,& A.V. Radyushkin*"Unraveling hadron structure with generalized parton distributions"* [arXiv:0504030]
- M.Guidal, H.Moutarde, M.Vanderhaeghen *"Generalized Parton Distributions in the valence region from Deeply Virtual Compton Scattering"* [arXiv:1303.6600]
- M.Polyakov, P.Schweitzer *"Forces inside hadrons: pressure, surface tension, mechanical radius, and all that"* [arXiv:1805.06596]


Wigner Distributions
W (x, k⊥, r⊥)
d²r⊥
d²k⊥
Transverse Momentum Dependent Distributions (TMDs)
Generalized Parton Distributions (GPDs)
x
s
xp
k⊥
y
z
δ z⊥~ 1/Q
xp
r⊥
z
x
y
d²k⊥
d²r⊥
dx
Parton Distribution Functions
Form Factors

TMD operator

Operator for transverse momentum dependent distribution

$$\langle p|\bar{T}\{\bar{q}(zn)[zn,\pm\infty n]\}\gamma^+ T\{[\pm\infty n,0]q(0)\}|p\rangle = p^+\int dx e^{ixzp^+} f_1(x)$$



TMD operator

Operator for transverse momentum dependent distribution

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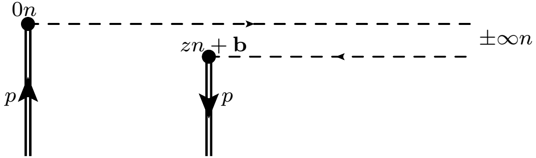

$$\langle p|\bar{T}\{\bar{q}(zn+\mathbf{b})[zn+\mathbf{b},\pm\infty n+\mathbf{b}]\}\gamma^+ T\{[\pm\infty n,0]q(0)\}|p\rangle = p^+\int dx e^{ixzp^+} F_1(x,\mathbf{b})$$

TMD operator

Operator for transverse momentum dependent distribution

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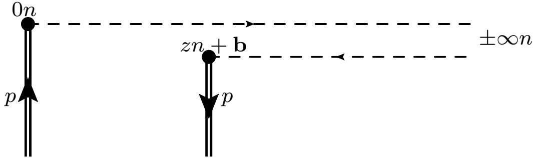

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Fourier image of function $F(x,\mathbf{b})$ has a meaning of distribution of transverse momentum (at collinear momentum x) [P.Mulders,J.Collins,...,93-95]

$$F_1(x,\mathbf{k}_T) = \int d^2\mathbf{b}\; e^{i\mathbf{b}\mathbf{k}_T}\;\; F_1(x,\mathbf{b})$$

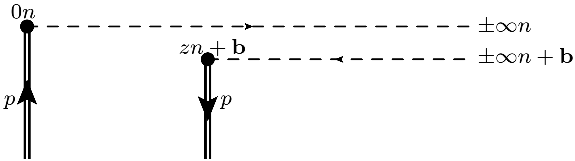
It is only the beginning of story ...

Break down of gauge invariance

- "Naive" TMD operator is not gauge invariant

$$\bar{q}(zn+\mathbf{b})[zn+\mathbf{b},\pm\infty n+\mathbf{b}]\gamma^+[\pm\infty n,0]q(0)\xrightarrow{\text{gauge transf.}}$$

$$\bar{q}(zn+\mathbf{b})[zn+\mathbf{b},\pm\infty n+\mathbf{b}]U^\dagger(\pm\infty n+\mathbf{b})\gamma^+U(\pm\infty)[\pm\infty n,0]q(0)$$



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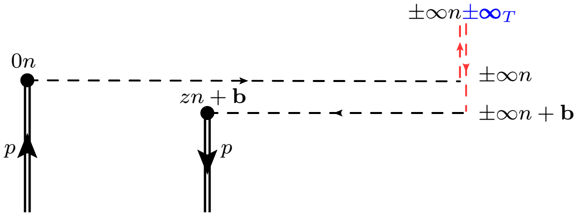
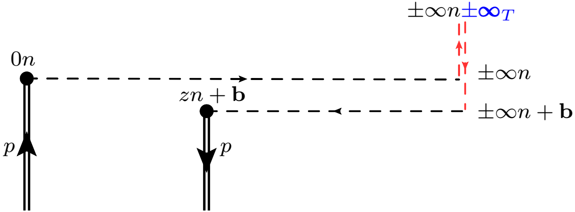


$$\langle p|\bar{T}\{\bar{q}(zn+\mathbf{b})[zn+\mathbf{b},\pm\infty n+\mathbf{b}][\pm\infty n+\mathbf{b},\pm\infty n\pm\boldsymbol{\infty}_T]\}\gamma^+T\{[[\pm\infty n\pm\boldsymbol{\infty}_T,\pm\infty n][\pm\infty n,0]q(0)\}|p\rangle=$$
$$p^+\int dx e^{ixzp^+}F_1(x,\mathbf{b})$$

[A.Belitsky,X.Ji,F.Yuan,0208038]

Break down of gauge invariance

- $$\bar{q}(zn+\mathbf{b})[zn+\mathbf{b},\pm\infty n+\mathbf{b}]\gamma^+[\pm\infty n,0]q(0)\xrightarrow{\text{gauge transf.}}\bar{q}(zn+\mathbf{b})[zn+\mathbf{b},\pm\infty n+\mathbf{b}][\pm\infty n,0]q(0)$$



like a staple

$$\langle p|\bar{T}\{\bar{q}(zn+\mathbf{b})[zn+\mathbf{b},\pm\infty n+\mathbf{b}][\pm\infty n+\mathbf{b},\pm\infty n\pm\boldsymbol{\infty}_T]\}\gamma^+T\{[[\pm\infty n\pm\boldsymbol{\infty}_T,\pm\infty n][\pm\infty n,0]q(0)\}|p\rangle=$$

$$\langle p|\bar{q}(zn+\mathbf{b})[\text{staple contour}]q(0)|p\rangle=p^+\int dx e^{ixzp^+}F_1(x,\mathbf{b})$$

[A.Belitsky,X.Ji,F.Yuan,0208038]

Transverse momentum couples to spin

$$\langle p, s|\bar{q}(zn+\mathbf{b})[\text{staple contour}]q(0)|p, s\rangle = p^+ \int dx e^{ixzp^+}\Big\{F_1(x,\mathbf{b}) + i\epsilon^{+-\mu\nu} b_\mu s_\nu M F_{1T}^{\perp}(x,\mathbf{b})\Big\}$$

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Transverse momentum couples to spin

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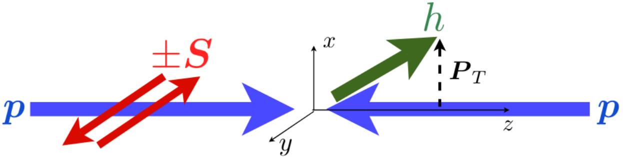
- $F_{1T}^{\perp}$ = Sivers function [D.Sivers, PRD 41 (1990)]
- Unpolarized quark in polarized hadron



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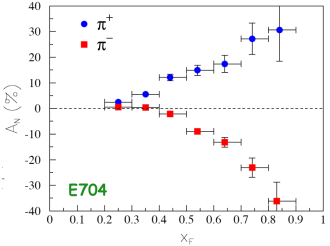
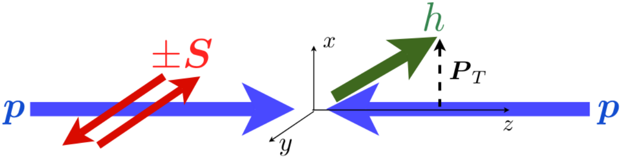
- $F_{1T}^{\perp}$ = Sivers function [D.Sivers, PRD 41 (1990)]
- Unpolarized quark in polarized hadron
- Sivers effect $p^\uparrow + p \to \pi X$

$$A_N = \frac{d\sigma^\uparrow(\mathbf{k}_T)-d\sigma^\downarrow(\mathbf{k}_T)}{d\sigma^\uparrow(\mathbf{k}_T)+d\sigma^\downarrow(\mathbf{k}_T)}$$



[E704,(1991)]



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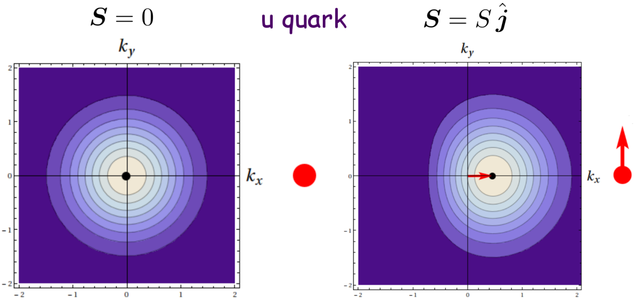
Partons distribution is asymmetric (even in unpolarized case)



[picture by A.Prokudin]

Zoo of TMD's

Transverse momentum distributions of leading order

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

Zoo of TMD's

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○ Have analog in "naive" parton model

Zoo of TMD's

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L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

- ○ Have analog in "naive" parton model
- ○○ Have not analog
- ○ Change orientation of spin
- ○ Produce spin

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Zoo of TMD's

Transverse momentum distributions of leading order

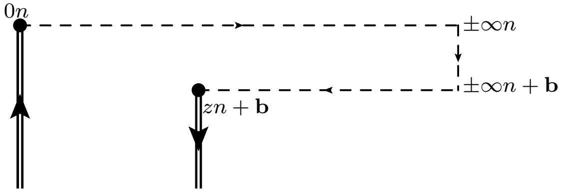
N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

- ○ Have analog in "naive" parton model
- ○○ Have not analog
- ○ Change orientation of spin
- ○ Produce spin

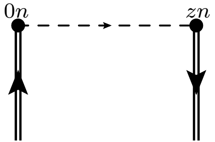
- 8 gluon TMDs
- 2 (or 8) TMD fragmentation function
- **non-perturbative evolution kernel**

"New" type of divergences → rapidity divergences

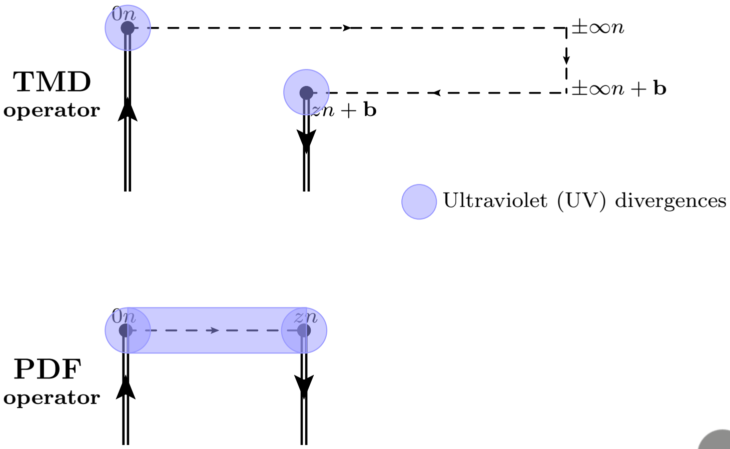
TMD
operator



PDF
operator



"New" type of divergences → rapidity divergences



“New” type of divergences → rapidity divergences

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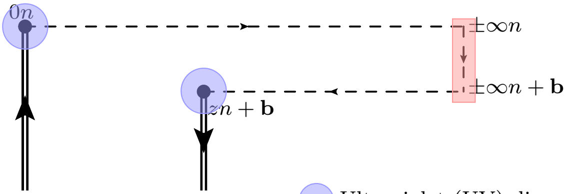
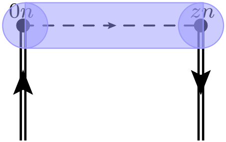


Ultraviolet (UV) divergences

Rapidity divergences

PDF operator



Rapidity divergences
are
renormalizable (in perturbation theory)
[AV,JHEP 1804 (2018)]

Renormalization of TMD operator

$$\Big[\bar q(zn+\mathbf{b})[\text{staple contour}]q(0)\Big](\mu,\zeta)=Z_{\text{UV}}^{\text{TMD}}(\mu)R(\mathbf{b};\mu,\zeta)\Big[\bar q(zn+\mathbf{b})[\text{staple contour}]q(0)\Big]^{\text{bare}}$$

- $Z_{\text{UV}}^{\text{TMD}}(\mu)$ UV renormalization constant
- $R(\mathbf{b};\mu,\zeta)$ rapidity divergence renormalization constant

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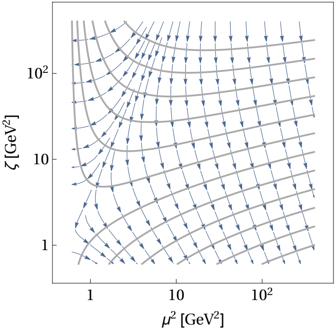

TMD evolution
is 2D evolution

$$\mu^2 \frac{dF(x,b;\mu,\zeta)}{d\mu^2} = \gamma_F(\mu,\zeta)F(x,b;\mu,\zeta)$$

$$\zeta \frac{dF(x,b;\mu,\zeta)}{d\zeta} = -\mathcal{D}(\mu,b)F(x,b;\mu,\zeta)$$

- Anomalous dimensions γ_F and $\mathcal{D}$ are known up to three-loops.
- Anomalous dimension are universal for all TMD distributions

Renormalization of TMD operator

$$\Big[\bar{q}(zn+\mathbf{b})[\text{staple contour}]q(0)\Big](\mu,\zeta) = Z_{\text{UV}}^{\text{TMD}}(\mu)R(\mathbf{b};\mu,\zeta)\Big[\bar{q}(zn+\mathbf{b})[\text{staple contour}]q(0)\Big]^{\text{bare}}$$

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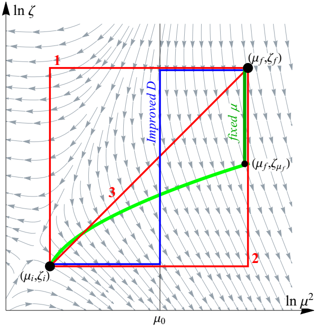


TMD evolution
is 2D evolution

$$F(x,\mathbf{b};\mu_f,\zeta_f) = \exp\left[\int_P \left(\gamma_F(\mu,\zeta)\frac{d\mu}{\mu} - \mathcal{D}(\mu,\mathbf{b})\frac{d\zeta}{\zeta}\right)\right]F(x,\mathbf{b};\mu_i,\zeta_i)$$

[I.Scimemi,AV,JHEP,1808(2018)]

- Path independence
- Unified picture of various evolution scenarios

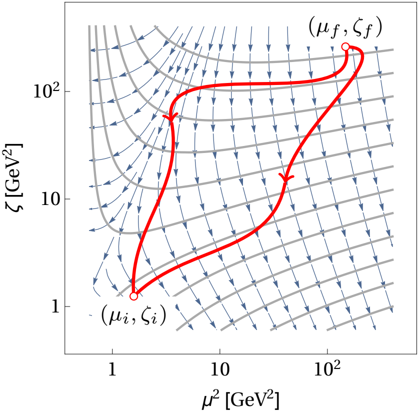
Evolution potential

Solution exist only if
integrability condition holds

$$\zeta \frac{d\gamma_F}{d\zeta} = -\mu^2 \frac{d\mathcal{D}}{d\mu^2}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0$$

$\vec{\mathbf{E}}$ is conservative field



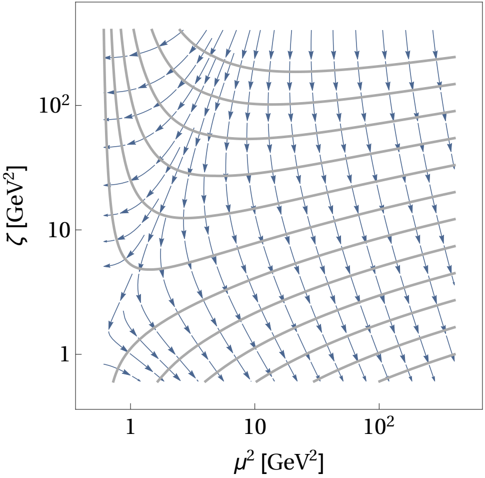
Conservative field is determined
by a potential

$$\vec{\mathbf{E}} = \vec{\boldsymbol{\nabla}} U$$

Evolution is a difference
between potentials

$$R[(\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left(U_f - U_i\right)$$

TMD distribution is not defined by a scale (μ, ζ)
It is defined by an equipotential line.



The scaling is defined by
~~a difference between scales~~
a difference between potentials

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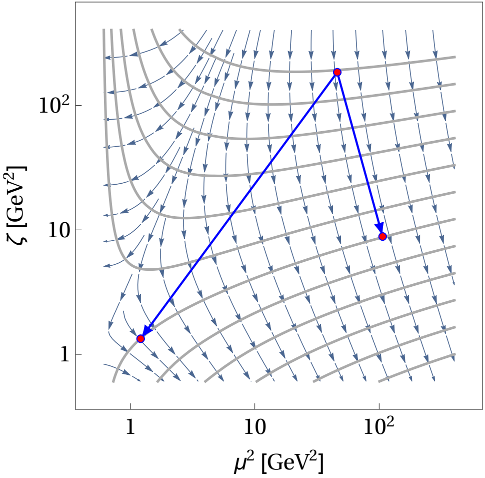
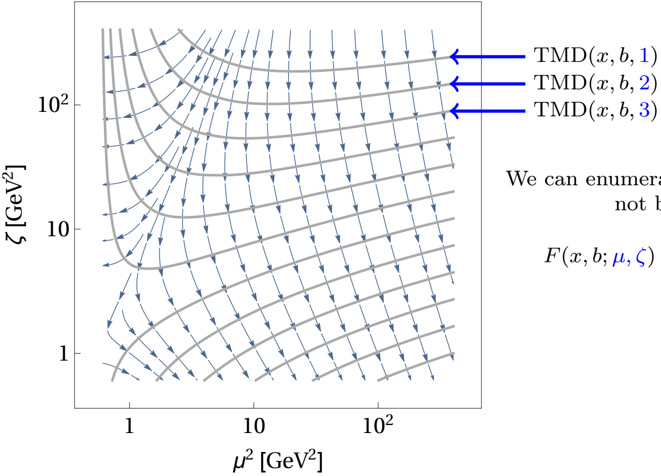


The scaling is defined by
~~a difference between scales~~
a difference between potentials

Evolution factor to both points
is the same
although the scales are
different by 10^2GeV^2

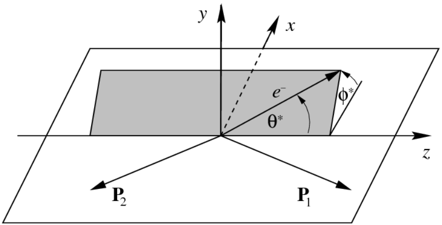
TMD distributions on the same equipotential line are equivalent.



We can enumerate them by a lines not by (μ, ζ)

$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

How to measure TMD distribution



To measure transverse momentum one needs:

- “light-like” hadron plane
- transverse momentum coupled to measured source
- p_T should be small $p_T \ll Q$, otherwise it is perturbatively generated

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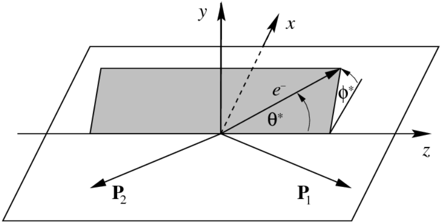
How to measure TMD distribution



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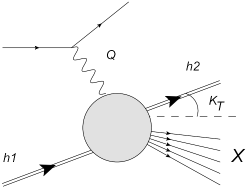
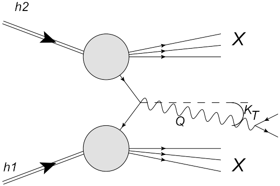
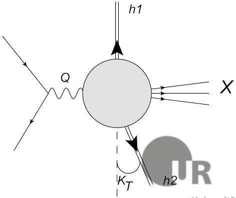
There are 3 main processes to extract TMD distributions







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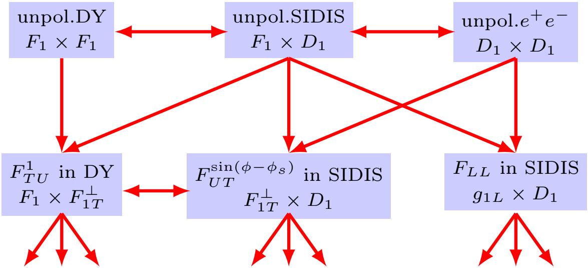
Complication

$$\frac{d\sigma}{dp_T^2 dQ} \simeq \sigma_0(Q) \int d^2\mathbf{b} e^{i\mathbf{b}\mathbf{p}_T} R[Q \to (\mu,\zeta)] F_1(x_1,\mathbf{b};\mu,\zeta) F_2(x_2,\mathbf{b};\mu,\zeta)$$

- Two distribution to extract F_1 & F_2
- + non-perturbative evolution $R \sim \exp(-\mathcal{D})$



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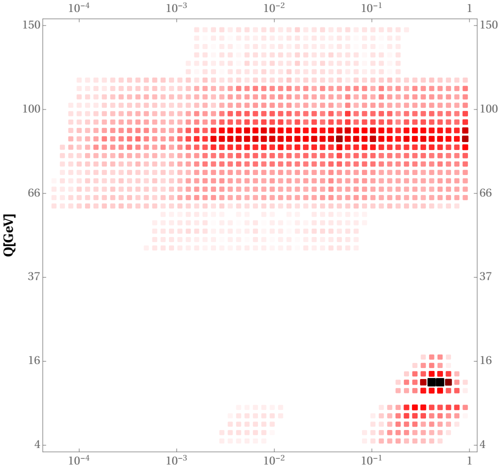
First NNLO extraction

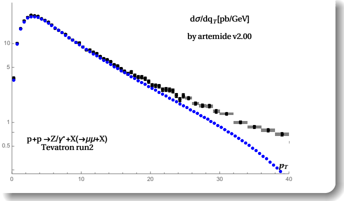
[V.Bertone,I.Scimemi,AV,1902.08474]

TMD factorization $\Rightarrow$ small-q_T/Q

$$\frac{q_T}{Q} < 0.2$$



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y} \sqrt{1 + \frac{q_T^2}{Q^2}}$$



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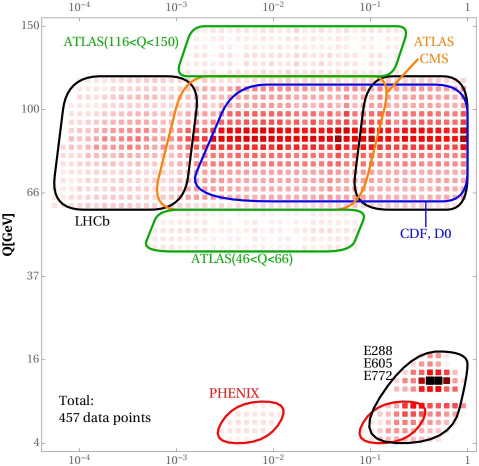


$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y} \sqrt{1 + \frac{q_T^2}{Q^2}}$$

High-energy: CDF, D0, ATLAS, CMS, LHCb
194 points

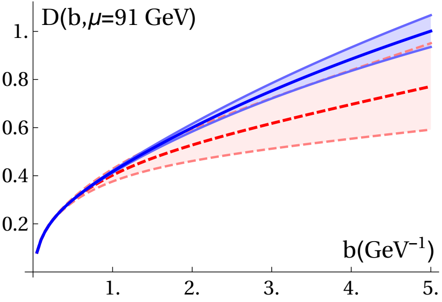
Low-energy: E288, E605, E772, PHENIX
263 points

Total: 457 points
$4 < Q < 150$GeV
$x > 10^{-4}$

Rapidity anomalous dimension

$\mathcal{D}(\mu, \mathbf{b})$

- Independent, and universal function of QCD
- The rapidity anomalous dimension is non-perturbative at large **b**
- Measures property of QCD vacuum (interpretation?)
- Appears in different tasks of hadron physics (jets, small-x)



Recent extraction from global fit of Drel-Yan processes

[V.Bertone,I.Scimemi,AV,1902.08474]

- - full data set

- - without LHC data

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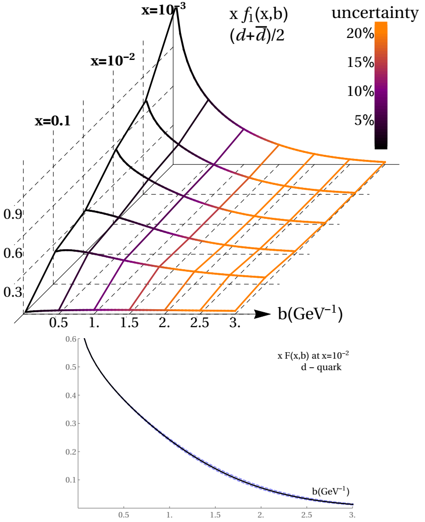
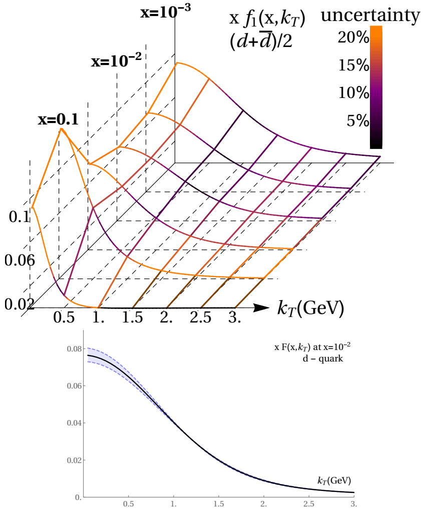
[V.Bertone,I.Scimemi,AV,1902.08474]

uTMDPDF in b-space



uTMDPDF in k_T-space



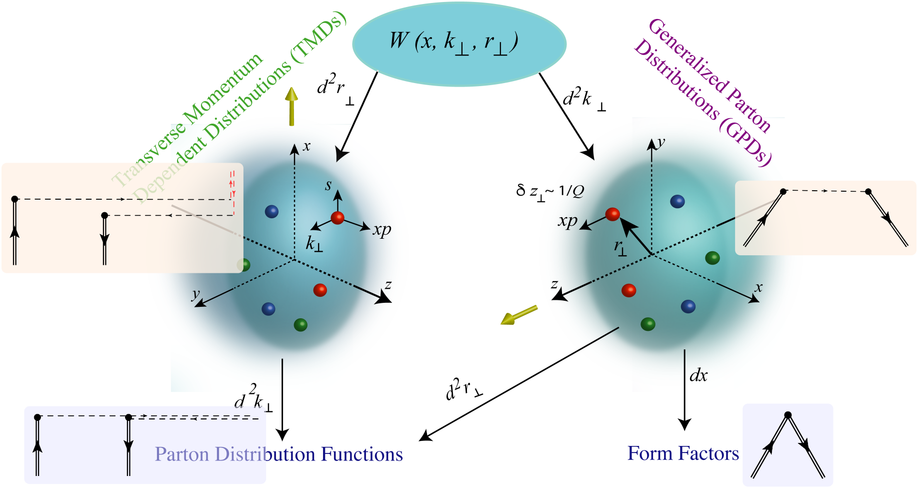
TMD state

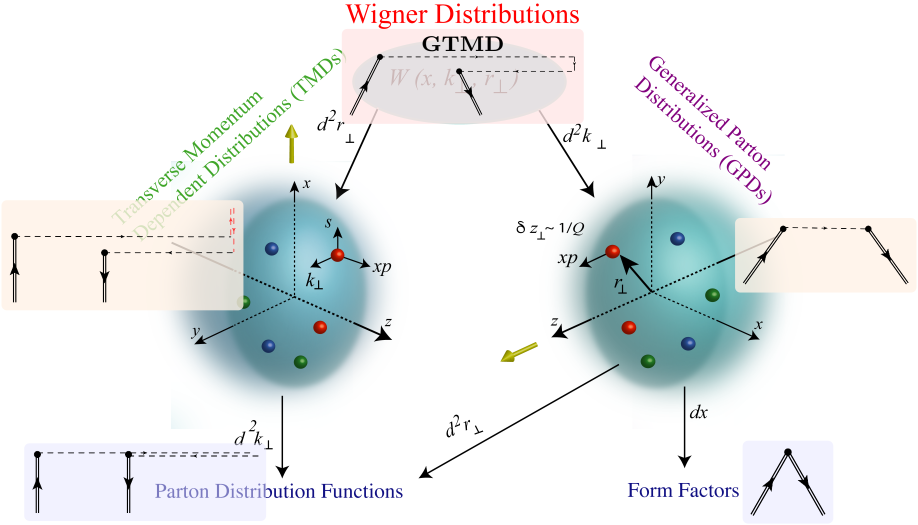
- Significant theory progress within last $\sim 5-8$ years
- Factorization theorem is finally proven
- First QCD-consistent extraction of TMDs
- More to come in following years

Further reading

- R.Tangerman, P.Mulders, *"Intrinsic transverse momentum and the polarized Drell-Yan process"* [arXiv:9403227]
- A.Bacchetta, *et al*, *"Semi-inclusive deep inelastic scattering at small transverse momentum"* [arXiv:0611265]
- R. Angeles-Martinez,*et al"Transverse Momentum Dependent (TMD) parton distribution functions: status and prospects"* [arXiv:1507.05267]
- I.Scimemi *" A short review on recent developments in TMD factorization and implementation"* [arXiv:1901.08398]
- I.Scimemi, A.Vladimirov *"Systematic analysis of double-scale evolution"* [arXiv:1803.06596]

Wigner Distributions



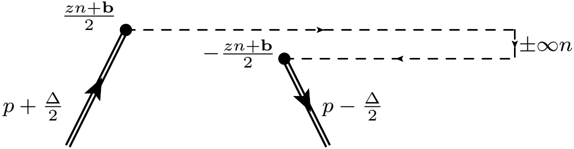

Wigner Distributions
GTMD
W (x, k⊥, r⊥)
d²r⊥
d²k⊥
Transverse Momentum Dependent Distributions (TMDs)
Generalized Parton Distributions (GPDs)
x
s
xp
k⊥
y
z
δ z⊥~ 1/Q
xp
r⊥
z
x
y
d²k⊥
d²r⊥
dx
Parton Distribution Functions
Form Factors

Generalized TMD distribution

Operator for transverse momentum dependent distribution

$$\left\langle p-\frac{\Delta}{2}\right|\bar q\left(\frac{zn+\mathbf{b}}{2}\right)\left[\text{staple contour}\right]q\left(-\frac{zn+\mathbf{b}}{2}\right)\}\left|p+\frac{\Delta}{2}\right\rangle = p^+\int dx e^{ixzp^+}\mathbb{F}_1(x,\mathbf{b},\mathbf{\Delta})$$

$+\ldots$



- All (main) properties are known (evolution, matching)
- No "simple" processes to measure

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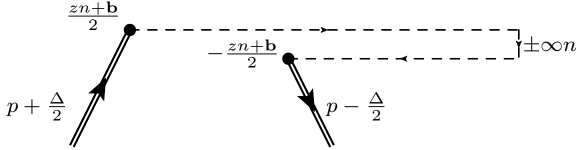

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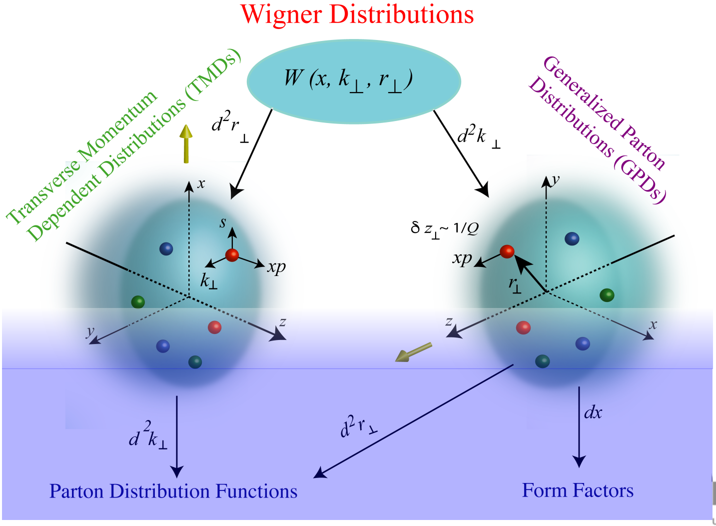
GTMD distribution $\Leftrightarrow$ **Wigner distribution**

$$\int d^2\mathbf{b}\ e^{i(\mathbf{b}\mathbf{k}_T)}\int d^2\mathbf{\Delta}\ e^{i(\mathbf{r}\mathbf{\Delta})}\mathbb{F}_1(x,\mathbf{b},\mathbf{\Delta}) = W(x,\mathbf{r},\mathbf{k}_T)$$

Conclusion



Wigner Distributions
W (x, k⊥, r⊥)
d²r⊥
d²k⊥
Transverse Momentum Dependent Distributions (TMDs)
Generalized Parton Distributions (GPDs)
x
s
xp
k⊥
y
z
δ z⊥~ 1/Q
xp
r⊥
y
z
x
d²k⊥
d²r⊥
dx
Parton Distribution Functions
Form Factors

Conclusion

Wigner Distributions

$W(x, k_\perp, r_\perp)$

Transverse Momentum Dependent distributions (TMDs)

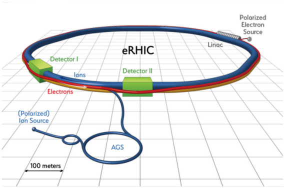
- Significant progress in theory
- Still many gaps (perturbative mathing, power corrections, etc)
- Plenty of measurements + more to come (COMPASS,JLab12,RHIC)
- First "consistent" extractions

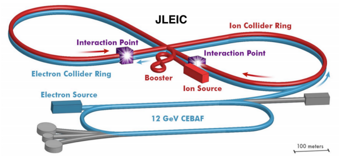
Generalized Parton Distributions (GPDs)

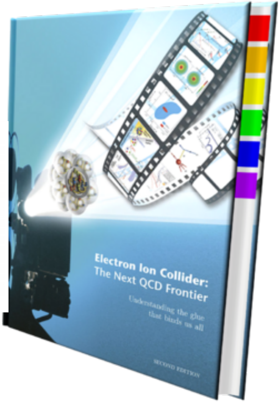
- Theory is developed long ago
- Involved cross-section ↔ amplitude decomposition
- Only recently consistent extraction of CFF from global analysis
- No GPDs (so far)

Nucleon tomography is area of pure QCD.
Progress in it also pushes all QCD-dependent areas.

Electron-Ion collider



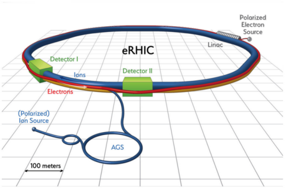


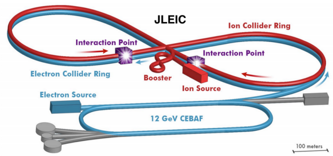


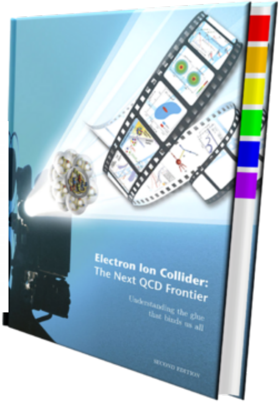
Dedicated QCD machine

- Proton/ion, polarized beam
- Large coverage of Q^2 (moderate values)
- Large dedicated program for **nucleon tomography**

Electron-Ion collider







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Thank you,
for attention!