Extraction of transverse momentum dependent parton distribution functions

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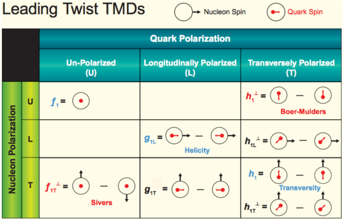
in collaboration with Valerio Bertone & Ignazio Scimemi
based on [1902.08474]

> Transverse momentum dependent distributions (TMDs) give access to detailed 3D picture of hadron insides.



- 2-8 TMD Fragmentation Functions
- gluons TMD distributions
- non-perturbative evolution kernel

Each of them is an independent function with rich structure that describes a particular aspect of hadron structure

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Extraction of TMDs is a cumbersome task

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❶ Numerically complicated calculation

TMD factorization is " naturally" formulated in b-space

$$\frac{d\sigma}{dq_T} \xrightarrow{\text{TMD factorization}} \sigma_0 \int d^2\mathbf{b} e^{-i(\mathbf{b}\cdot\mathbf{q}_T)} F_1(x_1, b) F_2(x_2, b) R_{NP}[b]$$

- ✚ In the integrand all functions multiplicative.
- ✚ Simple form of evolution factor.
- ✖ Slowly convergent Fourier integral.
- ✖ On top of it there are $2-5$ integrations.

Extraction of TMDs is a cumbersome task

❶ Numerically complicated calculation

❷ "Too much freedom"

TMD is a function of 2 variables $F(x, b)$

Collinear momentum fraction
$x \in [0, 1]$
typically $x > 10^{-3}$

Transverse distance
$b > 0$
all values involved

✖ Difficult to find a good parametrization.

$$F(x, k_T) = \int \frac{d^2\mathbf{b}}{(2\pi)^2} F(x, b) e^{-i(\mathbf{b}\cdot\mathbf{k}_T)}$$

Actually the transverse momentum appears
only after Fourier tranformation

Extraction of TMDs is a cumbersome task

❶ Numerically complicated calculation

❷ "Too much freedom"

❸ All TMDs + NP evolution are correlated in the data

TMD factorizable cross-section has a pair of TMDs

$$\boxed{\frac{d\sigma}{dq_T} \xrightarrow{\text{TMD factorization}} \sigma_0 \int d^2\mathbf{b} e^{-i(\mathbf{b}\cdot\mathbf{q}_T)} F_1(x_1,b) F_2(x_2,b) R_{NP}[b]}$$

Extraction is correlated

✖ Global analysis is requred to decorrelate TMDs

Extraction of TMDs is a cumbersome task

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❸ All TMDs + NP evolution are correlated in the data

Unpolarized TMDPDFs are best object to begin with

- The best avalible theory input (complete NNLO+)
- Source cross-sections: Drell-Yan/vector boson production
  - Large amount of data
  - A lot of presice data
  - Span huge range of energies

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Synopsis of theory input

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \sigma_0 \int bdb\, J_0(bq_T) H(Q,\mu) \sum_{ff'} z_{ff'} F_{f\leftarrow h}(x_1, b; \mu, \zeta_1) F_{f'\leftarrow h}(x_2, b; \mu, \zeta_2)$$

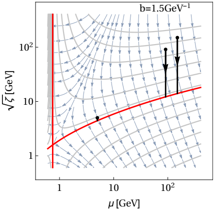
NNLO

$$\boxed{F(x, b; \mu, \zeta) = R[b; (\mu, \zeta)] F(x, b)}$$

N^{3}LO Γ_{cusp}
NNLO rest

NNLO
collinear matching

ζ-prescription and
“optimal” TMD definition

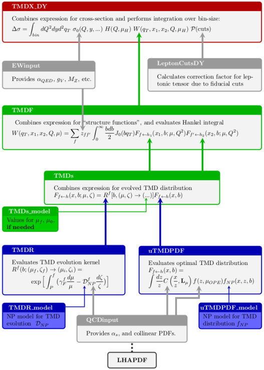


[I.Scimemi,AV,1803.11089]

We use the best available theory input: complete NNLO

Synopsis of numeric evaluation

Evaluation of DY cross-section by artemide



picture from [artemide manual]

To reach the required data precision one has to include extra numerics:

- Finite bin-size effects
- Fiducial cuts

It make calculation very numerically intensive:

- 3 bin-size integrations
- + Hankel transform
- + 2 NNLO Mellin convolutions
- + ? integrations for evolution

$\sim 10^6 - 10^7$ calls of PDF for a single data point

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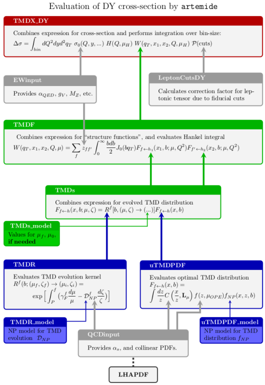
Synopsis of numeric evaluation



picture from [artemide manual]

ARTEMIDE

https://teorica.fis.ucm.es/artemide/

Package for TMD phenomenology

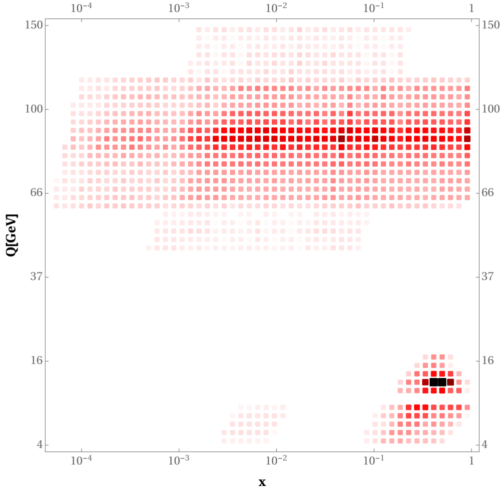
- Flexible definition of factorization scheme
  - Variety of evolutions (CSS,ζ-prescription, etc.)
  - All available PT: LO, NLO, NNLO,resummed
  - No restriction for NP models
- Fast code
- Constantly expanding
  - (unpol.)DY cross-sections
  - (unpol.)SIDIS cross-sections
- Various theory tools.

repository:
https://github.com/VladimirovAlexey/artemide-public

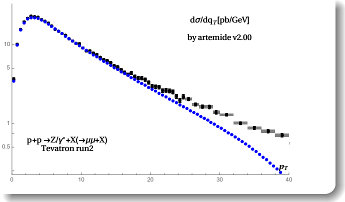
Synopsis on data

$$\text{TMD factorization} \Rightarrow \text{small-}q_T/Q$$

$$\frac{q_T}{Q} < 0.25 \qquad \& \qquad \left(\frac{q_T}{Q}\right)^2 < \frac{\delta\sigma_{\text{uc}}}{\sigma}$$



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y} \sqrt{1 + \frac{q_T^2}{Q^2}}$$



Synopsis on data

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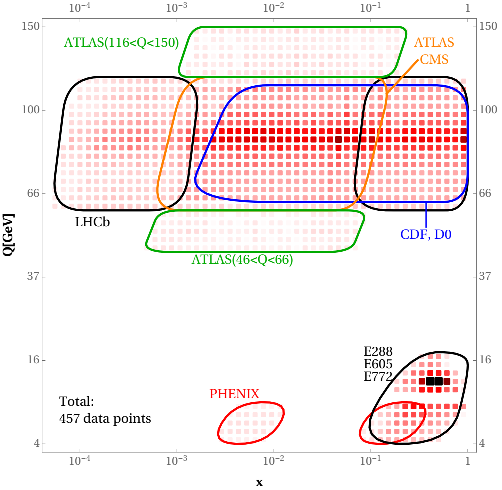


$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y} \sqrt{1 + \frac{q_T^2}{Q^2}}$$

High-energy: CDF, D0, ATLAS, CMS, LHCb
194 points

Low-energy: E288, E605, E772, PHENIX
263 points

Total: 457 points
$4 < Q < 150$GeV
$x > 10^{-4}$

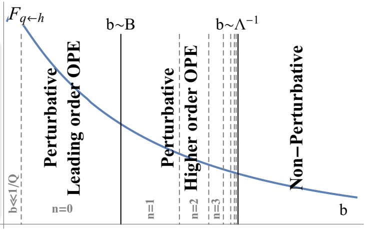
Non-perturbative model for TMD evolution

$$\mathcal{D}_{NP}(b,\mu) = \mathcal{D}_{\textbf{resum}}(b^*,\mu) + c_1 b b^*, \qquad b^* = b\Big/\sqrt{1+\frac{b^2}{B_{NP}^2}}$$

Non-perturbative model for TMD distribution

$$F_1(x,b) = \int_x^1 \frac{dy}{y} C\Big(\frac{x}{y}, \ln(b\mu_{OPE})\Big) f_1(y,\mu_{OPE}) f_{NP}(x,b)$$

$$f_{NP} = \exp\left(-\frac{(\lambda_1(1-x)+\lambda_2 x+\lambda_3 x(1-x))b^2}{\sqrt{1+\lambda_4 x^{\lambda_5} b^2}}\right)$$



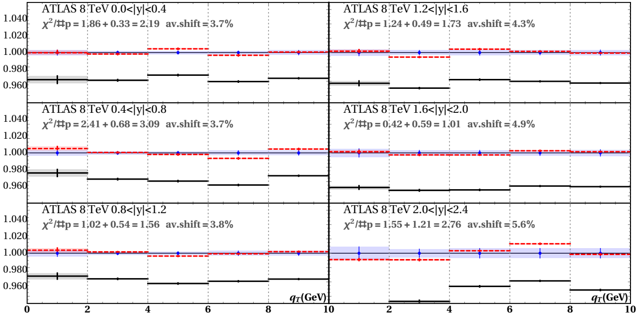
- **2+5 parameters**
- even function in b (required by OPE)
- $b \to 0$ NNLO perturbative
- $b \to \infty$ exp/Gauss (depending on parameters)

Quality of the fit

$$\chi^2/N_{\text{pt}} = 1.17 \qquad \Leftarrow \begin{cases} \textbf{Low energy data:} & 0.90 \\ \textbf{High energy data:} & 1.55 \end{cases}$$

Systematic undershooting

$$\text{nuisance parameters: } \chi^2 = \underbrace{\chi^2_D}_{shape} + \underbrace{\chi^2_\lambda}_{shift} \qquad \Leftrightarrow \qquad d = \text{“systematic shift”}$$



Quality of the fit

$$\chi^2/N_{\text{pt}} = 1.17 \quad \Leftarrow \begin{cases} \textbf{Low energy data:} & 0.90 \\ \textbf{High energy data:} & 1.55 \end{cases}$$

Systematic undershooting

nuisance parameters: $\chi^2 = \underbrace{\chi^2_D}_{shape} + \underbrace{\chi^2_\lambda}_{shift} \quad \Leftrightarrow \quad d =$ "systematic shift"

Data set	av.sys.	χ^2/N_{pt}	av.shift.
E288(200)	25%	0.86	41%
...	...	...	...
E772	10%	1.70	13%
ATLAS (8TeV) $\lvert y\rvert<0.4$	2.8%	2.19	3.7%
ATLAS (8TeV) $0.4<\lvert y\rvert<0.8$	2.8%	3.09	3.7%
ATLAS (8TeV) $0.8<\lvert y\rvert<1.2$	2.8%	1.56	3.8%
ATLAS (8TeV) $1.2<\lvert y\rvert<1.6$	2.8%	1.73	4.3%
ATLAS (8TeV) $1.6<\lvert y\rvert<2.0$	2.8%	1.01	4.9%
...	...	...	...
LHCb (7TeV)	1.7%	2.95	5.7%
LHCb (8TeV)	1.1%	5.54	5.7%
LHCb (13TeV)	3.9%	0.89	6.3%

The undershooting is mainly due to PDFs at large x

Data set	NNPDF3.1nnlo χ^2/N_{pt}	NNPDF3.1nnlo av.shift.		MMHT14nnlo χ^2/N_{pt}	MMHT14nnlo av.shift.
ATLAS (8TeV) |y|<0.4	2.19	3.7%	$x \sim 0.01$	3.98	7.6%
ATLAS (8TeV) 0.4<|y|<0.8	3.09	3.7%		3.87	7.1%
ATLAS (8TeV) 0.8<|y|<1.2	1.56	3.8%		2.32	6.2%
ATLAS (8TeV) 1.2<|y|<1.6	1.73	4.3%		2.04	5.3%
ATLAS (8TeV) 1.6<|y|<2.0	1.01	4.9%		1.55	5.0%
ATLAS (8TeV) 2.0<|y|<2.4	2.76	5.7%	$x \sim 0.1$	1.37	5.0%
...	...	...		...	...
LHCb (7TeV)	2.95	5.7%		1.70	4.9%
LHCb (8TeV)	5.54	5.7%		2.10	4.3%
LHCb (13TeV)	0.89	6.3%	$x \sim 0.3$	0.80	4.2%
...	...	...		...	...
E772	1.70	13%		2.4	-0.2%
E288(400)	0.80	27%		1.11	2%
E288(300)	0.93	36%		1.07	12%
E288(200)	0.86	41%	$x \sim 0.6$	0.62	15%
χ^2/N_{pt} total	1.17			1.39	

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ATLAS (8 [illegible]	[illegible]	[illegible]	$x\sim 0.1$	[illegible]	[illegible]
...					
LHCb (7T [illegible]	[illegible]	[illegible]		[illegible]	[illegible]
LHCb (8T [illegible]	[illegible]	[illegible]		[illegible]	[illegible]
LHCb (13 [illegible]	[illegible]	[illegible]		[illegible]	[illegible]
...					
E772	[illegible]	[illegible]		2.4	-0.2%
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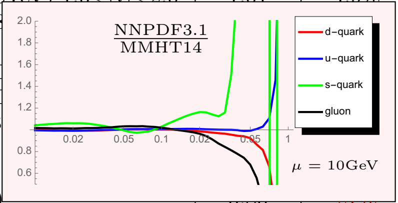

TMDPDFs are very sensitive to input collinear PDFs

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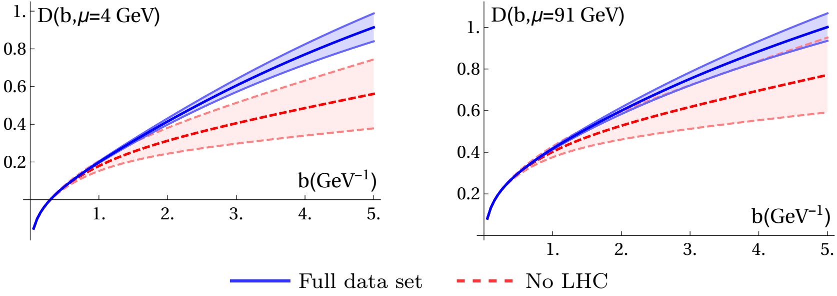
Results

Non-perturbative evolution kernel



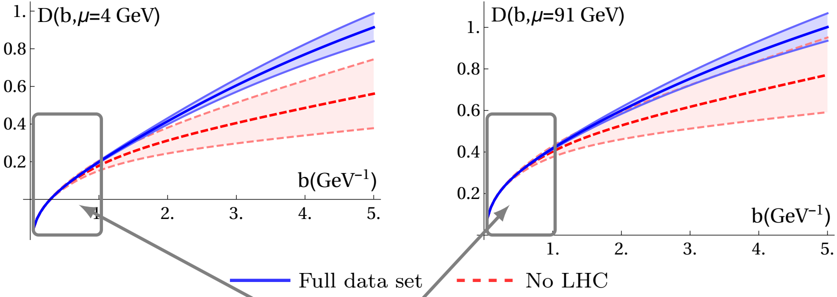
Error band by replica method

$$B_{\text{NP}} = 3.31 \pm 0.28$$
$$c_0 = 0.024 \pm 0.006$$

$$B_{\text{NP}} = 1.21 \pm 0.50$$
$$c_0 = 0.057 \pm 0.038$$

Results

Non-perturbative evolution kernel



LHC
sensitive region

eplica method

$$B_{\text{NP}} = 3.31 \pm 0.28$$
$$c_0 = 0.024 \pm 0.006$$

$$B_{\text{NP}} = 1.21 \pm 0.50$$
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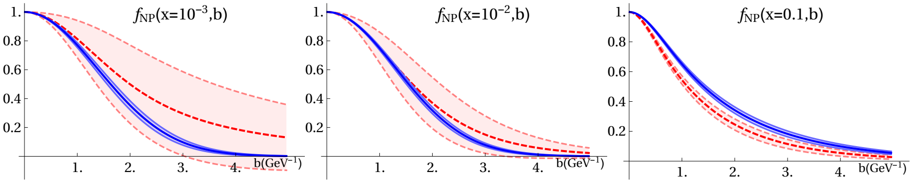
Results

unpolarized TMD PDF



Full data set   No LHC

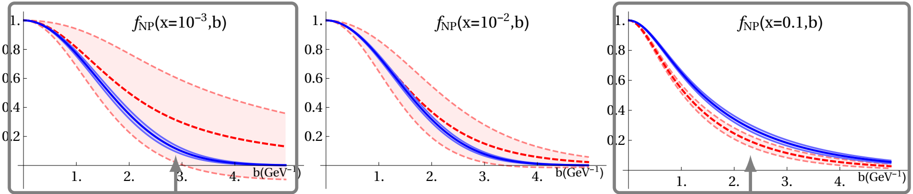
Error band by replica method

$$\boxed{\begin{aligned}\lambda_1 &= 0.26 \pm 0.02\\ \lambda_2 &= 8.18 \pm 1.00\\ \lambda_3 &= -4.7 \pm 1.37\\ \lambda_4 &= 300 \pm 89.0\\ \lambda_3 &= 2.44 \pm 0.12\end{aligned}}$$

$$\boxed{\begin{aligned}\lambda_1 &= 0.21 \pm 0.17\\ \lambda_2 &= 12.1 \pm 4.40\\ \lambda_3 &= -3.5 \pm 5.41\\ \lambda_4 &= 316 \pm 196.\\ \lambda_3 &= 2.11 \pm 0.28\end{aligned}}$$

Results

unpolarized TMD PDF



Full data set — No LHC

Constraint only by high energy data

Error band by replica method

Discrepancy ?PDF?

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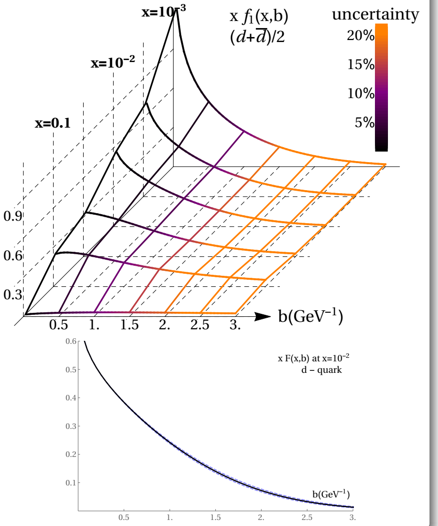
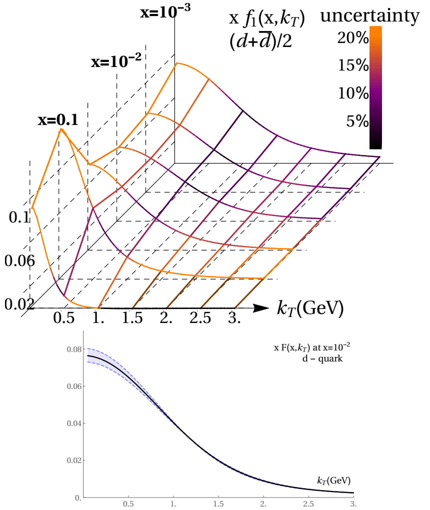
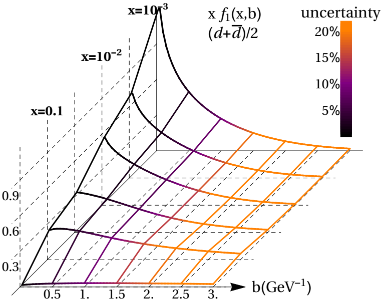
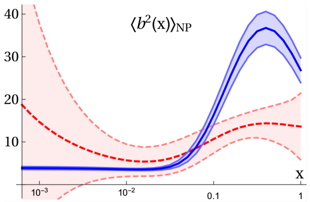
uTMDPDF in b-space



uTMDPDF in k_T-space







$$f_{NP} = \exp\left(-\frac{A(x)b^2+\ldots}{\sqrt{1+B(x)b^2+\ldots}}\right)$$

$$\boxed{\begin{array}{c} f_{NP} = \exp\left(-A(x)b^2\right) \\ B(x) \to 0 \end{array}}$$

$$\boxed{\begin{array}{c} f_{NP} = \exp\left(-\frac{A(x)}{\sqrt{B(x)}}b\right) \\ B(x) \gg 0 \end{array}}$$

- Narrow$_{\text{in b}}$ – Wide$_{\text{in } k_T}$ at smaller-x
- Wide$_{\text{in b}}$ – Narrow$_{\text{in } k_T}$ at larger-x

similar to [Baccetta, et al,1703.10157]

Conclusion

Now, we know uTMDPDFs and non-perturbative TMD evolution accurately

- Wide range of Q and x
- Extremely precise data from LHC
- Accurate treatment of experimental errors and their propagation
- ζ-prescription at work
- Artemide

Now, we have a solid foundation for further expansion

- Input for SIDIS (and further processes)
- Access to PDFs at large-x
- Non-perturbative TMD evolution for other processes (e.g. jets)
- Artemide