

# Extraction of unpolarized TMDs from Drell-Yan and SIDIS data

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This talk is mainly based on [Scimemi,AV,1912.06532] = SV19 fit

## Main features of SV19

- ▶ Joined description of DY and SIDIS
- ▶ NNLO matching and NNLO/ $N^3$ LO TMD evolution (NNLL' or  $N^3$ LL)
- ▶  $\zeta$ -prescription and independent definitions of non-perturbative elements

## artemide

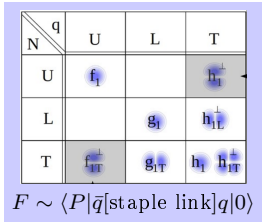
<https://github.com/VladimirovAlexey/artemide-public>



# TMD factorization formula (in $\zeta$ -prescription)

Rapidity anomalous dimension

$$\mathcal{D} \sim \langle 0 | \frac{\text{Tr}}{N_c} F_{+b} [\text{staple link}] | 0 \rangle$$



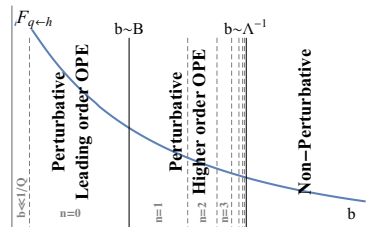
$$\frac{d\sigma}{dx dz dQ^2 d^2 \mathbf{q}_T} = \sum_{ff'} H_{ff'} \left( \frac{Q}{\mu} \right) \int d^2 b e^{i(\mathbf{b} \cdot \mathbf{q}_T)} \left( \frac{Q^2}{\zeta_\mu [\mathcal{D}]} \right)^{-2\mathcal{D}(b, \mu)} F_{f \leftarrow h}(x, b) D_{f' \leftarrow h}(z, b)$$

- ▶ Each data-point is a product (convolution) of **three independent non-perturbative** functions
- ▶ Functions do not “cross-talk” and could be modeled independently
- ▶ Each function is responsible for a separate kinematic variable
  - ▶ Rapidity AD:  $\mathcal{D} \rightarrow Q$  and  $b$
  - ▶ TMD PDF:  $F \rightarrow x$  and  $b$
  - ▶ TMD FF:  $D \rightarrow z$  and  $b$

Each TMD distribution is **independent** non-perturbative function

$$F(x, b) = \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p | \bar{q}(zn + b) [zn + b, -\infty n + b] \gamma^+ [-\infty n, 0] q(0) | p \rangle$$

$F(x, b)$  has too much parametric freedom



Matching to PDFs at small- $b$

$$F(x, b) = C_1 \otimes f_1(x) + \mathbf{b}^2 (C_2 \otimes f_1(x) + C_3 \otimes T(x)) + \dots$$

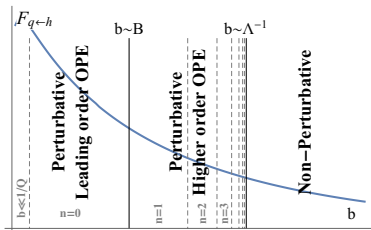
- ▶  $C_1$  is known at N<sup>3</sup>LO [Ebert, Mistlberger, Vita, 2006.05329]
- ▶ All  $b^2$ -powers with twist-2/3 [Moos, AV, 2008.01744]

Small- $b$  reduces parametric freedom

Each TMD distribution is **independent** non-perturbative function

$$F(x, b) = \int \frac{dz}{2\pi} e^{-ixzp^+} \langle p | \bar{q}(zn + b) [zn + b, -\infty n + b] \gamma^+ [-\infty n, 0] q(0) | p \rangle$$

The model for TMDs



$$F(x, b) = C_1 \otimes f_1(x) f_{NP}(x, b)$$

- ▶ **SV19 (& Pavia19)** uses  $C_1$  at NNLO
- ▶  $f_{NP}$  is subject of fitting
  - ▶ 5 parameters for TMDPDF
  - ▶ 4 parameters for TMDFF
  - ▶ **No flavor dependence**

$$f_{NP}(x, b) = \exp \left( - \frac{\lambda_1(1-x) + \lambda_2 x + x(1-x)\lambda_5}{\sqrt{1 + \lambda_3 x^{\lambda_4} b^2}} b^2 \right)$$

$$D_{NP}(x, b) = \exp \left( - \frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1 + \eta_3 (b/z)^2}} \frac{b^2}{z^2} \right) \left( 1 + \eta_4 \frac{b^2}{z^2} \right)$$

Each TMD distribution is **independent** non-perturbative function

$$\mathcal{D}(b, \mu) = \lambda \frac{ig}{2} \frac{\int_0^1 d\beta \langle 0 | F_{b+}(-\lambda n + b\beta) W_{C'} | 0 \rangle}{\langle 0 | W_{C'} | 0 \rangle}$$

[AV, Phys.Rev.Lett. 125 (2020)]

Small- $b$  OPE for CS-kernel

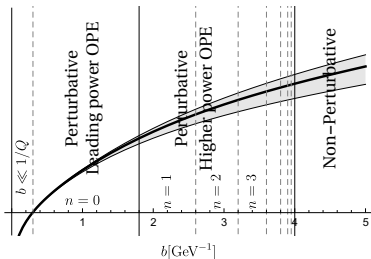
$$\mathcal{D}(b, \mu) = \mathcal{D}_0(b, \mu) + \mathbf{b}^2 \mathcal{D}_1 + \dots$$

- ▶  $\mathcal{D}_0$  is known at N<sup>3</sup>LO [AV, 1610.05791]

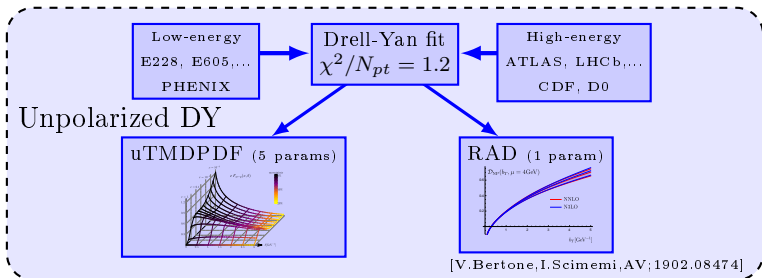
Model for CS-kernel

$$\mathcal{D}(b, \mu) = \mathcal{D}_{\text{resum}}(b, \mu) + c_0 b b^*$$

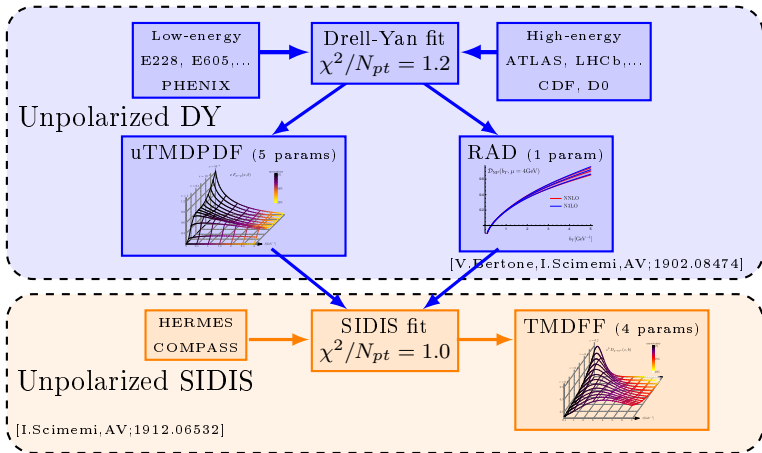
- ▶ SV19 (& Pavia19) uses  $\mathcal{D}_0$  at N<sup>3</sup>LO
- ▶ Linear asymptotic at  $b \rightarrow \infty$



# Universality & the chain of extractions

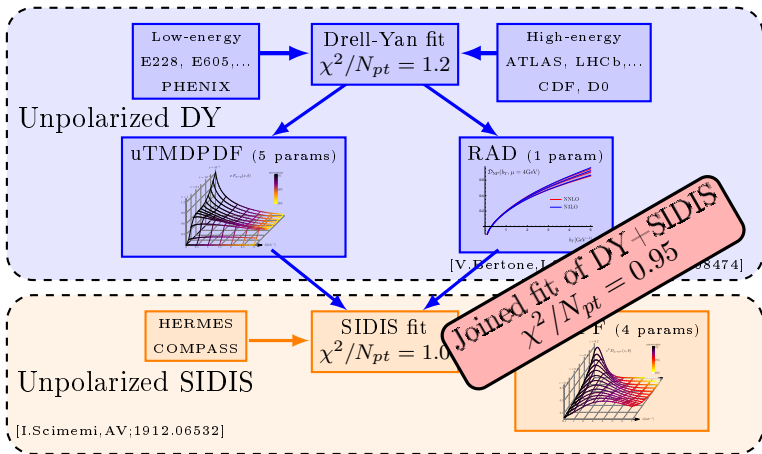


# Universality & the chain of extractions

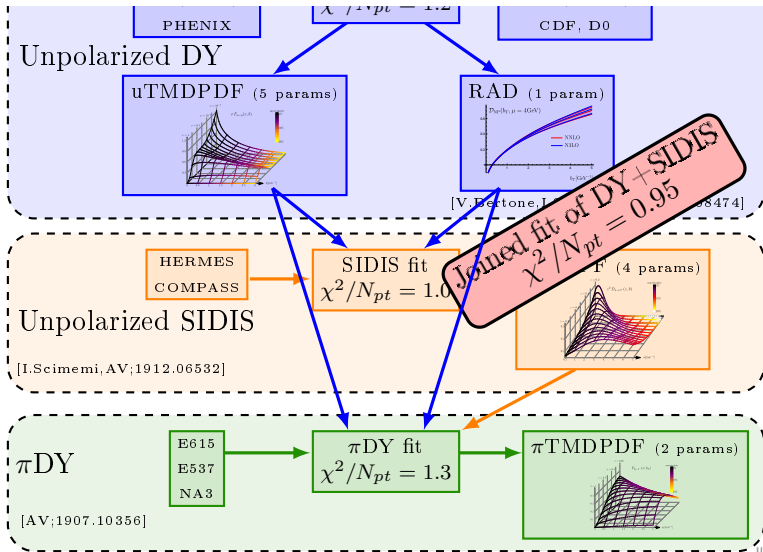




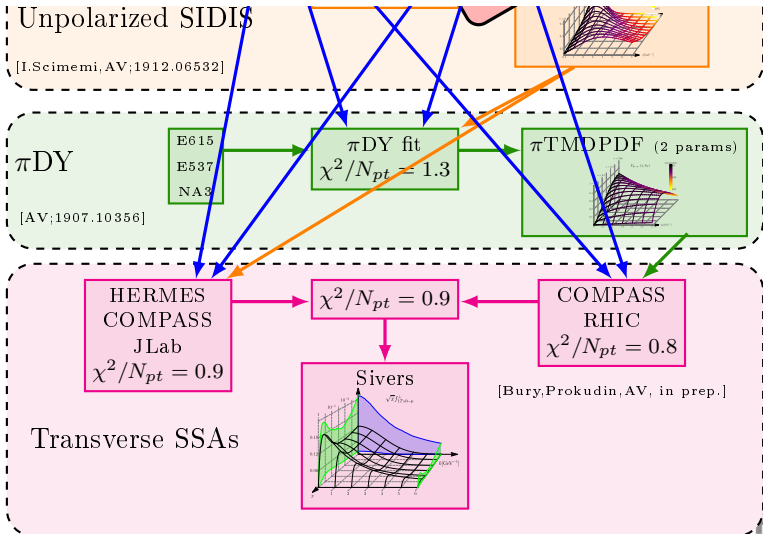
# Universality & the chain of extractions



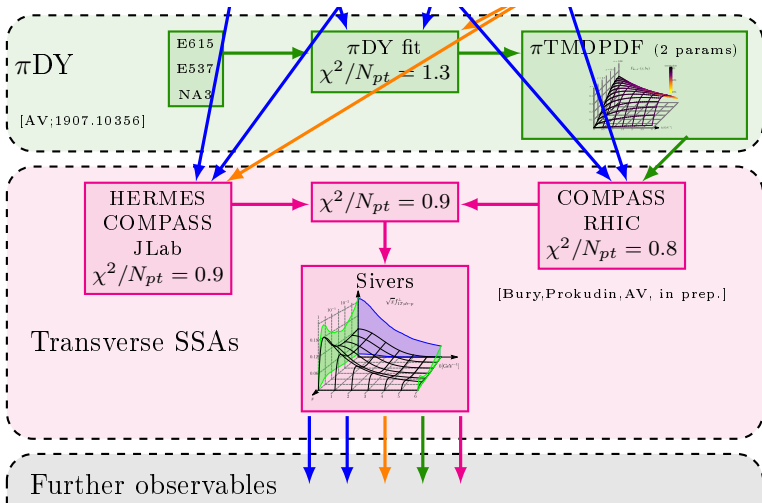
# Universality & the chain of extractions



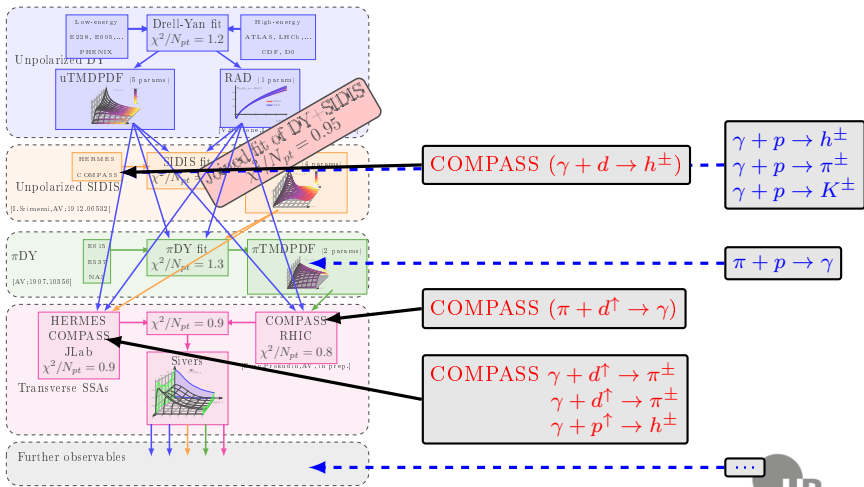
# Universality & the chain of extractions



# Universality & the chain of extractions



# Role of the COMPASS data



## There are plenty of details/questions

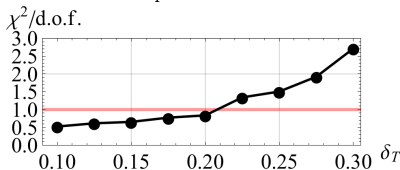
- ▶ Where is the limit of TMD factorization?
  - ▶ How to cut the data?
- ▶ Power corrections:
  - ▶ Induced (ATLAS, LHCb) (linear in  $q_T$  ?)
  - ▶ Kinematic
  - ▶ In the definition of collinear frame
  - ▶ Target mass and produced mass
- ▶ Universality and correlations
- ▶ ... many others ...
- ▶ What do we learn from it?



# Limits of TMD factorization

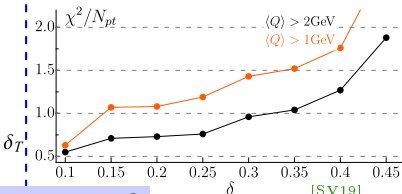
Test by inclusion of the data with  
 $q_T < \delta \cdot Q$

unpolarized DY



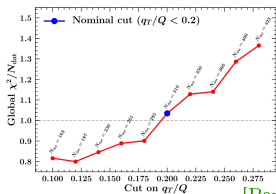
[I.Scimemi, AV, 1706.01473]

unpolarized SIDIS



[SV19]

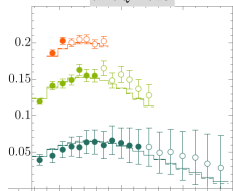
$q_T < 0.2 - 0.25Q$



[Pavia19]

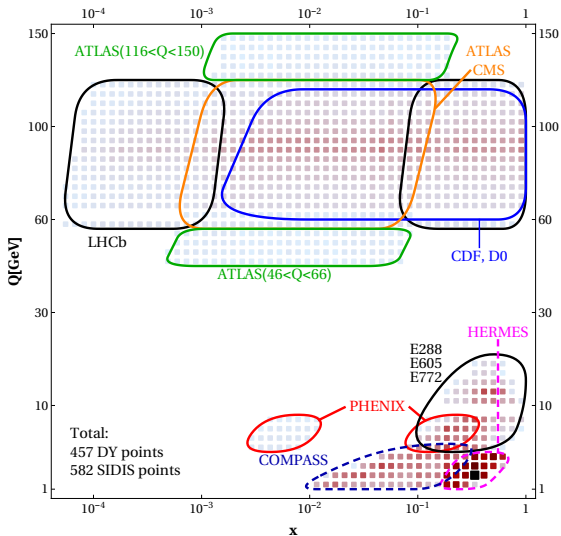
COMPASS

$7. < Q^2 < 16.$



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## Data survived after the cut



<b>High energy DY:</b>	194 points
<b>Low energy DY:</b>	263 points
<b>(Low energy) SIDIS:</b>	582 points
<b>TOTAL:</b>	1038 points

$$\text{NNLO: } \chi_{global}^2/N_{pt} = 0.95$$

$$\text{N}^3\text{LO: } \chi_{global}^2/N_{pt} = 1.05$$

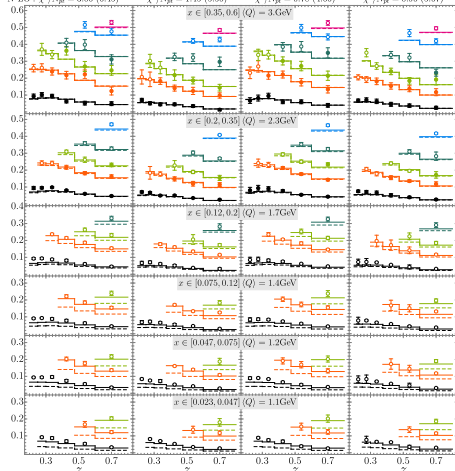
artemide v.2.02



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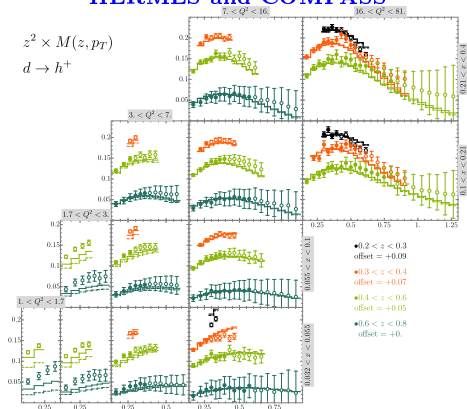
$p \rightarrow \pi^+$        $p \rightarrow \pi^-$        $d \rightarrow \pi^+$        $d \rightarrow \pi^-$   
 NNLO:  $\chi^2/N_{pt} = 2.20$  (2.64)       $\chi^2/N_{pt} = 1.12$  (2.31)       $\chi^2/N_{pt} = 0.57$  (1.46)       $\chi^2/N_{pt} = 0.74$  (1.91)  
 N<sup>3</sup>LO:  $\chi^2/N_{pt} = 3.06$  (6.45)       $\chi^2/N_{pt} = 1.45$  (5.56)       $\chi^2/N_{pt} = 0.78$  (4.66)       $\chi^2/N_{pt} = 0.96$  (5.67)



- $p_T \in [0., 0.15]$  GeV offset = +0.
- $p_T \in [0.15, 0.25]$  GeV offset = +0.05
- $p_T \in [0.25, 0.35]$  GeV offset = +0.1
- $p_T \in [0.35, 0.45]$  GeV offset = +0.2
- $p_T \in [0.45, 0.6]$  GeV offset = +0.35
- $p_T \in [0.6, 0.8]$  GeV offset = +0.45

Example of SIDIS data  
 No contradiction between  
 HERMES and COMPASS

$z^2 \times M(z, p_T)$   
 $d \rightarrow h^+$



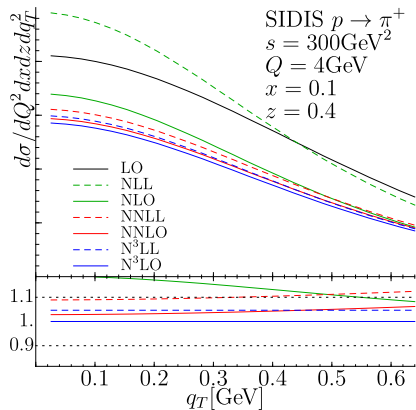
- $0.2 < x < 0.3$  offset = +0.09
- $0.3 < x < 0.4$  offset = +0.07
- $0.4 < x < 0.6$  offset = +0.05
- $0.6 < x < 0.8$  offset = +0.



## Possible sources of agreement/disagreement

### Perturbative corrections

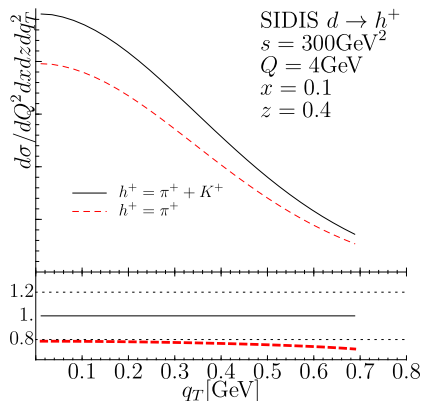
- ▶ At low- $Q$  perturbative corrections:
  - ▶ NLL  $\rightarrow$  NNLL  $\sim 40\%$
  - ▶ NNLL  $\rightarrow$  N<sup>3</sup>LL  $\sim 7\%$
  - ▶ NLL [Pavia17]  $\rightarrow$  N<sup>3</sup>LL [SV19]  $\sim 45\%$



## Possible sources of agreement/disagreement

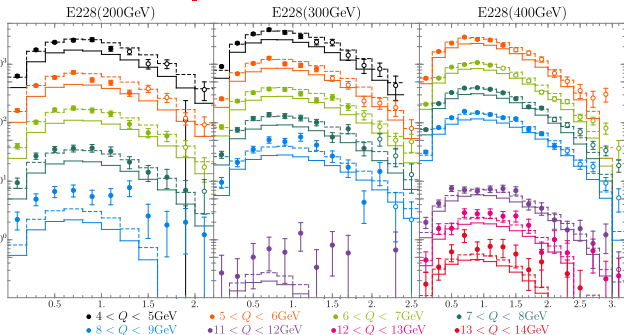
### Final state interpretation

- ▶ COMPASS measured  $\gamma + d \rightarrow h^\pm$ 
  - ▶ [Pavia17] and we identify  $h \equiv \pi$ .
  - ▶ [SV19]  $h^\pm \simeq \pi^\pm + K^\pm$
- ▶ Kaon channel gives  $\sim 20\%$  contribution
- ▶ Proton channel gives  $\sim 2-3\%$  contribution (neglected in SV19)



# Example of low-energy DY

**Some problem with normalization**

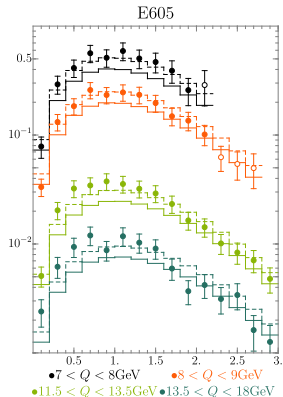


+30%

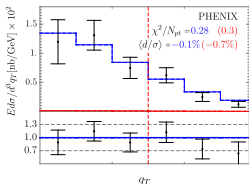
+25%

+15%

Norm.error of data 25%



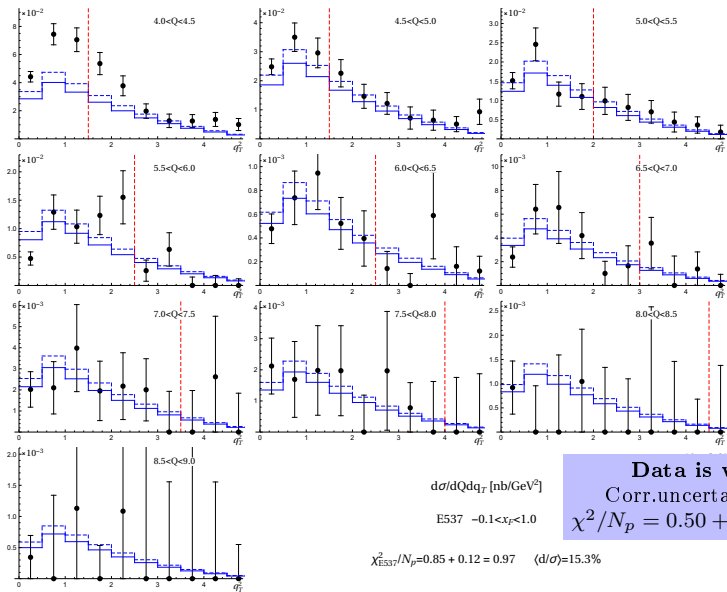
+15%



I bet that this discrepancy is due to large-x.







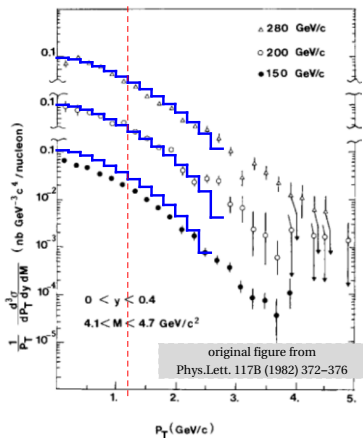
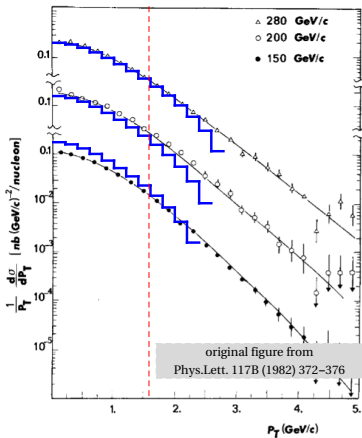
**Data is worse**  
 Corr. uncertainty = 8%  
 $\chi^2/N_p = 0.50 + 0.11 = 0.61$

$d\sigma/dQdq_T$  [nb/GeV<sup>2</sup>]

E537  $-0.1 < x_T < 1.0$

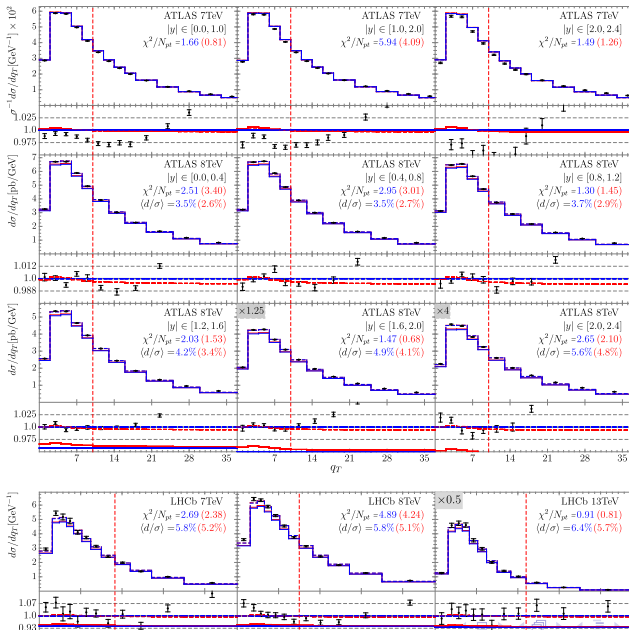
$\chi^2_{E537}/N_p = 0.85 + 0.12 = 0.97$   $(d/\sigma) = 15.3\%$





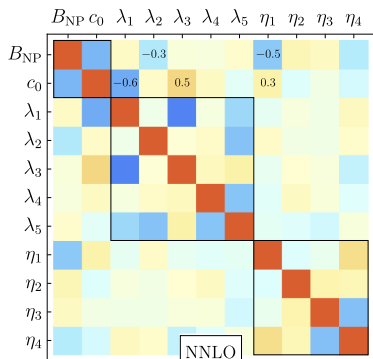
$\pi$ DY-data from NA3 and E537 does not show such anomalous behavior  
**Waiting for unpolarized  $\pi$ DY by COMPASS**

# LHC data within TMD factorization



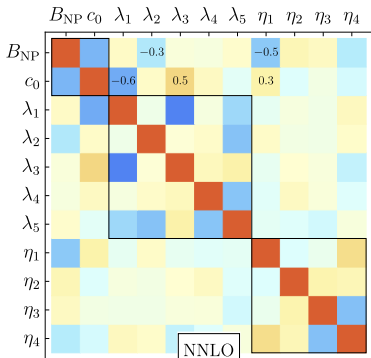


Evolution : 2 parameters  
 TMDPDF : 5 parameters + PDF  
 TMDFF : 4 parameters + NNFF



Different NP functions are almost decorrelated





Evolution : 2 parameters  
 TMDPDF : 5 parameters + PDF  
 TMDFF : 4 parameters + NNFF

Different NP functions are almost decorrelated

Fit quality essentially depends on the collinear input.

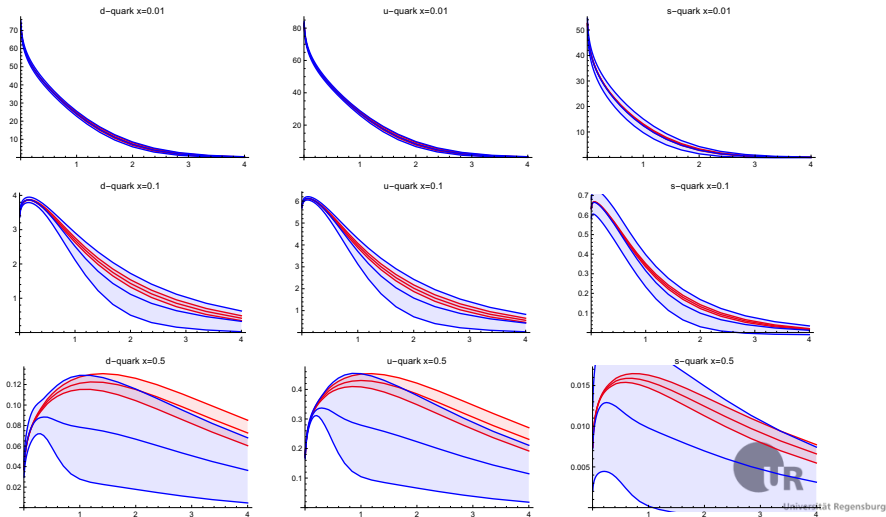
Vary NNPDF within the  $1\sigma$  band

$$\chi^2/N_{pt} \in [0.8, 6.]$$

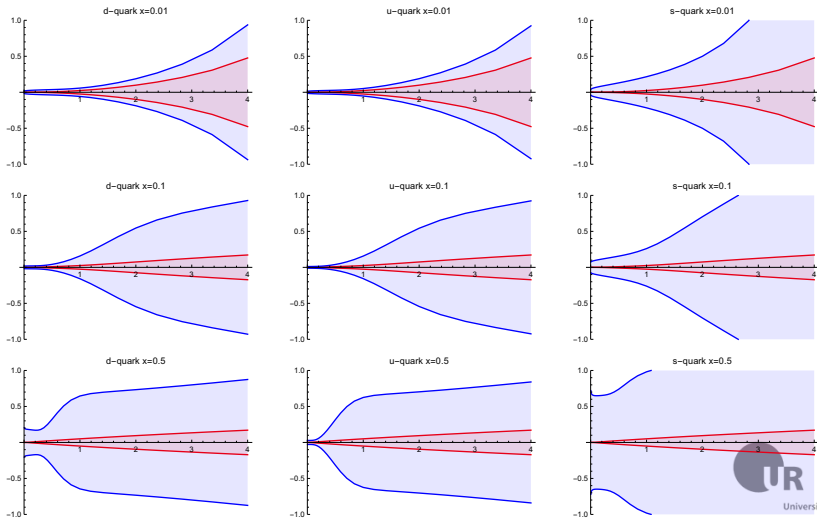
We cannot estimate accurately the PDF uncertainty.



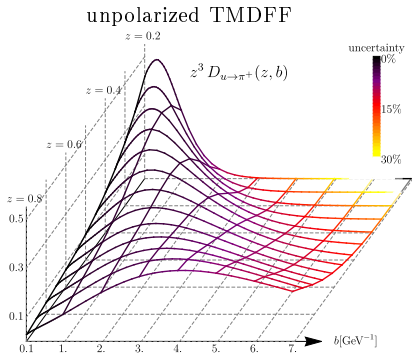
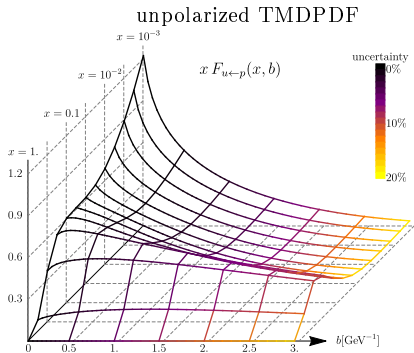
$$F(x, b) = C(x, b, \mu_{OPE}) \otimes \underbrace{f_1(x, \mu_{OPE})}_{\pm \delta f(\text{reweighted})} \underbrace{f_{NP}(x, b)}_{\pm \delta f_{NP}}$$



$$F(x, b) = C(x, b, \mu_{OPE}) \otimes \underbrace{f_1(x, \mu_{OPE})}_{\pm \delta f(\text{reweighted})} \underbrace{f_{NP}(x, b)}_{\pm \delta f_{NP}}$$



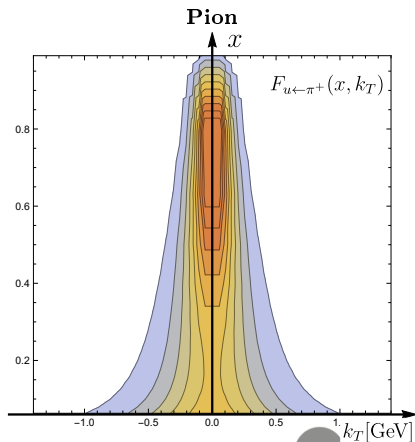
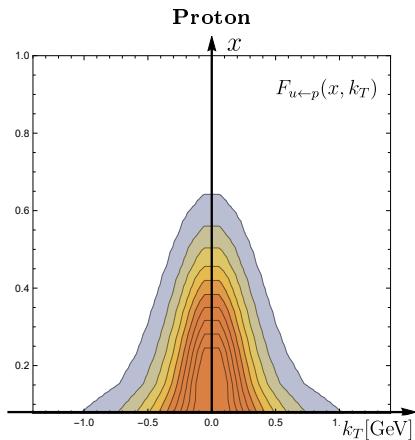
## unpolarized TMD-distributions



... systematic error 10-20%  
 ... evolution does not have this systematics

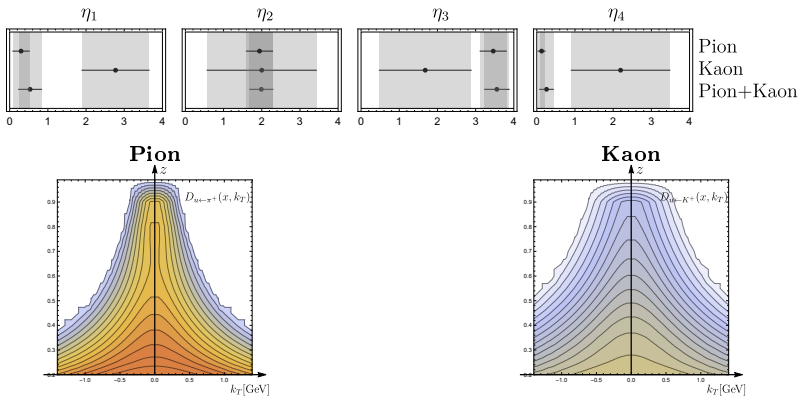


All distributions are different!



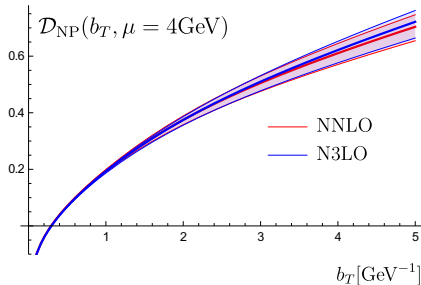
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In SV19 no flavor/particle dependence  
 What if I try to add it?



I see definite difference between  $\pi/K$  TMDFFs, however, uncertainties are large and current data could be fit with single TMDFF  
**COMPASS SIDIS with identified particles is very welcome!**

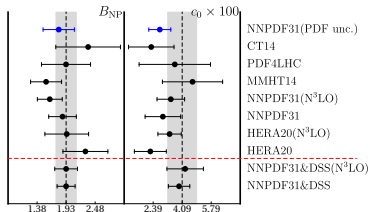
## Universal TMD evolution kernel



$$\mathcal{D}_{\text{NP}}(b, \mu) = \mathcal{D}_{\text{perp}}(b^*, \mu) + c_0 b b^*,$$

$$b^* = b / \sqrt{1 + b^2 / B_{\text{NP}}^2}$$

- ▶ Linear asymptotic at  $b \rightarrow \infty$
- ▶ RAD is independent on PDF set







## CS-kernel is the window to a QCD vacuum!

CS-kernel measures the properties of QCD vacuum [AV,2003.02288]

$$\mathcal{D}(b, \mu) = \lambda \frac{ig}{2} \frac{\int_0^1 d\beta \langle 0 | F_{b+}(-\lambda n + b\beta) W_{C'} | 0 \rangle}{\langle 0 | W_{C'} | 0 \rangle}$$

### Power correction

$$\mathcal{D}(b, \mu) = \mathcal{D}_0 + \mathbf{b}^2 \mathcal{D}_2 + \dots, \quad \mathcal{D}_2 \simeq \frac{\pi^2}{72} \frac{G_2}{\Lambda_{\text{QCD}}^2} \simeq (1. - 5.) \times 10^{-2} \text{GeV}^{-1}$$

	Pavia17	SV19	SV17	Pavia19	BLNY(03/14)
$\mathcal{D}_2 \times 10^2$	$2.8 \pm 0.5$	$2.9 \pm 0.6$	$0.7^{+1.2}_{-0.7}$	$0.9 \pm 0.2$	$20 - 35$

### Amazing function...

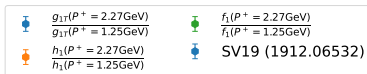
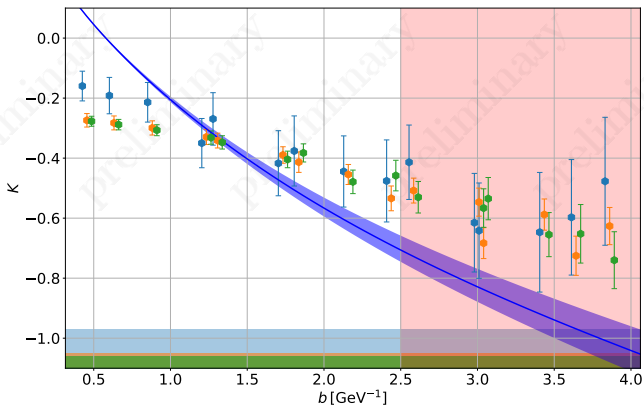
- ▶ String tension
- ▶ Confining potential
- ▶ ...

# First measurement of CS-kernel on lattice

based on [AV,Schafer,2002.07527]

see also [Ebert,Stewart,Zhao,1811.00026]

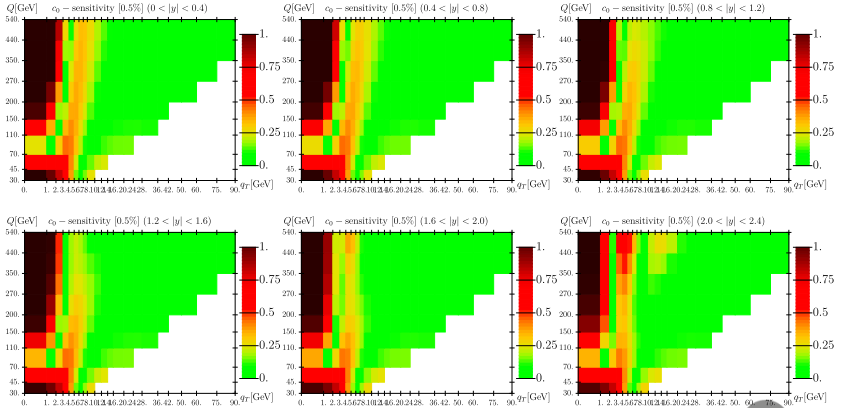
## Regensburg lattice group



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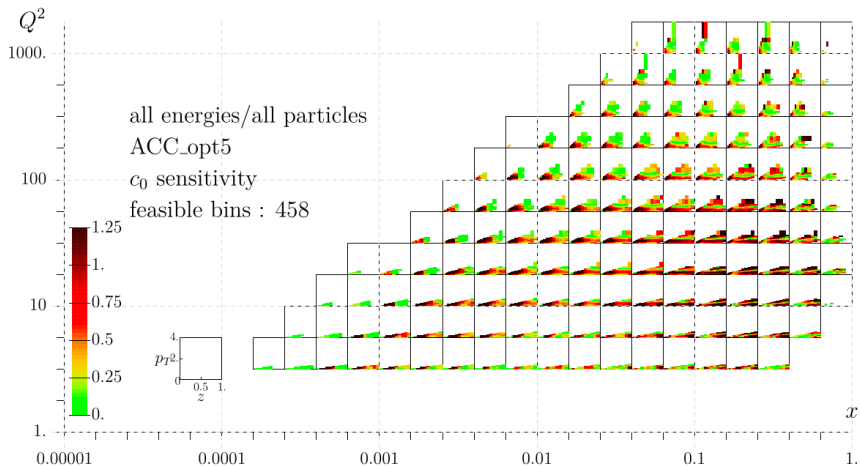
# Where to measure CS-kernel?

## LHC

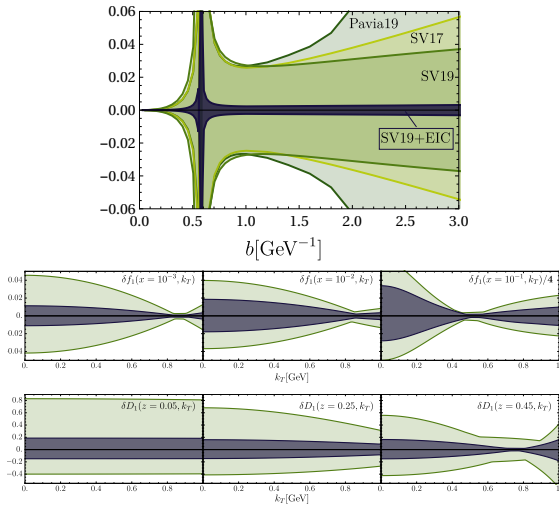


# Where to measure CS-kernel?

## EIC



# Expected impact of EIC



Unpolarized TMD distributions are known  
But this is only the first step

### Problems to solve in the nearest future

- ▶ Uncertainties
- ▶ Flavor dependence
- ▶ Issue(s) with normalization of certain data

### What to expect

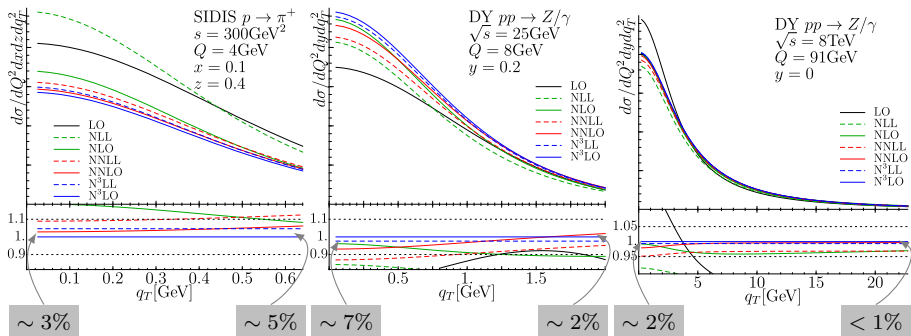
- ▶ Inclusion of new observables (COMPASS your turn!)
- ▶ New tools (TMDlib)
- ▶ Lattice
- ▶ Non-perturbative modeling
- ▶ ...

# Backup slides





In  $\zeta$ -prescription:  $\mu \sim Q$   
 Matching scales  $\mu_{\text{OPE}}$  are intrinsic for each function



Difference between NNLO and N<sup>3</sup>LO is not that important



# TMD factorization for SIDIS

Proof of factorization is done in the Breit frame



$$q_T^2 = \frac{p_T^2}{z^2} \frac{1 + \gamma^2}{1 - \varsigma^2}$$

$$x_1 = -x \frac{z}{\gamma^2} \left( 1 - \sqrt{1 + \gamma^2 \left( 1 - \frac{q_T^2}{Q^2} \right)} \right), \quad z_1 = z \frac{x_1}{x} \frac{1 + \sqrt{1 - \varsigma^2}}{2 \left( 1 - \frac{q_T^2}{Q^2} \right)}$$

$$\gamma = \frac{2Mx}{Q}$$

$$\varsigma = \gamma \frac{m}{zQ}$$

In practice:  $q_T < 0.25Q$

- ▶ Most part of data is not TMD factorisable.
- ▶ Low  $z$ 's are not accessible
- ▶ H1, ZEUS data have no TMD points, too low  $z$ .

## Test of importance of power correction

**These are not all power corrections,  
but only those that we know how to account**

include $(m/Q)$	yes	<b>no</b>	yes	yes	<b>no</b>	<b>no</b>
include $(M/Q)$	yes	yes	<b>no</b>	yes	<b>no</b>	<b>no</b>
include $(q_T/Q)$ in kinematics	yes	yes	yes	<b>no</b>	<b>no</b>	<b>no</b>
include $(q_T/Q)$ in $x_S, z_S$	yes	yes	yes	yes	yes	<b>no</b>
$\chi^2/N_{pt}$	1.00	1.00	1.09	1.06	1.16	1.31

Most important corrections are  $\frac{M}{Q}$  and  $\frac{q_T}{Q}$  from the rotation Breit  $\rightarrow$  Lab

